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AN ANALYSIS INTO THE ERRORS MADE WHEN SOLVING SIMULTANEOUS LINEAR EQUATIONS AT ORDINARY LEVEL AT ONE SCHOOL IN ZVIMBA DISTRICT, MASHONALAND WEST PROVINCE.

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ABSTRACT

Errors made by pupils when solving simultaneous linear equations were analysed using a qualitative survey design. A sample of forty-four Ordinary Level pupils was randomly selected from the population of Ordinary Level pupils, comprising of Form Three and Form Fours, at a certain school in Zvimba District, Mashonaland West Province. A test was administered to the sample. Errors omitted were noted. Possible causes of these errors were investigated. Teacher questionnaires which comprised of nine closed and one open question were given from Fennema-Sherman mathematics scale. Interviews were also held to compliment the results. Results revealed that many pupils were making substitution, arithmetic omission of brackets and elimination errors and effective teaching and learning methodologies need to be implemented to easy these misconceptions. According to this study elimination and substitution errors hinder pupils’ good performance in solving the equations. The study recommends that teachers should allocate more time on introducing the topic of simultaneous linear equations in order for pupils to articulate the perquisite concepts for solving linear simultaneous equations. Further, questions on removing of brackets need to be examined and reemphasised. More so teachers should clearly emphasise the significance of brackets when substituting expressions /numbers. Further studies in this area should concentrate on the classroom observations on the teaching and learning methods for simultaneous linear equations.
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CHAPTER ONE

1.0 INTRODUCTION

This chapter is an effort to introduce the reader to the preliminary issues that will be covered in the study. The chapter covers background to the study, statement of the problem, research objectives, and significance of the study. Assumptions, delimitations, limitations and definition of terms are also highlighted in the chapter. On background of the study, the researcher discussed on what really stimulates the desire to carry out the research. The research questions will also be stated in this chapter and shall be answered at the end of the research. Finally, on limitations and delimitations, the researcher will highlight some of the cases which might reduce the correct outcome of the research and give some ways to limit the cases.

1.1 BACKGROUND OF THE STUDY

For the past fifteen years the researcher, has been involved in teaching mathematics at Ordinary level, and has witnessed pupils losing marks unnecessarily when solving simple linear simultaneous equations. Pupils made multiple errors emanating from procedures used when solving simple linear simultaneous equations. Furthermore, evidence from other teachers over the years has consistently confirmed that teaching of simple linear simultaneous equations heavily relies on the laid down procedures in our commonly used textbooks; New General Mathematics and Focus on Mathematics. Even though pupils often solve simple linear simultaneous equations, their algebraic errors/mistakes remain hanging making it difficult to solve the equations.

According to Salau (2007), mathematics is one of the compulsory subjects in education. This point out that mathematics is crucial to the advancement of the scientific world. Furthermore, Mandebvu (2010) stated that the world over and Zimbabwe in particular expects its future generations to have passed mathematics as to improve technologically. Woods (2005) articulated that mathematics competence is a critical determinant of educational and career options available to the scientific world. The increased demand of the subject has necessitated the research to find ways to minimize the errors made by the pupils as to improve the pass rates in mathematics.

Kasanda (1997) researched on factors influencing pupils’ performance in mathematics. Furthermore, the Zimbabwe Schools Examination Council (2013-2017) syllabus incorporates the topic in its curricular and expects pupils to solve simultaneous linear equations using elimination, substitution and graphical method, but analysis of past examinations reveal that elimination and substitution methods are commonly asked. More so, most of the examination questions for simultaneous equations appear in Mathematics Paper One for both June and November Examinations. The researcher found it worth studying as most of Chief Examiners Reports indicated that pupils poorly performed in simultaneous equations. This motivated the researcher to find out more about ways in which pupils can solve the simultaneous equations easily by
minimising errors. The other drive that motivated the researcher is the desire for the improvement of mathematical teaching and learning skills in order to minimise errors made by pupils when solving simultaneous equations.

1.2 STATEMENT OF PROBLEM
For long, the poor performance of pupils in solving simultaneous equations in ZIMSEC Mathematics Paper One Chief Examiners Report (2012) have been indicated but limited research has been done in Zimbabwe to reduce the problem. The researcher also, observed that this problem is common in many schools in Zimbabwe. Most pupils lose valuable marks in their final examination at Ordinary Level. In this study, the researcher will analyse the errors made by pupil and find possible ways to enhance the teaching and learning of the topic, simultaneous linear equations.

1.3 AIM OF THE STUDY
The study seeks to analyse the common errors and misconceptions pupils have when solving simple linear simultaneous equations and how these can be minimised.

1.4 RESEARCH OBJECTIVES
The researcher should be able to:
- determine errors made by pupils when solving simultaneous equations,
- find out how pupils make the errors,
- develop effective methods of teaching simple linear simultaneous equations as to minimise the errors.

1.5 RESEARCH QUESTIONS
On the conclusion of this research, the following questions would have been answered
- What errors are made by pupils when solving simultaneous equations?
- What difficulties do pupils face in solving simultaneous linear equations?
- What are the causes of these errors?
- How can these errors be minimised?

1.6 ASSUMPTIONS OF THE STUDY
The assumptions of the study are that the sample selected will fairly represent the population. The respondents will fully cooperate with the researcher. The Ordinary Level pupils have done simple linear simultaneous equations using graphical method, change of the subject of the formula, linear equations and algebra. These aspects of solving linear simultaneous equations are included in the Zimbabwe Junior Certificate syllabus which the researcher had assumed.
1.7 SIGNIFICANCE OF THE STUDY
The study is of great value to Zimbabwe as a nation and in particular to Ministry of Primary and Secondary Education to the growth of mathematics. The results of the study will inform teachers, textbook writers and curriculum planners and other interested stakeholders to extend their understanding of how errors in solving linear equations can be recognised and caringly engaged. The research will also be beneficial to Zimbabwe Schools Examination Council (ZIMSEC) in achieving some of the stated objectives of the mathematics syllabus, particularly to simultaneous linear equations.

The present study will extend existing knowledge about the solving of linear simultaneous equations and its recommendations will enable teachers and pupils to become knowledgeable with the teaching and learning of the topic. More so it will enable pupils to reduce their errors/mistakes in solving simple linear simultaneous equations.

The findings of the study would benefit the researcher to uncover the critical aspects that many researchers were not able to explore in analysis of errors pupils make when solving linear simultaneous equations. As the researcher studies the errors made by pupils, he will become knowledgeable and enhance his teaching methods in solving the linear simultaneous equations. The completion of the study would be key aspect in the attainment of the Bachelor’s Degree in Education by the researcher because the research project is a pre-requisite of the programme.

1.8 DELIMITATIONS OF THE STUDY
The research is going to be confined to a certain school in Zvimba District, Mashonaland West. The study will be concentrating on errors made by pupils using algebraic methods of solving simultaneous equations. The research is confined to Ordinary Level pupils of form threes and form fours classes only.

1.9 LIMITATIONS OF THE STUDY
The sample size of one school which comprises of one hundred and eighty pupils may not be a true representative of the situation obtaining on the ground. The results of the research may not be generalised to the whole population since the conditions may differ. For instance, we have satellite schools which may face problems due to learning conditions, may differ from established schools. Family and work commitments might affect the smooth carrying out of the study. The researcher has restricted his study to simultaneous equations without looking at word problems leading to formation of simultaneous linear equations. The researcher will not conduct the interview with the pupils and also the teaching of simultaneous equations will rely heavily on the data provided by the questionnaire and interviews, not on the data actually observed in the classroom teaching.
1.10 DEFINITIONS OF TERMS

**Error:** An error is regarded as a mistake in the process of solving linear simultaneous equations using of either elimination or substitution method.

**Simple linear simultaneous equations:** are two equations written in the form

\[
ax + by = c \\
dx + ey = f,
\]

where \(x\) and \(y\) are unknowns to be found its numerical values and \(a, b, c, d, e\) and \(f\) are real numbers with at least one of \(a\) or \(b\) and \(d\) or \(e\) being non zero. The simple linear simultaneous equations are considered solved when a solution is found that satisfy both simultaneous equations.

**Substitution method:** is a method used to solve linear simultaneous equations by making one variable in one equation the subject of the formula and substituting the resulting expression into the other equation

**Elimination method:** In this study the term means the method used to solve linear simultaneous equations by getting rid of one variable by multiplying the equations with a constant to make the coefficient of the variable to be removed the same and then add or subtract the equations.

1.11 SUMMARY

This chapter deliberated on the preliminary issues and background information on the subject of research and gave a general overview of some of the issues that the study addresses. The next Chapter reviews related literature on the subject of study. The theoretical impetus that informs the study is discussed in the same chapter.
CHAPTER TWO
REVIEW OF RELATED LITERATURE

2.0 INTRODUCTION
This chapter attempts to look at the theoretical framework of the study. It also reviews related literature to determine errors in solving simultaneous linear equations. The related literature is important as it guides the research topic on how some errors and misconceptions were explained in the context of this study and other studies concerning analysis of errors in solving simple linear simultaneous equations. Literature review on the causes of errors in solving simultaneous equations will also be looked at followed by the ways in which these errors can be reduced.

2.1 THEORETICAL FRAMEWORK OF THE STUDY
The theoretical framework of this research is compelled by the Constructivists (Piaget, 1968 and Skemp, 1976) and Pedagogical Content Knowledge (Shulman, 1986). The researcher presumes that this will assists him to analyse pupils’ errors when solving simultaneous linear equations at Ordinary level in Zimbabwe context.

2.1.1 Constructivism in learning mathematics
Constructivists, Piaget (1972) and Skemp (1976), viewed learning as knowledge not constructed solely from experience but rather, a mixture of experience and new present knowledge structures. Piaget (1972) articulated that mental structures or schemata are constructed through interaction by processes called assimilation and accommodation. Assimilation is a process whereby new ideas are fitted to what a child already knows. On the other hand, accommodation is a process whereby the existing schemata have to be restructured to fit new information. Once a concept is formed, it is stable and resistant to change. A pupil’s existing concept will therefore determine what he or she learns from experience or instruction (Piaget, 1972).

Skemp (1976) theorised that a concept is activated in the mind when an example of it is encountered. He pointed out that in order to develop good concepts; good examples of the concept are required. Skemp (1976) said that mathematics learning is cumulative, new concepts gained is linked to the previous concepts. Hence, if a pupil is unable to "assimilate" and "accommodate" this creates a gap in the learning of the concept, and in turn, leads to mathematical errors or misconceptions.

According to Skemp (1976:9),’ making errors in computation is not utterly bad. It is a significant part of the learning process if these errors are dealt with diagnostically’. He therefore implies that most of the pupils’ errors are not of an accidental character, but are attributable to individual problem solving strategies and
rules from previous experience in the mathematics classroom, incompatibility with teachers' instruction or techniques, or pupils observed patterns and inferences during instruction.

2.1.3 Pedagogical Content Knowledge
A new way of thinking about the content knowledge necessary for the teaching was popularised by Shulman (1986). He categorised the knowledge into two domains, teaching content knowledge and pedagogical content knowledge. According to Shulman (1986, 14), ‘teaching content knowledge is for the teaching and organisation of knowledge in the mind of the teacher while Pedagogical content knowledge is the knowledge of generic principles of classroom organisation and management and the likes’. He further divided content for teaching into three categories namely subject matter content knowledge, curricular knowledge and pedagogical content knowledge.

Shulman (1986) proceeded to explicitly explain each type of knowledge. He said content knowledge includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that pupils of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. If those preconceptions are misconceptions, which are often the case, teachers, need knowledge of the effective strategies most likely to be fruitful in reformulating the understanding of the learners. Therefore, teaching involves showing pupils how to solve problems, answering pupils’ questions, and checking pupils’ work, it demands an understanding of the content of the school curriculum (Shulman, 1986).

Shulman and his associates sought to “represent content understanding as a special kind of technical knowledge key to the profession of teaching Shulman (1986) goes further by saying that subject matter is more than knowledge of fact; it requires knowledge of both the substantive structures and synthetic structures of the subject domain. Shulman (1987,9) defined pedagogical content knowledge (PCK) as ‘that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding’. Shulman (2002) further said that the transformation of subject matter knowledge into pedagogical content knowledge should be a significant focus in teacher education. They implied that, teachers should select their subject matter for teaching to pupils carefully considering the ability of their class. The selection should foster easy analyses and understanding of mathematical ideas by pupils. The subject matter knowledge encourages teachers to understand common learner conceptions, misconceptions, difficulties, errors and interests in the field of mathematics.

In teaching and learning, Shulman (1987) emphasised the need for teachers to carefully analyse their pupil’s errors, make an effort to understand the errors and establish the causes of these errors. After finding the errors, teachers need to employ correct teaching methods as to deepen their learners understanding of mathematical concepts as well as to improve their pupils reasoning methods. Teachers require specific
knowledge about errors and the methods of response to these errors in order to achieve what they set out to do. Pedagogical content knowledge can therefore enable the teacher to recognise whether pupils’ conceptions are truly misconceptions, establish their source and assist to unravel the patterns of their misunderstanding (Shulman, 1986)

2.1.4 Causes of errors in solving mathematical problems.

According to Confrey (1990), Piagetian studies in the custom of conceptual change focuses on conceptions in mathematics and science. The research explored on pupil’s conceptions at the development of understanding of particular mathematical and scientific concepts over time. Piaget’s major theory was that pupil’s awareness is a process not a state. Thus pupil’s knowledge needs to be examined in view of its developmental relations. Hence Piaget looked at pupil’s conceptions not misconceptions. Confrey (1990) says that researchers in the custom of systematic errors have acknowledged that pupils hold mini theories about mathematical and scientific ideas. Pupils are much attached to their misconceptions due to the fact that they are actively constructed from their experiences. According to Radatz (1979) it is for this reason that they stumble to leave the misconception up. He went on to say that many causes of errors and misconceptions in mathematics can be identified by looking at mechanisms used in processing, obtaining, retaining and remembering the mathematical tasks. Radatz (1979) said that these errors are due to lack of mastery of prerequisite skills, incorrect associations, application of unrelated system or strategies and dealing out with iconic presentations.

Barrera et al (2004) looked into errors caused by lack of meaning and postulates that such errors can be classified into three stages which are errors originating from arithmetic, errors originating in use of formulas or procedural errors and structural errors which are due to properties. Fischbein and Barash (1994) further elaborate that algorithmic knowledge is the ability to use theoretically justified procedures, formal knowledge is the ability to use definitions, theorems, axioms and proofs whereas intuitive is self-evident cognition. Thus pupils experience some errors and misconceptions in these three areas of knowledge. However intuitive knowledge often hinders and manipulates the formal understanding and or the use of algorithm ways. Errors also occur when solving linear simultaneous equations schema is mistakenly applied despite the correct intuitive way of thinking.

According to Radatz (1980:12), pupils' errors are casually determined, and very often systematic. Systematic errors are usually a consequence of pupils’ misconceptions. These can include failure to make connections with what they already know. Pupils may connect pattern with a misconception and thereby learn an erroneous procedure. Ashlock (2002) elaborated this when he said that misconceptions and erroneous procedures are results of over generalisation and over specialisation of rules in an effort to make sense of new information. Unless pedagogical actions are taken or interventions done by teachers, some of these errors will persist for a very long time. This has motivated the inception of this research so as to reduce
the errors through impacting effective teaching and learning methods of retaining the concepts of solving linear simultaneous equations.

Ashlock (2002) postulated that analysing pupils’ errors may reveal the erroneous problem-solving process and thus provide information on the understanding of the attitudes towards mathematical problems. Pupil’s errors are usually persistent unless the teacher finds proper teaching methods on how concepts can be retained by learners. By examining each of pupils’ written work, teachers would be able to look for patterns and hence find possible causes for errors and misconceptions. Subsequently, teachers will develop strategies which can be used to encourage pupils to reflect on their understanding. Concepts and schemata are stable once they are formed and are held to be resistant to change. Skemp (1976) suggested that good examples of concepts are required in order for concepts to be understood properly by pupils.

Ashlock (2002) articulated that pupils are not always successful in acquiring or developing correct conceptual structures which resulted in misconceptions. Misconceptions and errors must not be seen as obstacles, but must be regarded as an opportunity to reflect and learn. Skemp (1987) encouraged teachers to recognise pupils’ misconceptions; prescribe appropriate instructional strategies to be more diagnostically oriented in order to avoid any subsequent major conceptual problems. Thus, teachers are should use varied examples as to enable pupils to overcome their misconceptions early.

The errors highlighted above by different scholars, have motivated the researcher to carry out the study. Hence effectively execute proper methodology to minimise the occurrences of these discussed misconceptions and mistakes when solving simultaneous linear equations.

2.2 RESEARCH AND MISCONCEPTION WITH SIMULTANEOUS EQUATIONS

Researches in error analysis in mathematics have examined and analysed the errors pupils have been making for long (Zacharia E, 2010; Gunawarden E, 2011; Hood S, 2013 and Salman,) but studies in simultaneous linear equations in Zimbabwe have not been extremely done and very limited documentation of such available studies is available. Zacharia (2010) found that not only do pupils lack conceptual understanding of simple linear simultaneous equations but so do pre-service teachers. A teachers’ mathematical knowledge has a strong impact on pupils’ understanding. So it is important to try and mend this learning gap and find where pupils are making mistakes when it comes to linear simultaneous equations and adapt how they teach the topic (Zacharia, 2010).

Swan (2006) observed that teachers teach topics separately; tend to follow textbooks sequences when teaching maths concepts and teachers jump topics as the need arises. Zacharia (2010) and Salman (1997) provide different ways teachers can introduce and teach mathematical concepts. In order to strengthen pupils’ conceptual understanding of linear simultaneous equations it is important to examine where the pupils’ weaknesses lie in which the researcher will look into.
2.2.1 Treatment of Simultaneous Linear Equations in Textbooks

Welder (2008) found that misconceptions were a product of teaching methods and teaching materials. In support of that Shulman said that teaching material are written and rarely consider the removal of errors experienced by the research. The researcher was motivated by Welders and Shulman findings, it is therefore important to initially review what commonly used textbooks (New General Mathematics by Channon et al, 2011 and Focus on Mathematics by Lewis, 2015) believe is the best way of solving linear simultaneous equations.

A common approach among Form Three and Form Four textbooks is a geometric approach (substitution and elimination methods) of solving simultaneous linear equations. This is necessitated by the demands of examination questions set on linear simultaneous equations particularly in ZIMSEC Mathematics Paper one. The curricular demands the use of graphical, substitution and elimination methods. Of late, public examination papers mostly require candidates to use algebraic methods; substitution and elimination. Subsequently, most of the mathematics textbook writers in Zimbabwe's' solve linear simultaneous equation using the substitution method; which is the solving of one equation for a single variable and using that equation to substitute into the second equation thereby introducing the method of elimination; which is the manipulation of equations in a way that their addition causes the removal of one of the variables (Channon et al, 2011 and Lewis, 2015).

Lewis (2015) introduced the chapter of linear simultaneous equations by using the substitution method. One of the unknown, $x$, of equation was made the subject of the formula and the expression was substituted into equation into the second equation. The resultant obtained $y=2$ as the solution. The value of $y$ was substituted into the first equation which yielded the numerical value of $x$. 

\begin{align*}
\text{Example 2} \\
\text{Use the substitution method to solve the equations} \\
x + 2y &= 4 \\
2x - 2y &= 11 \\
x + 2y &= 4 \\
2x - 2y &= 11 \\
\text{From (i) } x &= 4 - 2y \\
\text{Substitute (iii) in (ii)} \\
2(4 - 2x) - 2y &= 11 \\
8 - 4y - 2y &= 11 \\
-6y &= 3 \\
y &= \frac{3}{6} \\
y &= \frac{1}{2} \\
\text{Substitute in (iii)} \\
x &= 4 - 2 \times \left(\frac{1}{2}\right) \\
x &= 4 + 1 \\
x &= 5 \\
\text{Check in (ii)} \\
2x - 2y &= 2 \times 5 - 2 \times \left(\frac{1}{2}\right) \\
&= 10 + 1 = 11 \\
\text{The solution is } x &= 5, \ y = -\frac{1}{2} 
\end{align*}
The author emphasised to pupils the need to use substitution if one of the unknown coefficient is a unit. No further explanation is given on how to solve the equations. The second method used was the elimination method and five steps in using the method were outlined. More so, after eliminating the first variable, the value of the second unknown was obtained by substituting into either equation (i) or equation (ii). There is a combination of substitution and elimination methods to solve the linear simultaneous equations advocated by the writer.

Another writer of Mathematics textbooks Channon et al (2011) in his Ordinary Level Book 3 introduced the methods of solving linear simultaneous equations as a continuation of work already covered at Zimbabwe Junior Certificate Level. The equations were solved by the combination of substitution and elimination methods. He initially eliminated the unknown x in order to solve for y value. Then the value of y was substituted in equation (i) to obtain the numerical value of x.
An assumption has been made that pupils solved the equation at Junior Level, but the syllabus requires the Junior Level to cover graphical method of solving simultaneous linear equations (ZJC Syllabus 1991). The research has however found a gap to research on the missing elements in the widely used textbooks in Zimbabwe for decades now as an initial misconception pupils have. Determination of errors is of great value to curriculum developers and teachers in designing textbooks and other materials for teaching and learning. Teachers do not have good understanding why pupils make mistakes. Clarkson (2008) argues that learners making mistakes is natural part of learning.

Gunawardena (2011) suggested that teachers need knowledge of effective strategies fruitful in reinforcing pupils understanding. Teachers need to analyse their pupils’ errors and causes of their errors and cultivate corrective measures and methods of teaching as to deepen their understanding of mathematical concept.

Research reports have ascertained that pupils usually experience difficulties when solving simultaneous equations. The Zimbabwe Schools Examination Council Chief examiners' report (2012) strongly highlighted the poor performance of pupils when solving simultaneous linear equations. It further explained that pupils’ errors emanate from carelessness errors, which usually prevent them from obtaining full marks. In carrying out the study the researcher intends to establish how these errors can be mitigated.

**2.3 HOW TO REDUCE MATHEMATICAL ERRORS AND MISCONCEPTIONS**

Gunawardena (2011) says that errors in solving problems cannot be easily dislodged since the misconception and errors are deeply seated in pupils thinking process. This is largely due to the fact that
they become attached to the notions they constructed during the learning process. He further highlighted that pupils might appear to have overcome problem only to resurface at a later stage. Gunawardena suggests that to effectively eliminate the errors and misconceptions, the pupils must actively participate in the concepts. The National Council of Teaching Mathematics (NCTM) (2007) agrees with this notion as it points out that the process does not entirely depend on the teacher. This implies that pupils have to be aware of the errors they are bound to make. In each case through practice and awareness, they tend to attach the correct procedures in their problem solving situations.

Teachers also play an important role in the total elimination of errors and misconceptions made by the pupils. NCTM (2007) suggest that teachers should practice classroom learning environments that help the pupils to develop both conceptual and procedural knowledge so that they construct correct conceptions right from the beginning. Gunawarden (2011) has high hopes that a powerful strategy of exposing pupils’ error and misconception is through a self-explanation procedure. This has an effect in that when pupils are asked to tell or write what they perform better and what they perform badly, pupils understanding is supported by this process since it is by pupils own admission, pupils centred and collaborative. Teachers need to allow pupils to explain their thinking processes as they give better insights to teachers of pupils thinking. This gives pupils some allowances and opportunities to arrange their own misunderstandings. However, it is superlative to a small group than large groups. This is helpful process of learning as it eliminates errors since when teachers listen to their pupils; some schemas for understanding the diversity of pupils thinking are developed. Pupils also refine and revise their mathematical thinking. Such an action shifts the role of the teacher as an assessor of pupil’s ideas to pupils as assessors of their own emerging facts.

Woodward (2004) says that another way of dealing with errors is to uncover them and find the schema where deep seated misconceptions lie. This initiates methods to transform the misconceptions and restructure the incorrect representation. It is crucial that the teachers should assess the error of the pupil and engage with the pupil in a way that corrects the concept. This can be done through taking the pupil’s answers to classroom for discussions. In this manner, pupils tend to get a better understanding not only of their errors but also of their peers as well. This is in agreement with the National Council of Teaching of Mathematics (2000) which says employing diagnostic methods of teaching is central to constructivist methods which calibrate the teacher’s instructions based on pupils needs.

2.4 ADEQUACY OF LITERATURE

The researcher, in a bid to get the general view of the context of the problem, read a number of researches carried out by other researchers. A number of them looked at the problem or its sub-problems in detail. Most of the researches were mainly looking at errors in solving the simultaneous equations at junior level and advanced level; using the graphical method at Junior level. The simultaneous equations looked are quadratic and linear equations. Mamba (2012) did the study on learners’ errors at Advanced Level when
solving algebraic tasks and found that carelessness of pupils’ workings was the major cause of poor performance. Salman (2012) found that substitution method was the most common method used to solve linear simultaneous equations. Other researchers on errors of simultaneous equations were Salman (2010) and Mamba (2012) concur the existence of the problems since most authors study on the context finding the different aspects of algebraic tasks.

On the other hand, though some authorities did not actually carry out the research, they have contributed some of the possible causative agencies of the problem under study. To mention a few these, include Hood.S.M (2013) in her topic “A study on solution techniques used by eighth Grade mathematics pupils when solving system of equations.” Gunawrden (2011) in his research on Secondary School Students’ Misconception in Algebra’ This further includes ZIMSEC reports (2012) on pupils’ examination poor performance when solving simultaneous equations.

The purpose of the review was achieved since the researcher got a direction to follow and to extend the parameters. Most of the researchers were putting more emphasis on secondary sources though in some cases they referred some information to researches carried out. The literature used was basing their evidence on their findings although there was a tendency of generalising. In some were exactly in line with the research at hand only that they differ in wording whilst some were focusing on sub-problems related to the big problem at hand.

In all the studies mentioned above, logical conclusions were made from both internal and external validity of results. Only a few were consistent which cropped a bias in how they were inconsistent with researches due to methodologies which distorted their findings and their conclusions to some extent.

There were many important results from this study. The results to focus on are the ones that shed light on the objectives of this study: to determine which approach to solving systems of linear equations pupils use most when given multiple choice answers and if pupils tend to solve problems differently depending on their major. It was found that substitution was the approach most used to solve linear simultaneous equations. As a result of this, the researcher is left with a gap to study on these errors which cause pupils to perform poorly when solving simultaneous equation. The researcher will carry out the research with greater hope that with his awareness of these difficulties, teachers can plan more effective teaching and learning experiences that recognise and anticipate the potential misconceptions that may arise in their pupils thinking.

2.5 SUMMARY
In this chapter theoretical framework and learners’ errors and misconception was discussed and clarified what other researchers have studied and concluded. Literature review has shown the types of errors committed by the pupils when solving simultaneous linear equations and the frequency of occurrences discussed by previous researchers. The researcher feels more research should be done to determine how
pupils make the errors, how do they add and subtract algebraic terms and develop effective ways to minimise the errors of simultaneous linear equations. Having outlined the theoretical framework and literature review, the researcher proceeded to articulate research methodology and design.
CHAPTER THREE
RESEARCH METHODOLOGY

3.1 INTRODUCTION
This chapter highlights strategies adopted in this study as a means of developing valid and reliable data. The researcher will describe the research design. The target population will be given and reasons for choosing the population shall be provided. The researcher will further establish the nature of the sample to be worked with and explain the sampling procedure. The research instruments used in the study are tests, questionnaires and interviews. The procedures of collecting data shall be clearly stated.

3.2 RESEARCH DESIGN
Kombo and Tromp (2006) assert that a research design can be thought as the structure of the research. Similarly, Polit and Hungler (1999) state that a research design is a blue print for conducting a study with maximum control over factors that may interfere with the validity of the findings. In other words, the research design clarifies what data is required, what methods are going to be used to collect and analyse this data, and how all of this is going to answer the research questions. The research design for this study is a descriptive survey. Cohen and Manion (2005) assert that the descriptive survey method looks at the phenomenon with intense accuracy and then describes exactly what the researcher sees. Descriptive surveys also present a chance to combine quantitative and qualitative data. According to Cohen and Manion (1995:83), the quantitative survey design does not involve manipulation of subjects but simply measures subjects as they are in order to generate generalisations and to add to existing knowledge. Qualitative research design will also be used during interviews. The validity of the information collected will depend on the honesty of the respondents. However, Kumar (2008) theorized that the descriptive method lacks predictive power because the researcher may discover and describe the present but is unable to predict the future. In support of Kumar, Bell (1996) affirms that biases may occur, either in the nature and accuracy of the responses that are received which may be misrepresented or lack of response from intended participants.

3.3 POPULATION
“Population is the total number of units from which data can be collected such as individuals, artefacts, events or organisations” (Patton, 1997). According to Winter (2007) “Population consist of people assigned on the basis of common characteristics”. Research population is generally large collection of individual that is the main focus of a scientific query. It is for the benefit of the population that the researches are done. It is from the population where a sample is drawn. The population, in this research, covered Ordinary level pupils at one of the schools in Zvimba District, Mashonaland West Province and all teachers at the school. The population will be made of one hundred and eighty pupils (112 male and 68 female) and all teachers.
3.4 SAMPLE
According to Punch (2010) a sample is a smaller group which is studied, drawn from the population in a deliberate or targeted way according to the logic of research. Similarly, Polit et al (2001) asserts that sample is a subset of the population. With reference to the above definitions the researcher’s sample refers to the people who are going to be studied from the total population. In this study a sample of forty-four pupils and six mathematics teachers.

3.5 SAMPLING PROCEDURE
Webster (2015) defines sampling as the act, process or technique of selecting a representative part of a population for the purpose of determining parameters or characteristics of the whole population. The sample must be representative of the population from which it was drawn and it must have good size to warrant statistical analysis. The main function of the sample is to allow researcher to conduct the study to individual from the population so that the results of their study can be used to derive conclusions that will apply to the entire population. The sampling procedure to be used in the study is probability sampling. Probability sampling system is one in which every individual has a known chance of being selected or the chance can be calculated.

There are many methods of probability sampling, the researcher will only use one of its which is stratified sampling. Stratified sampling involves dividing the population into homogeneous non-overlapping groups (i.e., strata), selecting a sample from each group, and conducting a simple random sample in each stratum (Webster, 2015). Pupils will be grouped in some homogeneous groups in terms of ability which are fast, average and slow learners. In this way each group will be proportionally represented through random sampling.

3.6 RESEARCH INSTRUMENT
In this research tests for pupils, questionnaires and interviews for teachers will be employed.

3.6.1 Tests
In this study, the researcher will use pre-test and post-test as the instruments for data collection. The pre-test will be administered to forty-four pupils of mixed abilities to determine the errors made by pupils in solving linear simultaneous equations. The pre-test will enable the researcher to identify and find the causes of the errors pupils make when solving linear simultaneous equations. In the pre-test pupils had to show knowledge of basic algebra in solving equation problems. The test shall be constructed using items from frequently used textbooks (New General Mathematics and Focus on Mathematics) and past examination papers. Aspects to be tested will include: removing of brackets, solving of linear equations and linear simultaneous equations
The pre-test will review the types of errors pupils incur and their misconceptions when solving simultaneous equations. This will allow the researcher to identify the exact errors of pupils and deduce some misconceptions from their workings. The test will also guide the interviews to be done to teachers.

Thereafter, the researcher will administer a post-test as to determine the effectiveness of the suggested methodology as to reduce the errors made by the pupils when solving simultaneous equations. The pre-test and post-test will basically have same items constructed from past examination papers and textbooks that are regularly used by the pupils in the school.

3.6.2 Questionnaires

Dyer, (1995) puts it that a questionnaire consists of a list of pre-set questions. Questionnaire is the principle method that will be used to collect data. The questions will be designed in such a way that they can solicit information concerning the type of errors and their causes in solving simultaneous. Mathematics teachers at the school will be given the questionnaires in order to contribute to the study. The questionnaire will consist of structured and unstructured questions. Unstructured items consisted of questions which the respondent had freedom of responses. The questionnaires will be administered on the target group. Six questionnaires for sampled mathematics teachers will be used.

The questionnaire allows the researcher to collect data from a much larger sample, thereby rendering conclusions more valid and reliable because data gathered become more representative for the population (Leedy 1985). The instrument is also efficient in that it requires less time if properly designed and administered. Dyer, (1995) observed that the questionnaire also enables the researcher to allow respondents time to mediate on specific issues on the questionnaire. In this way the researcher will be guaranteed of the research ‘best judgments.

However, some respondents may also fail to honestly respond to questions in an attempt to please the researcher. Open-ended questions can generate large amounts of data that can take a long time to process and analyse. In this study; the researcher limited the space available to respondents so that their responses were concise. Respondents may answer superficially especially if the questionnaire takes a long time to complete. The researcher made an attempt to reduce the number of questions on the questionnaire. Teachers may not be willing to answer the questions. They might not wish to reveal the information or they might think that they will not benefit from responding perhaps even be penalised by giving their real opinion. Kumar (2011) postulates that, the layout of a questionnaire should be easy to read and please the eye and the sequence of the questions should be easy to follow. A questionnaire should be developed in an interactive style. This means that respondents should feel as if someone is talking to them.

In this study, the respondents will be told why the information is being collected and how the results are going to be beneficial to their teaching and learning of pupils. The respondents will be asked to reply
honestly and will be told that if their response is negative this is just as useful as a more positive opinion. Furthermore, the questionnaire will be made anonymous.

3.7 INTERVIEWS.
The interview method of collecting data involves presentation of oral-verbal stimuli and reply in terms of oral-verbal responses. This method can be used through personal interviews and, if possible, through telephone interviews (Christensen: 2008). The instrument shall be used to get information mainly from mathematics teachers. An interview guide will be used. A small pilot study will be used to revise questions in the guide which may be unclear, unable to solicit desired information or produce negative reactions.

When well conducted, it can produce in depth data. This method is also flexible because the interviewer can adapt the situation to each subject by establishing rapport; a trust relationship so that the interviewer can obtain information which respondents would not give in a questionnaire. The interview may also result in more accurate and honest responses since the interviewer can explain and clarify both the purpose of the research and individual questions (Christensen: 2008). The interviewer can also follow up on incomplete or under-responses by asking additional probing questions. Reasons for particular responses can also be obtained.

Among the important weaknesses, mention may be made of the following: It is a very expensive method, especially when large and widely spread geographical sample is taken. Oppenheim (1994) maintains that this method is relatively more-time-consuming, especially when the sample is large and recalls upon the respondents are necessary. Robson (1994), the presence of the interviewer on the spot may over-stimulate the respondent, sometimes even to the extent that he/she may give imaginary information just to make the interview interesting. Patton (1990) articulated that interviewing at times may also introduce systematic errors. To mitigate above problems in the study, the researcher will limit the number of teachers to be interviewed as to reduce cost and time. More so researcher will tell the respondents the benefit of the study to the teaching and learning of simultaneous equation to encourage respondents to give accurate information.

3.8 DATA COLLECTION PROCEDURES
Weiersma (1995) says data collection is a term used to describe a process of preparing and collecting data. The purpose of data collection is to obtain information to keep on record, to make decisions about important issues to pass information to others. This study will involve forty-four pupils from Ordinary level classes. The pupils have to sit for a thirty-minute pre-test and post-test with six items. All pupils will have studied the topic linear simultaneous equations, algebraic expressions and equations.

The researcher has a confirmation letter that he is a student at the Bindura University of Science Education. Permission shall be sought from the school head and head of mathematics department at the school to
administer the tests, questionnaires and interviews as to collect data for analysis purposes. The researcher will indicate that the research information would only be used for partial fulfilment of the required degree programme. The respondents will be assured of their anonymity and confidentiality of the information they provide.

Form Three pupils will be grouped according to their mathematical ability, fast, average and slow learners. Fifteen pupils will be selected from each group using the stratified random sampling. The main objective of grouping is to have a fair representative of the errors emanating from all groups to be looked at. The pre-test and post-test will be administered to the sampled pupils.

For interviews, three teachers will be selected from the pool of ordinary level mathematics teachers. The researcher shall explain to the interviewee the reasons for the interview and why. The researcher will follow the questions on the interview schedule.

3.9 DATA PRESENTATION AND ANALYSIS PROCEDURES

3.9.1 Data Analysis
According to Dooley (2003) data analysis is coding schemes in summarizing collected data. Thus, data analysis is a practice in which raw data is ordered and organized so that useful information can be extracted from it. Data shall be analysed both qualitatively and quantitatively. There are a variety of ways in which people can approach data analysis, and it is easy to manipulate data during the analysis phase to push certain conclusions or agendas. For this reason, the researcher shall pay attention when data analysis is presented, and to think critically about the data and the conclusions to be drawn.

The researcher in the study will use tallies, graphs and charts to summarise the gathered data. Dooley (2003) asserts that qualitative data analysis is a systematic process which involves selecting, categorizing, comparing, synthesising and interpreting to provide explanations of the data collected. Quantitative data analysis is the process of using statistical methods to describe, summarise and compare data. However, in this study the researcher will dwell much on qualitative data analysis.

Data collected from the open ended questions and content analysis will be qualitatively summarised using descriptive statistic. Response received from the questionnaires shall be organized, tabulated and analysed using frequencies, percentages and presented in form of frequency, percentages, tables and figures. On the other hand, data from close ended questions and interviews shall be quantitatively analysed.

3.10 Summary
The preceding chapter discussed the research methodology chosen by the researcher. Descriptive survey is going to be used in the research to explore aspects of errors made when solving linear simultaneous equations. Stratified sampling will be carried out to come up with a sample of forty-four pupils of mixed abilities. Data collection instruments discussed includes tests, questionnaires, and interviews. The researcher
also highlighted the rationale for using both quantitative and qualitative methods in the study. The next chapter looks at data analysis, presentation and discussion.
CHAPTER FOUR
DATA PRESENTATION AND ANALYSIS

4.1 Introduction
This chapter presents the results from all the respondents who took part in the study. The findings of the study are presented in form of tables, bar charts, pie charts and figures to help the researcher summarize the findings of the study. This chapter therefore, examines in detail the findings on the analysis of errors when solving linear simultaneous. The chapter begins by presenting the results from the test administered to determine the existence of the errors and proceeds to present the findings from questionnaires for teachers and analyze the interview results.

4.2 Analysis of Test Questions

Question 1(a)
The question on removing of brackets indicated that pupils are having difficulties in multiplying every term in the bracket by the outside term. Pupils who multiplied every term failed to simplify completely leaving their answers in the form -+ and 4-6+-3x. Most pupils misinterpreted -3(2) as the addition of numbers to get -5. The researcher grouped the errors as distributive, simplification, and arithmetic errors. The table below shows the frequency of how the errors occurred.

<table>
<thead>
<tr>
<th>Error</th>
<th>Distributive</th>
<th>Arithmetic</th>
<th>Simplification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>18</td>
<td>12</td>
<td>9</td>
</tr>
</tbody>
</table>

From the above errors, it can be seen that distributive errors constitute 46.15% of the errors committed by pupils. This indicates that pupils lack the lowest order basics in solving simultaneous linear equations. Thus this distributive error also directly affects 30.77% of arithmetic errors and also affecting their simplification of linear equations.

Figure 4: Pupils written script
From the working above, the pupil didn’t know how to remove the brackets and also had problems in simplifying the directed numbers. The pupil was not sure whether the multiplication of a negative number and a positive number yields a positive or negative. As a result, the answer was not simplified. The ZIMSEC Chief Examiners Report (2012) indicated that this leads to the loss of marks.

**Question 1(b)**

Expressing $x$ as the subject of the formula proved difficulty to 65.9% of the pupils who attempted the question. The majority of pupils failed to divide everything by 2.

![Figure 5: Bar Chart Representation of answers](image)

The above data gave a range of 14 which is too large a number. The mean score of respondents 50 % showing that performers and non- performers are equal in numbers.

![Figure 6: Pupils written script](image)
The pupil was expressing x in terms of y and made a multiple of mistakes. The y term was transposed incorrectly and failed to divide every term by 2 on the left hand side. This error was also indicated in the research of Hood (2013) in his study on algebraic errors of grade twelve mathematics students in South Africa. Even though pupils were doing higher level mathematics, subject of the formula is difficult to some pupils.

**Question 1(c)**

---

**Figure 7: Pupils written script**
Figure 8: Pie Chart Diagram showing errors

56.9% of the pupils failed to remove the brackets. This is a major hindrance when solving linear equations with brackets which the researcher agrees with Hood (2013) who stated that pupil’s failure to distribute correctly is caused by their lack of knowledge on algebraic manipulations. Another error noted was the failure to transpose the numbers when solving equations. This constitutes 29.5% of the total respondents.

Question 2(a)

Research finding shows that 46.15% of the pupils were failing to correctly substitute by omitting the brackets when it comes to expressions, thereby failing to obtain one of the correct answers of simultaneous equation. Common wrong substitution was $3x-5-x=7$ instead of $3x-(5-x)=7$. This omission of bracket leads to obtain an incorrect answer of $x=6$, then substituted in the second equation to obtain again incorrect solution $y=-1$. Since mathematics is a cumulative subject, pupils’ failure to remove brackets has a bearing on their final answer. This agrees with Skemp (1976) who articulated that new concept gained is linked to the previous concepts. The error was identified as omission of brackets.
The table above indicated the number of pupils who made the errors of omission of brackets, distributive and algebraic. 46.15% of respondents committed the error of omission of brackets. An example of omission of brackets discovered is shown below.

From the above pupils’ written work, the respondents substituted the expression for $y$ into the other equation. The pupils undermined the importance of brackets in the expression. Furthermore, they treated the subtraction of $5-x$ as multiplication to obtain $5x$. This caused a gross error to the solution of the equation. A combination of omission of brackets and algebraic manipulation errors were noted in the pupils’ written work.
47.73% of the respondents indicated that most pupils prefer substitution method to solve simultaneous linear equations. 20, 45% preferred elimination method. The former who used the substitution method encountered the errors. The errors included pupil’s failure to remove brackets (distributive), omission of brackets and the algebraic error in one way or the other. On the other hand, Lewis (2015) and Channon et al. (2010) textbooks advocated for substitution and a combination of the two methods most as indicated by the pupils’ choice of methods. Shulman (1987) postulated that the textbook the pupils use as their resource material affects their learning of concepts. The example below shows how one respondent solved the simultaneous equation using substitution method:

![Figure 11: Pupils written Script using substitution method](image)

On the other hand, some pupils preferred elimination methods to solve the simultaneous linear equations. From the findings of the test administered it was shown that, respondents who preferred elimination method to solve simultaneous equations, 44.44% failed to eliminate the unknown. The following written script is an example of a pupil who failed to eliminate the unknown:

<table>
<thead>
<tr>
<th>Method</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Substitution</td>
<td>21</td>
</tr>
<tr>
<td>Elimination</td>
<td>9</td>
</tr>
<tr>
<td>Elimination and Substitution</td>
<td>14</td>
</tr>
</tbody>
</table>
From the above example, shows that pupil failed to multiply every term in the equation $x - y = 5$ by a constant 3. The confusion goes further by incorrectly subtracting the two equations, thereby obtains two leading wrong equation of $-y = 7$ and $-y = 5$, furthermore subtract the equations to get $-y = 2$. Radatz (1980) said pupils' errors are usually a consequence of pupils’ misconceptions. These can include failure to make connections with what they already know. Pupils may connect pattern with a misconception and thereby learn an erroneous procedure.

**Table 3: Frequency of Pupils making the subject**

<table>
<thead>
<tr>
<th>Subject</th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>7</td>
<td>15</td>
</tr>
</tbody>
</table>

31.81% pupils made $x$ the subject of the formula to obtain $x = 5 - y$. % who made $y$ the subject of the formula had problems in balancing the equation to obtain $y = 3x - 7$. The errors would proceed to the omission of brackets mentioned in question 2(a) leading to incorrect answers.

**Question 2 (c)**

**Table 4: Frequency of Errors with Preferred Method**

<table>
<thead>
<tr>
<th>Method</th>
<th>Subject error</th>
<th>Elimination error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>12</td>
<td>32</td>
</tr>
</tbody>
</table>
27.27% pupils who used substitution method either made subject errors or omission of brackets errors. 72.72% pupils who used elimination method were facing difficulties in getting rid of one of the unknown. This was due to addition, subtraction and multiplication errors. These errors were typically made by pupils in their test scripts. 43.18% pupils were erroneously adding $2y + (-3y)$ to obtain $5y^2$.

![Figure 13: Pupils written script](image)

The pupils’ script indicated that pupil failed to make $x$ the subject. The pupil further substituted the expression correctly but incorrectly distributed the term 3 inside the bracket. This leads to obtain incorrect answer which was used to find the corresponding value of $y$. The value for $y$ will automatically yield an incorrect answer. ZIMSEC Chief Examiners Report (2012) reported that this error causes a candidate to obtain a zero score. This method is heavily emphasised by the two authors Lewis (2015) and Channon et al (2010) reason why most pupils used it.

It is therefore the thrust of this study to find ways on how to reduce the number of errors encountered by pupils so as to improve their performance in solving the simultaneous equations.

### 4.3 INTERVIEW WITH TEACHERS

The results of the interview with teachers’ showed the same findings as those highlighted on the pupils’ written work. These results were on types of errors and challenges faced by pupils. The interview reflected that the following are the causes of errors in solving simultaneous equations.

#### Table 5: Shows Errors indicated by Teachers

<table>
<thead>
<tr>
<th>Error</th>
<th>Processing error</th>
<th>Elementary error</th>
<th>Careless error</th>
<th>Simplification error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>5</td>
<td>6</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

83.33% of teachers said that pupils also encounter errors in the process of their workings. 66.67% of teachers highlighted that lapse in concentration caused careless errors as they present their solutions. 83.33%
of teachers said that pupils may have the correct working but fail to simplify their solution at the last stages. One of the teachers said

‘pupils can get the first solution of the equations but fail to substitute the numerical value in the second equation. This is caused by some pupils thinking other social issues. Maths need 100% concentration!’

100% of teachers indicated that pupils lack elementary mathematical knowledge. The interview results agree with Radatz (1980) who said that these errors are due to lack of mastery of prerequisite skills, incorrect associations, application of unrelated system or strategies and dealing out with iconic presentations.

Most teachers further pointed out that some pupils suffer from, ‘number phobia’. The situation becomes horrible to such pupils during mathematics lessons. More so 50% of teachers agreed with Radatz (1980) when he said that pupils tend to learn mathematics by rote learning hence if a different situation arise pupils become stuck. This is because pupils regard solving simultaneous equations as a tedious task and look at each problem given as totally different from what they learnt.

**Errors in solving simultaneous equations**

From the interview and test administered at a certain school in Zvimba District, Mashonaland West, the researcher has made the following findings:

(i) Omission of brackets-46.15% of pupils failed to use and omit brackets when substituting expressions and negative numbers which culminate into the causing of errors in solving simultaneous equations.

(ii) Problems of distribution of terms over brackets-38.46% of pupils fail to distribute term outside the bracket to every term inside the brackets.

(iii) Arithmetic error-15.38% pupils erroneously add and subtract algebraic terms when solving simultaneous equations making it difficult eliminate the variables and to solve the resultant equation after elimination.

(iv) pupils failed to multiply each term of the equation by the constant chosen, making the error before eliminating the unknown.

One of the teachers interviewed indicated that

Solving of simultaneous linear equations using both methods causes errors, in the sense that the first value of the first unknown has a great bearing on the accuracy of the second value. Failure of the pupil to obtain the first solution will automatically cause the other value to be incorrect.

The results of the interview showed that the errors of pupils can only be reduced if teachers are teaching for understanding of the concepts rather than for the examination purposes. As one teacher indicated ‘as long as
we need results no matter how we will rush the pupils to finish the syllabus, all teaching methods are useless here. We need results only!

Most teachers on pupils’ errors to address their learning needs indicated that this will help them to master the solving techniques of equations as teachers and pupils will iron out the grey areas together. Everyone shall be versed with the areas of difficulties others face. This enable teacher to revisit their teaching methods and be more learners centred rather than teacher centred.

4.4 QUESTIONNAIRE RESULTS FOR TEACHERS

Teachers Qualification

66.67% of mathematics teachers are degree holders majoring in Mathematics and 16.67% have diplomas in Education from teachers’ colleges. Among teachers with degrees in mathematics, 25% holds a post graduate qualification in Education. Shulman (1986) argued that pedagogical content knowledge should be twined with the subject content knowledge as to teach simultaneous equations effectively. This contradicts with the findings of the study as most of the teachers do not have the pedagogical skills required for effective teaching. Figure 1 below, shows the teachers’ qualification

<table>
<thead>
<tr>
<th>Qualification</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diploma in Education</td>
<td>1</td>
</tr>
<tr>
<td>Degree in mathematics</td>
<td>4</td>
</tr>
<tr>
<td>Master’s in Education(Mathematics)</td>
<td>0</td>
</tr>
<tr>
<td>Post Graduate in Education</td>
<td>1</td>
</tr>
</tbody>
</table>

Gender of mathematics teachers

The study showed that 100% of mathematics respondents are male. Therefore, there is need for gender parity.
The interview with teachers also indicated that the 100% of teachers put a lot emphasises on the addition and subtraction of directed numbers. 50% were in need of linear equations being taught first. 33.33% of teachers indicated removing of brackets. The results of teachers agreed with the findings on the errors encountered in pupils’ written work when the test was administered. In contrast, the authors of the textbooks Channon et al (2010) and Lewis (2015) don’t have the revision aspects of the stated topics.

The study indicated that 83.33% of the respondents prefer the New General Mathematics by Channon et al (2010) to be used as a basic textbook for mathematics when teaching simultaneous equations. This agrees
with Shulman (1987) who said that teachers follow the laid down procedures and transfer the errors to pupils.

Types of errors mostly made by pupils

**Table 7: Shows type of errors**

<table>
<thead>
<tr>
<th>Types of Errors</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elimination</td>
<td>6</td>
</tr>
<tr>
<td>Substitution</td>
<td>5</td>
</tr>
<tr>
<td>Transposing</td>
<td>3</td>
</tr>
<tr>
<td>Removing of brackets</td>
<td>4</td>
</tr>
<tr>
<td>Omission of Brackets</td>
<td>4</td>
</tr>
<tr>
<td>Incomplete multiplication</td>
<td>3</td>
</tr>
</tbody>
</table>

The table above indicated that 100% of teachers indicated that pupils have difficulties in eliminating the variables. 83.33% of teachers indicated that pupils have difficulties in substituting the expression obtained from the subject, 50% of teachers’ omission of brackets and not multiplying every term in the equation by a constant when eliminating. These stated errors by the teachers’ tallies very well with what has been indicated by the pupils’ written work.

![Figure 16: Bar Chart Representation of errors](image)
Table 8: Marks Obtained by Pupils

<table>
<thead>
<tr>
<th>Marks</th>
<th>100%</th>
<th>Less than 100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>3</td>
<td>41</td>
</tr>
</tbody>
</table>

Since the sample drawn consists of mixed abilities, 6.82% of the pupils obtained full marks from the given problems, indicating that they are exceptionally good. The 93.18% of pupils made at least one of the above mentioned errors.

100% teachers disagreed that the simultaneous equation is a difficult topic. This contradicts with the information obtained from pupils’ written work indicating that teachers are failing to impact proper pedagogy in order to sail in the same footing with the pupils. This goes hand in glove with Shulman (1986) who advocated for the twining of content knowledge and pedagogical knowledge.

Possible Ways of dealing with errors

Table 9: Possible Ways of Dealing with Errors

<table>
<thead>
<tr>
<th>Possible Way</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Checking of solution</td>
<td>6</td>
</tr>
<tr>
<td>Consistency with one method</td>
<td>4</td>
</tr>
<tr>
<td>Sequencing of concepts</td>
<td>5</td>
</tr>
<tr>
<td>Reinforcement of preliminary</td>
<td>3</td>
</tr>
<tr>
<td>Unhurried Teaching approach</td>
<td>2</td>
</tr>
</tbody>
</table>

Teachers suggested possible ways on how to deal with the errors pupils face when solving simultaneous equations and their responses are shown in the table above. 100% of teachers suggested that pupils must be taught on how to check their solution. Lewis (2015), Channon et al (2010), Ashlock and Gunawardan agreed with the method suggested. Gunawarden (2011) further emphasised that pupils should be actively involved in the process of solving simultaneous equations. 66.67% of teachers advocated for the consistency with the method of solving simultaneous equations. That means, if the elimination method is chosen by pupil must find both numerical values of the unknown using the same method. 50% of teachers indicated the need for revision or reinforcement of the preliminary concepts related to solving simultaneous equations. 33.33% teachers said that pupils must be taken through the teaching and learning process gradually and carefully and not be rushed through the syllabus. Some teachers, contradicted in the interview saying that we teach for examination, therefore the need to hurry otherwise you will be a’ useless teacher’
83.33% teachers ask for the proper sequencing of topics leading to the solving of the simultaneous equations. National Council of Teaching Mathematics (2007) suggested that the teacher is at the centre of error elimination; hence pupils build conceptions through the way they are taught. All these findings agree that to eliminate errors there should be a shift to learner centred approaches.

4.5 DATA INTERPRETATION
In teaching and learning, Shulman (1987) said that teachers need to carefully analyse pupils’ errors and make great effort to understand the errors. Teachers need to employ correct teaching method as to avoid these errors. Therefore, errors determined by the study teachers should use it to adopt new teaching methods for the teaching and learning of simultaneous equations. Pupils must understand the concepts of simultaneous linear equations without difficulties.

The study has attempted to determine the errors and comment on misconceptions pupils encounter in solving simultaneous equations. This will act as a document for change in the teaching and learning the topic with minimum errors. It is hoped that teachers will employ effective teaching methods outlined for the teaching of simultaneous equations to avoid potential misconceptions.

If there is a person who would like to continue with this study, they should use the teaching methods and pupils’ attitudes as the causes of errors in solving the simultaneous. Interviews for pupils should be carried out and actual classroom teacher observation whilst teaching the topic.

4.6 SUMMARY
The purpose of this chapter was to highlight the findings of the study. The chapter highlighted the results obtained from test, questionnaire and interview to ascertain the errors pupils make when solving the simultaneous equations. The data was presented using tables, bar charts and pie charts and also a discussion on the research findings were presented. However, the preceding chapter will look at the summary, conclusion and recommendations of the study.
CHAPTER FIVE
SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction
This chapter will summarize the findings of the study. It will discuss the implications of the findings for teaching and learning of simultaneous linear equations at Ordinary Level. The suggestions for further research in this area will also be given.

5.2 Summary
In this study, the researcher wanted to establish and analyse the errors made when solving simultaneous linear equations using algebraic methods at Ordinary Level and determine ways on how to minimise these errors to enhance pupils’ understandings when solving the equations.

The research was based on Form Four pupils at one of secondary schools in Zvimba District, Mashonaland West Province. The class was chosen on the backdrop that the pupils had already covered the topic and they were quite versed with all the methods of solving simultaneous methods. Another reason was that they were preparing for their final ZIMSEC mathematics examinations and had the zeal to learn more about the topic.

The data were collected in various ways including individual teacher interviews, classroom written tests and teacher questionnaires. The data collected from the questionnaires and tests were tabulated and presented graphically as bar charts and pie charts. It was then processed and summarised. The interpretations and the analysis of the data validated the researcher’s assumption that Ordinary Level pupils have done simple linear simultaneous equations using graphical method, change of the subject of the formula, linear equations and algebra.

5.3 Conclusion
The findings of this research appear to highlight the common errors made by pupils when solving simultaneous linear equations. This study, therefore will suggest teaching and learning methods that will help pupils to reduce errors in solving simultaneous equations in particular and mathematics in general.

Teachers and textbook writers, greatly influence the way pupils carry out mathematical computations, as a result they should advocate for simple to complex approaches as propounded by Piaget (1976). The teaching of simultaneous linear equations need to be done in connection with other topics related to it such as, subject of the formula, algebraic computations, use of brackets, removing of brackets and linear equations. The aforementioned topics must be revised in one or two lessons for pupils to grasp them before teaching simultaneous equations. This agrees with Radatz (1979) who said that errors are due to lack of mastery of prerequisite skills and incorrect association of topics. Therefore, as teachers there is need to
allow the pupils master the required skills before teaching the new content and it becomes easier for pupils
to ‘accommodate’ the concepts.

The study results show that pupils faced problems when removing brackets. Therefore, it is imperative for
teachers and textbook writers to introduce the topic and chapter by explaining it at the beginning. Few
examples must be given as to reinforce the aspects on brackets and algebraic computation. Furthermore,
teachers need to stress on the addition and subtraction of directed numbers and simplification of algebraic
terms. Therefore, teachers need to find good approaches to ensure pupils prerequisite skills in solving
simultaneous equations.

More so mathematics teachers need to work out several problems using either elimination or substitution
method only as the combination of the methods can lead unnecessary loss of marks to average pupils. If the
pupils are good in algebraic computation, then elimination method must be used or vice versa. Teachers
should encourage pupils to check their solutions. Most of the textbooks used, Channon et al (2011) and
Lewis (2015) encouraged pupils to check their solutions. Teachers should make an effort to do likewise.

Furthermore, as postulated by Shulman (1986) teachers should combine the pedagogical and content
knowledge to better pupils understanding. The teachers should not make assumptions on prerequisite
knowledge of pupils as this affect their understanding of a new concept. Therefore, test of understanding of
previous concepts is needed in order to investigate pupils’ difficulties in learning new concept. In this study
all the assumed knowledge must be taught in one or two lessons for pupils to understand the solving of
equations.

The results of this study highlighted the errors or misconceptions made by the pupils when solving linear
simultaneous equations. Teachers, textbook writers and pupils should put great effort as to minimise the
errors as to improve their performance when solving simultaneous equations. The researcher will suggest
possible methods on how to reduce the occurrence of these errors. Shulman (1986) suggested that pupils
should avoid misunderstanding by learning difficulty concepts from simple to complex for easy
conceptualisation.

5.4 RECOMMENDATIONS
Educators and textbook writers can take it as a cornerstone and put great effort to assist their learners and
readers to reduce errors in learning simultaneous equations. Teachers need to use teaching methods that
enable pupils to easily solve simultaneous linear equations. The ideas suggested to reduce errors in solving
equations will help pupils forever in understanding simultaneous linear equations in Zimbabwe.

It is a good strategy to spend time at the beginning of the lesson when introducing the topic of simultaneous
linear equations in order to have pupils articulate the prerequisite concepts for solving linear simultaneous
equations before looking at the methods. Questions on removing of brackets need to be examined and
reemphasised. The authors of commonly used textbooks leave the job to educators to arrange the topics in order of simplistic.

More so teachers should clearly emphasise the significance of brackets when substituting expressions /numbers. Brackets allow pupils to distribute number outside the bracket with all terms inside it. If this is understood by pupils, teacher need to proceed to look at the solving of simple linear equation with brackets which will lead to solving of simultaneous equations using algebraic methods.

The Chief Examiners Report (2012) indicated the poor performance of pupils to solve simultaneous equations. Pupils tend to loose unnecessary marks by failing to obtain one the solutions correct. The research had noted that most widely used textbooks advocate for the combination of methods to solve simultaneous equations. From the questions given to pupils, errors committed by pupils using these methods are almost half. However, pupils should be introduced to either elimination method or substitution method to solve linear simultaneous equations depending on the strength of the pupils on directed numbers and subject of the formula.

For the substitution method pupils must be consistent with their method of substitution. A combination will definitely lead to emergence of unnecessary errors which causes loss of marks. Teachers and pupils should adopt exclusive substitution method. Pupils are encouraged to be consistent with the method chosen to avoid unnecessary errors.

5.5 SUMMARY

In this chapter, a summary of the findings of the study has been presented. The implications of the study in the present context have been discussed. Finally, suggestions for further research in the area have been provided. The major finding of the research was that teachers were not instilling positive attitudes to pupils towards learning mathematics because of their poor teaching methods. Therefore, in-service training for teachers is mainly recommended.
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APPENDIX 1
QUESTIONNAIRE TO TEACHERS

My name is Gore Rowayi, student of Bindura University of Science Education. I am carrying a study on the errors which Ordinary Level pupils make when solving linear simultaneous equations. May you kindly complete the given questions? Do not write your names on the questionnaires. All your answers are recognised. Thank you.

1. May you indicate by ticking the appropriate box?
   Female
   Male

2 Indicate your teaching experience
   0-5 years
   6-10 years
   11-15 years
   Above 15 years

3 Highest academic level:
   ‘O’ Level
   ‘A’ Level
   Others (specify)

4 Highest level of professional qualification:
   Diploma in Education
   Degree in Education
   Master’s in Education
   Others (specify)

5 What subject matter do you emphasise most when teaching mathematics?
   A. Directed numbers
   B. linear equations
C. Removing of brackets
Other (specify)……………………………………………………………………

6. Which method of solving linear simultaneous equations do your pupils mostly prefer?

A  Substitution
B  Elimination
Other (specify)……………………………………………………………………

8 Which textbook do you encourage your pupils to use when solving linear simultaneous equations.

A  New General Mathematics book3
B  Focus on Mathematics B

9 What type of errors do pupils mostly make when solving simultaneous linear equations?

Substitution  
Elimination  
Other (specify) ………………………………………………………………………

10 Any suggestions on how pupils’ errors in solving simultaneous equations can be minimised.

………………………………………………………………………………………………………………
………………………………………………………………………………………………………………
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APPENDIX 2

INTERVIEW QUESTIONS TO TEACHERS

1. What time of the year do you normally teach linear simultaneous equations?

2. Which area on solving linear simultaneous equations do your pupils faces most challenges?

4. What type of errors do the pupils really make?

5. What approaches do you use in teaching the linear simultaneous equations?

6. Do you feel you have all the necessary resources to effectively teach the topic?

7. What do you think are the cause of such errors?

8. How can pupil’s errors be used in addressing their learning needs?
APPENDIX 3

Student Number: __________

TEST
TIME 30 MINUTES

Instructions
➢ Answer all questions
➢ Show all working neatly on the space provided

1) (a) Remove brackets and simplify
   \[ 4 - 3(2 + x) \]

(b) Make x the subject of the formula
   \[ 2x + y = 15 \]

(c) Solve the equation
   \[ 7 - 3(x + 4) = 4 \]
2. Solve the following linear simultaneous equations

(a) \[3x-y=7\]
   \[y=5-x\]

(b) \[x-2y=7\]
   \[3x-y=1\]

(c) \[3x+2y=10\]
   \[2x-3y=11\]