SECONDARY SCHOOL STUDENTS ERRORS AND MISCONCEPTIONS IN
ALGEBRA WITH SPECIFIC REFERENCE TO A SCHOOL IN MASHONALAND
EAST, ZIMBABWE. 2013-2016

BY

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APPROVAL FORM

The undersigned certify to have read and recommended to the Bindura University of Education for acceptance, a research project entitled “Secondary School Students Errors and Misconceptions in algebra submitted by Dodzo Wongayi in partial fulfilment of the requirements of MSCED (Maths) Degree.

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DECLARATION
I declare that: Secondary School students’ errors and misconceptions with particular
reference to a school in Mashonaland East is my own work which has not been submitted
before for any degree or examination in any other university and that all the sources used or
quoted have been indicated and acknowledged as complete reference.

Student signature ........................................
ACKNOWLEDGEMENTS

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DEDICATIONS

This piece of work is dedicated to my husband and my children Alicia, Alex and Alexio.
ABSTRACT

This study was carried at a school in Mashonaland East Province in Zimbabwe. This study aimed to investigate secondary school students’ categories of errors and misconceptions in algebra with a view to find the origin of those errors and to make recommendations on classroom instruction. A purposive sampling was used to include the best class of form threes and fours. A descriptive survey was used to summarise sample data. Mixed methods research was used to collect data. Tests and interviews were used to obtain the data. Answers from the tests exposed categories of errors and misconceptions in variables, algebraic expressions, equations and word problems. Interviews were used for clarity of the misconceptions. The major sources of misconceptions emanated from other mathematical errors from other topics. The most common errors were found in word problems followed by equations, variables and algebraic expressions respectively. Recommendations were made to teachers and stakeholders to make some researches on errors and misconceptions on other topics other than algebra. Teachers and other stake holders were recommended to investigate further on the possible solutions to these misconceptions.
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CHAPTER 1

Introduction

This chapter is focusing on the background of the errors misconceptions held by secondary school pupils in the learning of algebra. It explains why it is important to carry out this research and clarifies what exactly is looked at. The key questions which were addressed by this research are outlined. Significance and purpose of the study are other important aspects which were looked into by this chapter. Delimitation and limitations were outlined in this chapter. Definition of key terms and organisation of the study made the last sections of chapter 1.

1.1 Background of the study

Mathematics is generally linked to with the development of any nation in the world. Mathematics as a discipline opens and shuts more doors for men and women than any other content area we have got (Burton, 1999). Whether it is in science, engineering or technology, Mathematics is one of the core subjects to be offered to all students to the tertiary levels of education (Salau, 1995). In Zimbabwe both primary and secondary teacher training colleges have ordinary level of mathematics as requirement for enrolment to prospective teachers. Employers also have Mathematics as a prerequisite for employment contracts. The compulsory nature of Mathematics with the assumption that the knowledge of the subject is essential to all members of all society makes it essential in life.

Despite the relative importance of Mathematics, it is very disappointing to note that the student’s performance in the subject in both internal and external examinations has remained
consistently poor. This concurs with Zimbabwe Schools Examination Council and its stakeholders which stated that poor results in Mathematics deny many people the opportunity to have careers in science related courses.

Zimbabwe Junior Certificate (ZJC) syllabus (1990) specifies the need for pupils to learn to represent situation that involve algebra. These pupils begin learning of algebra when they enter early secondary school. Reports conducted by mathematics educators such as Herscovicks (1996), indicate that at Z.J.C level, pupils experience serious problems in understanding pre-algebraic concepts. Miles (1999) shared similar sentiments by suggesting that there is a possibility that some pupils who have learning problems in mathematics have difficulties in algebra not because of their difficulties in mathematical-based but because of language-based which delay their assimilation and comprehension of algebraic-mathematical instruction. However, the constructivist’s theory of learning indicates that effective learning can only take place when pupils are given a chance to grapple with the problems. Thus pupils should reflect on their solutions and procedures, and then teachers check the reasonableness of their results.

According to Michael (2001), some fundamental mathematical misconceptions often originate from earliest years of schooling, but may persist at higher level. Thus misconceptions can be inherited from primary mathematics to secondary or even up to university. Kendeou and Vanden (2005) pointed out that students can enter into a classroom having misconceptions that have the potential to derail new learning. This can justify why it is important to carry out a research on the misconceptions held by learners in the teaching and learning of mathematical concepts.

‘An error’ is a concept of deviation from what is correct, right or true. If the student does not understand the mathematical concepts meaningfully, it is natural they will be committing errors in performing the operations (Chamundeswara, 2014). He continues to say that errors
are made due to misunderstanding of the concepts. Researchers has found out that in algebraic addition and subtraction there is lack of grasp of essential principles, concepts and process concerned under the prescribed syllabus (Gunawardena 2011). According to the researchers experience in teaching, students experience great difficulty in their effective learning of the algebra in spite of their best efforts being put in. Therefore teachers should analyse the hard spots and by which teaching of the algebra can be made more effective. Hence a need is felt to analyse the conceptual errors encountered in mathematical operations in algebra among students at the secondary level.

Greens and Rubenstein (2008) observed that, pupils in lower levels of secondary schools are struggling with algebraic concepts and skills; this may result in discontinuing of higher level. Since there is a decline in number of female students pursuing mathematics and science courses as according to Eshiwani (1985), there is a chance that struggle in algebra can be one of the reasons. The researcher is therefore interested in errors and misconceptions made by female students in algebra.

The errors pupils make when attempting to solve algebraic problems have been studied for several years by many researchers. According to Michael (2001), some fundamental mathematical misconceptions often originate from earliest years of schooling, but may persist at higher level. Reports conducted by mathematics educators such as Herscovicks (1996) also indicate that ZJC level pupils experience serious problems in understanding pre-algebraic concepts. As a result pupils perform badly in topics which involve algebraic tasks up to high levels of secondary school. However if research could characterise students’ errors and misconceptions it would be possible to design effective instruction. This research there aims to identify errors which are held by pupils at ordinary level.
According to Gilbert, Osborne and Fensham (1982), there are certain areas in mathematics where pupils are making some misconception but little attention has been given to them. According to Gunawardena (2011), there are considerable number of studies on pupils’ errors and misconceptions in arithmetic but comparatively there are few studies addressing the pupils’ misconceptions in algebra. This implies that there is need for more researches in algebraic concepts to find out the possible causes of such misconceptions. Since mathematics is now a pre requisite in most tertiary institutions, as mentioned above, investigations critical mathematical topics become more necessary than before.

According to Fennemma (1996) mathematics has been viewed as a subject favouring male students. They posit that there were differences between female and male students and learning of mathematics tend to exist particularly in activities that required complex reasoning; that the differences increased about the onset of adolescence. Africa Academy of Sciences in collaboration with the Association for the Development of Education in Africa has the same sentiments on issues of women’s performance in mathematics (Gunawardena 2011). Many researches has found out that the differences in mathematics achievement that occur in the early years are in favour of females whereas differences in mathematics achievements as well as in attitudes favouring males generally occur during the secondary school years.

However the compulsory nature for mathematics makes it clear that female, like male; need it in their private life, working life, socio-economic and political life of the country of which they are citizens (Cockcroft, 1982). In this study the researcher is therefore interested in errors and misconceptions in algebra made by female students.

1.2 Statement of the problem
Algebra is a powerful solving tool; therefore it is central to pupils’ ability to do mathematics. Therefore teachers should impart and reinforce a good understanding or acquisition of algebraic concepts and thinking skills. It is imperative, therefore that an investigation was carried to identify and critically analyse the errors and misconceptions that pupils encounter in understanding and solving algebraic problems.

1.3 Purpose of the study
Knowledge of pupils’ errors and misconceptions will help teachers to properly organise instructional methods and to gain insight into the student thinking process which will reveal their knowledge schema (Gunawardena 2011). This research will contribute towards the generation of new knowledge on pupils’ errors and misconceptions in algebra. This involves knowing pupils errors and misconceptions in algebra and their possible sources. This knowledge will be useful to pupils, teachers and aspiring textbook writers.

1.4 Objectives

- to Identify categories of errors and misconceptions in variables
- to Identify categories of errors and misconceptions in algebraic expressions
- to Identify categories of errors and misconceptions in equations
- to Identify categories of errors and misconceptions in word problems
- to determine possible sources of misconception

1.5 Research questions

The study will address the following research questions:

1. What are students’ categories of errors and misconceptions in solving problems related to variables?
2. What are students’ categories of errors and misconceptions in solving problems related to algebraic expressions?
3. What are students’ categories of errors and misconceptions in solving equations?
4. What are students’ categories of errors and misconceptions in solving word problems?
5. What are the possible sources of misconceptions in algebra?
1.6 Significance of the study

A few studies address the issue of students’ misconceptions in algebra as compared to a number of researches on students’ errors and misconceptions in arithmetic. Many pay attention to some isolated concepts such as variables, equations, or expressions. Little has been made to study the interrelated nature of the misconceptions in more than one conceptual area. Testing each item in the recipe separately will not give a complete sense (Gunawardena 2011), More detailed exploration of the misconceptions is a crucial prerequisite for any further attempt to improve the quality of mathematics education. The results of this study will inform teachers, curriculum planners, textbook writers, and other stakeholders to broaden their understanding of how errors and misconceptions in algebra can be identified and thoughtfully engaged. Thus, if researchers can know and describe the ways of students’ understanding in algebra in a detailed way, it will be easier for teachers and researchers to design effective methods to improve students’ understanding. Word problems introduce a context where the above three components can link to a solution model. The objectives of my research are to determine errors and misconceptions so that these sources can be eliminated through properly organized instructional methods.

1.7 Assumption of the study

The researcher assumed that students have common errors and misconceptions in algebra which cause them to perform badly in mathematics.

1.8 Limitations of the study

The sample size, sample frame and the sampling method to be used make it difficult to generalise the results to the whole population of female students in Zimbabwe. Since the key methods of data collection is analysis of pupils’ solutions, the researcher might wrongfully interpret the pupils’ misconceptions and their possible sources. This is likely going to affect the authenticity of the findings. This effect could be minimised by interviewing pupils orally to explain how they obtained their answers. Another possible source of weaknesses is the oral interview; pupils might not feel free to give their views about their answers face to face with the teacher and others being reluctant to explain their answers. As a result, this would affect
the reliability and validity of the findings. In this case the researcher will try to be as friendly as possible to the learners so that they would feel free to participate.

1.9 Delimitations
The study would be conducted at a school in Mashonaland East Province of Zimbabwe. The school has four maths teachers out of a staff complement of 35 teachers and an enrolment of 600 girls with two Form three and two form four classes each with an average of 45 pupils. In this study, only form 3 and 4 pupils will be involved. Form three pupils were chosen in order to identify some of the misconceptions they inherited from form one and two and form four pupils were chosen so that misconceptions inherited from form one, two and three may be identified.

1.10 Definition of terms
Misconception: Refers to a student's problem or difficulty in understanding key Concepts.
Error: mistakes made by learners as a result of carelessness, misinterpretation of symbols, lack of awareness or inability to check the answer given.
Conceptual errors: mistakes made when the learner does not understand the properties or principles in a concept.
Procedural errors: these occur when learners skip directions or misunderstand directions, but answer the question or the problem anyway.

1.11 Organisation of the dissertation

In this dissertation there are five chapters. However, before the first chapter this dissertation shall have a section of the preliminaries. The contents of each chapter is summarised below.

Preliminaries. This section shall constitute the following; the title, that is the topic of the study, dedication, a concise summary of the whole project that is the abstract, acknowledgments, table of contents, list of tables and list of figures.

Chapter 1 This chapter includes the introduction and the contextual background to the misconceptions in algebra; this is the justification of carrying out the research on the concept algebra, statement of the problem, research questions, significance of the study, and
delimitations of the study that is the physical and contextual boundaries of the study. The sample under study and the theoretical boundaries of the study under subsection delimitations of the study are also discussed. Limitations of the study are other aspects which will be looked at in this chapter. Assumptions of the study, organisation of the dissertation and the definition of key terms of the study are also part of this chapter.

**Chapter 2** The next section is the review of the related literature. The components of this chapter include the introduction to this chapter. The chapter will touch on the sources of algebra error and misconceptions, and learning and errors and misconceptions. Categories of errors and misconceptions of different conceptual areas are also part of this section.

**Chapter 3** This shall constitute the methodology section of this study. Quantitative and qualitative methods which will be used in this study will be described. The discussion will focus on the validity and reliability of the methods to be used. Also to be in chapter, will be the discussion of the sample and data analysis method as well as the ethical considerations.

**Chapter 4** This shall constitute both qualitative analyses of the main findings. Record of the pupils’ misconception as well as the percentage of the errors and interviews will be analysed qualitatively.

**Chapter 5** This chapter will present summary, conclusions and recommendations of the dissertation. In this section, results from the tests and interviews are summarised and recommendations will be made.

### 1.12 Summary

Chapter 1 established the background of the study. The purpose and significance of the study were stated. Research questions were posed; assumptions of the study were discussed. Also key terms were defined to show that the research has direction and meaning. Presentation of the project was finally laid out.
CHAPTER 2

LITERATURE REVIEW

2.0 Introduction

The focus of this chapter is to provide a brief discussion on errors and misconceptions in algebra; this involves highlighting sources of errors and misconceptions in algebra. The chapter will also look at categories of errors and misconceptions in algebra.

2.1 Explanation of concepts: Errors and Misconceptions

Hansen (2006) explains errors as mistakes made by learners as a result of carelessness, misinterpretation of symbols and texts, lack of relevant experience or knowledge related to a Mathematical topic, learning objective or concept, lack of awareness, or inability to check the answer given. She found that misconceptions lead to errors. Drew (2005) defines misconceptions as the “misapplication of a rule, an overgeneralization or under-generalization or an alternative conception of the situation”. Mistakes displayed due to misconceptions learners have about a topic indicates incorrect interpretation of a Mathematical idea as a result of a student’s personal experience or incomplete observation. Luneta and Makonye (2010) in their study defined an error as a mistake, slip, blunder, or inaccuracy and deviation from accuracy.

Hodes and Nolting (1998) proposed four types of errors and explain them as follows;

a) Careless errors: mistakes made which can be caught automatically upon reviewing one’s own work.

b) Conceptual errors: mistakes made when the learner does not understand the properties or principles covered in a lecture or lesson.

c) Application errors: mistakes that learners make when they know the concept but cannot apply it to a specific situation or question.
d) Procedural errors: these occur when learners skip directions or misunderstand directions, but answer the question or the problem anyway.

### 2.2 Acquiring mathematical knowledge

Higgins et.al (2002) argue that mistakes made when acquiring mathematical knowledge, may indicate alternative ways of reasoning, and such mistakes should not be dismissed as “wrong thinking” but be seen as necessary stages of conceptual development. Piaget (1972) in Pimm (2009), explained development as comprising of four stages: that is, the sensory-motor, pre-operational, concrete operation, and formal operation stages. Piaget argued that it is through these operational stages that we can understand the development of knowledge. For instance, the formal operation stage shows that the learners can also reason on hypothesis and not only on objects. Piaget talked about three developmental processes of how children progress conceptually from one stage to another namely: assimilation referring the manner in which learners transform incoming information so that it fits within their way of thinking, accommodation as a stage where a learner receives new information which is quite different from existing knowledge which one then tries to re-construct and re-organize ideas, and lastly, equilibration referring to the keystone of developmental change between the learner’s cognitive system and the external world. Equilibration is the stage in which learners begin to realize the errors and misconceptions they have developed and further use these mistakes to restructure their existing knowledge.

### 2.3 Learning and Students' Errors and Misconceptions

Learning is a continuous process that involves active participation of students even though instruction clearly affects what students learn. According to constructivists point of view Piaget (1972) and Skemp (1976) in Sarwadi and Shahril (2014) viewed learning as; knowledge is not constructed solely from experience but rather, a blend of experience and present knowledge structures. Mental structures or schemata are constructed through interaction by processes called assimilation and accommodation Piaget (1976). The process by which ideas are fitted to what a child already knows (existing schema) is called assimilation. Accommodation, on the other hand, is a process whereby the existing schemata have to be restructured to fit new information. Once a schema or concept is formed, it is stable and resistant to change. A
student's existing schema or concept will therefore determine what he or she learns from experience or instruction (Piaget (1976) in Sarwadi and Shahril 2014). Skemp's theory (1976) in Sarwadi and Shahril (2014) suggests that a concept is activated in the mind when an example of it is encountered. He pointed out that in order to develop good concepts; good examples of the concept are required. Mathematics learning is cumulative, that is, new knowledge gained is linked to the previous knowledge. Hence, if a student is unable to "assimilate" and "accommodate" this creates a gap in the learning of the concept, and in turn, leads to mathematical errors or misconceptions (Sarwadi and Shahril 2014). Making errors in computation is a significant part of the learning process if these errors are dealt with diagnostically. According to Erlwanger (1973) in Sarwadi and Shahril (2014) most student errors are not of an accidental character, but are attributable to individual problem solving strategies and rules from previous experience in the mathematics classroom, incompatibility with teachers' instruction or techniques, or students observed patterns and inferences during instruction.

Students' errors are causally determined and very often systematic Radatz (1980) in Sarwadi and Sharil (2014). Systematic errors are normally caused by student misconceptions. These include failure of students to connect new information with what they already know. According to Ashlock (2002) students may connect pattern with a misconception and thereby learn an erroneous procedure. He posited that misconceptions and erroneous procedures are results of overgeneralisation and overspecialisation of rules in an effort to make sense of new information.

Errors will persist for a very long time unless pedagogical actions are taken or interventions done by teachers. According to Borasi (1990) there are beliefs held by students that inhibit learning from errors and one of the beliefs is they cannot learn from the mistakes. According to Vinner (1990) another belief is that mathematics consists of disconnected rules and procedures. Borasi (1990) posits that students who hold such beliefs perceived mathematics as not meant to make sense.

Student errors are unique and they reflect their understanding of a concept, problem or a procedure Sarawadi and Sharil (2014). They further said that analysing student errors may reveal the erroneous problem-solving process and thus provide information on the understanding of and the attitudes towards mathematical problems. According to Radatz (1980) in Sarawadi and Sharil (2014) upon analysing performance tests in solving text problems,
erroneous patterns demonstrated by students are due to other language difficulties, inadequate understanding of texts, or incorrect number determination. Teachers would be able to look for patterns and hence find possible causes for errors and misconceptions by examining each of students’ written work. Hence teachers will be able to develop strategies which can be used to encourage students to reflect on their understanding. Concepts and schemata are stable once they are formed and are held to be resistant to change Piaget (1976) in Sarawadi and Sharil (2014).

Students are not always successful in acquiring or developing correct conceptual structures which resulted in misconceptions (Shahril 2005). Misconceptions and errors must not be seen as obstacles or ‘dead ends’, but must be regarded as an opportunity to reflect and learn (Sarwadi and Shahril 2014). Therefore teachers should recognise these misconceptions, use appropriate instructional strategies in order to avoid any subsequent major conceptual problems.

2.4 Sources of Misconceptions and Errors

Luneta and Makonye (2010:36) found the teaching and learning of Mathematics to be so difficult and ineffective, that they suspect poor performance in mathematics to be correlated with the learner errors and misconceptions.

Battista (2001) states that the way in which learners construct knowledge is dependent on the cognitive structures learners have previously developed. Thus, there are conceptions and preconceptions that learners of different ages and backgrounds bring with them to Mathematics classrooms, and if preconceptions are misconceptions, teachers need knowledge of strategies most likely to be fruitful in reorganizing the learners’ understanding. Shulman (2005) argues that the ability to identify learners’ misconceptions is based on teacher pedagogical skills or teacher competence. In other words, the teacher’s main focus is not mainly on classroom management, preparing good lessons and presenting well-structured tasks, but also on the quality of questions about the content of lesson, and explanations given to learners.

Higgins, Ryan, Swam and Williams (2002) found that possible causes of mistakes learners make may be due to lapses in concentration, hasty reasoning, memory overloaded or failure to notice important features of a problem. Bell (1993) found the main cause for many students to be that they appear to understand a concept at the end of a unit, but do not retain it after a few
months. In other way, they lack long term learning. In contrast, students with long term memory do not forget the acquired knowledge, and are able to apply it in real life situations. He suggests that a diagnostic teaching strategy can help promote long-term learning and transfer from the immediate topic to wider situations. He further argues that students see scores and not weaknesses, because they often want to know if their answer is correct or what score they got on a test, but don’t want to go beyond scores to look into why they got the score they did. Yet paradoxically, this is one of the many ways to improve scores and acquire new knowledge. This what he regarded as learners being more dependent on instrumental understanding which is the following of mathematical rules and procedures without understanding, as compared to relational understanding which is knowing what to do in Mathematics and the reasons behind that.

2.5 Types of misconceptions

Procedural knowledge is the ability to carry out a series of actions in order to solve a problem (Radtz, 1979). Thus procedural misconception generally refers to a wrong idea on how to carry out an action to solve a problem. Conceptual understanding involves knowing the structure or rules of algebra or arithmetic such as the associativity, commutativity, transitivity as well as the closure property (Fischbein, 1994). Conceptual misconception is therefore when these applications are wrongly applied.

2.6 Errors and misconceptions in variables

Pupils have some difficulties in comprehending variables. Letters have different meanings in different context and when they represent entities then it becomes a problem to the learner (Kieran 1990). Davis (1975) in Gunawardena (2011) suggested an example, a + b, he said this this represents the procedures of adding a and b and the object a + b taken as one quantity. An example can be that if a=3 and b=2 then a + b can be taken as a procedure of adding 3 and 2 but if there are a black balls and b white balls then the total number of balls will be a + b and is taken as a quantity. According to Davis (1975) this is referred to as process product dilemma where there is no clear distinction between the two.

Philip (1999) grouped variables as 1) letters as labels for example f and y 3f=1y: 3feet and 1 yard 2) as constants pi, exponent and c 3) as unknowns 5x-9=11 4) as generalised numbers a +
b = b + a 5) as varying quantities for example y = 9x. Students will therefore find it difficult to identify the correct group for the variable.

However many students actually hold erroneous concepts about variables. The current study focuses on three of the common major misconceptions about variables that students experience. The first misconception Kuchemann (1978) found that some students consistently ignored variables. He gave an example “add 4 to n+5”. 68% answered correctly “n+9” 20% gave the incorrect answer 9, suggesting they simply ignored the variable n altogether. The second type of misconception students treat variables as a label this was shown by Stacey and Macgregor (1997) when they presented more than 2000 middle school students the following problem: “David is 10 cm taller than Con. Con is h cm tall. What can you write for David’s height?” The correct answer is 10 + h, where in 10 is added to the number or quantity denoted by h. Yet many students treated the variable as a label associated with the name of an object or example C + 10 = D. Based on other research findings, interviews with individual students, and coding of students’ informal or written explanations, Stacey and Macgregor (1997) interpreted this answer to reflect ‘C’ as meaning ‘Con’s height’ and D as meaning ‘David’s height’. Another similar erroneous concept is seen when students interpret the variable as an abbreviated word (e.g., response of D h where the abbreviation stands for the words David’s height). This misconception of construing a variable as a label for an object is reflected also in the classic error to the “Students and Professors” problem, which reads as follows: “Write an equation, using the variables S and P to represent the following statement. ‘At this university there are six times as many students as professors.’ Use S for the number of students and P for the number of professors.” An erroneous understanding that S is a label for an object (students), as opposed to a variable (number of students), led 37% of a sample of students entering college to incorrectly answer the question as 6S = P (Rosnick, 1981). When asked to explain this answer, students stated that they believed the answer was 6S = P because S was a label for students. (The correct answer is S = 6P where S stands for number of students.) This misconception reasoning on this “student and professor” problem was prevalent also among students already in college (Clement, Lochhead, & Monk, 1981). Another example of the misconception of a variable as a label for an object/entity is seen when students, who are given the question “In the expression t + 4, what does it represent?,” answer with “time” instead of “any number”. Finally, a third type of misconception is when students believe a variable is a specific unknown (Kuchemann, 1978; Stacey & Macgregor, 1997). In this case, students do not fully understand that a variable can represent multiple values; rather they believe it can only
represent one fixed value. For example, when asked how many values p represents, students assume p can only hold one value, as opposed to many values.

2.7 Errors and misconceptions in algebraic expressions

This is when one or more letters are used in an expression. Agnieszka (1997) observed that pupils associate letters with real objects. He gave an example of $8a$ associated with 8 apples and he said there is no problem when they are asked to simplify $2a + 3a$ meaning to say even if they think that $a$ is for apples, they will add to get $5a$ which according to them will be 5 apples. However, the author suggests that, they will likely to face difficulties in simplifying $3a + b + a$. In this case they may assign $b$ to be another object say bananas, and they do not know whether the total will be bananas or apples. As a result their answers will likely to be $5a$ or $5b$ or $5ab$. Booth (1988) noted that, pupils perceive open algebraic expressions as incomplete and often try to finish by over simplifying. For example $a + b$, seems not to be complete to the pupils and they may try to simplify it to $ab$. Thomas and Tall (1991) feels that, the explanation for this misconception is the tendency in many arithmetic problems to have a final digit answer. Examples can be $2 + 7 = 9$; $7 \times 8 = 56$; In this case the possible sources of such misconceptions are the textbooks. Most elementary mathematics text books have tasks written in this way. Martz (1980) suggested that many common errors in simplifying algebraic expressions seem to be a result of application of the correct but inappropriate rules.

2.8 Errors and misconceptions in equations

Orton (2011) refers an “arithmetical” equation having the form;

$$ax + b = c$$
And “algebraic” equation having the form;

\[ ax + b = cx. \]

Filloy and Rojan (2003) classify these linear equations whose analysis is in the light of their theories. Having recognized “algebraic” rather than “arithmetical” type of equation, Orton (2011) suggest that pupils face further problems because of the dual nature of the equal signs. Martz (2009) views syntactic similarity between semantically different statements. For instance;

\[ 4x + 12 = 4(x + 3) \]

and

\[ 3x + 3 = 2x + 7 \]

are viewed as a serious obstacle in algebra. Hall (2012), suggests that an equation;

\[ 4x + 12 = 4(x + 3) \]

Is a tautology because the left hand side and the right hand side are syntactically equivalent whereas the second equation;

\[ 3x + 3 = 2x + 7 \]

Is not tautology and requires a new procedure to solution of the equation. Martz (2009) described a major structural confusion which can arise in solving linear equations. Maverech and Vitschak (2013) concluded that, from the pupils they tested had a poor understanding of the meaning of the equal signs. Hence an explicit way may help to give many pupils a more structural understanding of linear equations.
Sleeman (2013) suggests that confusion is going from arithmetic to algebra. For example, Martz (2009) argues that:

\[
3 \frac{3}{4} \quad \text{and} \quad 3 + \frac{3}{4};
\]

Is not unreasonable that the student should interpret algebraic expression:

\[
3x \quad \text{as} \quad 3 + x.
\]

Therefore, there is plenty of room for confusion and misinterpretation in the initial stage of algebraic equations. Problems in solving algebraic equations are related to some errors pupils faced. Carry, Lewis and Bernard (2007) introduce that, ‘other inverse error” in which;

\[
7x = 1
\]

Becomes

\[
x = 1 - 7.
\]

Carry et al (2007) suggest that additive inverse have been employed instead of multiplicative inverse thus referred to as “other inverse error.” Hall (2012) assumes that inverse error might be related to deletion error, where

\[
3x - 3
\]

Becomes simply x because as the pupil may see 3 and -3 as inverses and cancels them out.

Sleeman (2013) points that pupils view \(3x\) as \(3 + x\). Hall (2012) suggest that deletion error and other inverse errors may possibly be reduced in frequency by heightened teacher emphasis which reinforces the idea of inverses. Sleeman (2013) proposes that other inverse error is not in the present literature but justifies its inclusion in the study.
Another identified error is called transposing error. Kieran (1992) points out that, emphasis on symmetry is absent in the procedure of transposing. Hall (2012) suggests an evidence of transposing errors in pupils. These problems exist when blindly applying the change side – change sign rule in equations involving denominators such as:

\[ \frac{x}{2} + 3 = 10 \]

It implies that

\[ x + 3 = 20 \]

Heirber and Carpenter (2009) point the reason why transposing errors exist. Problems rise due to over generalization of equations such as:

\[ \frac{x}{2} = 3 \]

this follows that

\[ x = 6. \]

Generally errors in equations can be categorised as other inverse error, deletion error and transposing error

**2.9 Errors and misconceptions in word problems**

Translating word problems into mathematical equations posits problems which are affected by several factors. This does not only include mathematical skills, but also linguistic skills and verbal skills—particularly in the English language, since schools in Zimbabwe uses English as the language of instruction. Another obstacle in problem-solving is that a pupil is expected to have good foundations in mathematics (Cathcart, Pothier, Vance, & Bezuk, 2000).
Furthermore, in any type of word problem, a problem-solver uses his/her prior knowledge in obtaining an accurate solution and answer. Truly, understanding a word problem in problem-solving is a complex method that involves several determining factors, which either affect or influence the process in which solvers find the correct answer.

There are three significant translations in solving algebraic word problems (Lesh, 1987). Students translate a word problem formulated as a sentence in English to an algebraic sentence (equation), then from the algebraic sentence to an arithmetic sentence, and finally from an arithmetic sentence back to the original problem situation. In order to obtain the right answer, it is significant for solvers to first analyse, comprehend, and translate thoroughly the English problem structure into a mathematical equation. However, most students do not recognize the similarities and differences between arithmetic and algebra which impose a problem in translation. When students encounter such difficulty, they resort to other conceptions which they believe will help them solve the math problem regardless whether the method is sensible or not (Van Ameron 2002).

Students’ mistakes on misplaced numerical coefficients are not simple errors due to carelessness but rather are difficulties brought about by a thought process called reversal error (Clement 1982). He posited that reversal error is an offshoot of student word problem translators to treat numerical variables, as if they stood for objects rather than numbers. For example, when a student is asked to comprehend a statement like “for every two students ($S$), five books ($B$) are given”, he will think of the variables as either students or books but not as numerical quantities of an unknown. Hence, such assimilation may lead to a reversal error (e.g., $5S = 2B$ or $2S = 5B$) in representation by either syntactic method or semantic method. Clement (1982) described the two conceptual sources of reversal errors as follows:

1) **Syntactic word order matching process** is when the student simply assumes that the order of key words in the problem statement will map directly into the order of symbols appearing in the equation.

   In the above given example on students and books, a student who assimilated the problem through syntactic method will translate it into $2S = 5B$ because two is near the word “students”
in the same manner that five is near “books”. For many problem-solvers, this method is convenient and handy, because it gives them a “sound” equation without much thinking.

(2) Semantic static comparison process is achieved when a student relates real life experiences in reasoning out for the translation that has occurred. There is some semblance of reason in this approach as an intuitive symbolization strategy, but the approach is very literal attempt to compare the sizes of the two groups.

In the above given example on students and books, a student who assimilated the problem through the semantic method will answer and will reason that there should be more books than students. Thus, five is multiplied to $S$ to show a greater quantity.

Using the above mentioned thinking processes, many problem-solvers write equations which are not based on any mathematical reasoning (Clement 1982).

### 2.10 Research Gap

Although there are many causes of student difficulties in mathematics, the lack of support from research fields for teaching and learning is noticeable, Gunawardena (2011). A few studies address the issue of students’ misconceptions in algebra as compared to a number of researches on students ‘errors and misconceptions in arithmetic. If research could characterize students’ errors and misconceptions, it is possible to design effective instructions. Research on student errors and misconceptions is a way to provide support for both teachers and students. The existing research is mostly about identifying and explaining causes for a particular concept. Many pay attention to some isolated concepts such as variables, equations or expressions, little has been made to study the interrelated nature of the misconceptions in more than one conceptual area (Gunawardena 2011). Problems of this nature are particularly worthy of investigation as there is still a lack of robust research in identifying students’ misconceptions for more than one conceptual area collectively, Gunawarden (2011). If researchers can identify students’ difficulties collectively in more than one area, it will be easier to identify the systematic patterns of errors that spread through the areas and make suggestions for remediation.
2.11 Summary

The chapter was all about the findings obtained by other researchers on algebraic misconceptions. Explanation of concept and acquiring mathematical knowledge are some of the aspects which were included in this chapter. The chapter also touches on sources of misconceptions and this was followed by the classifications of misconceptions in procedural and conceptual. Finally misconceptions in different conceptual areas which are variables, algebraic expressions, equations and word problems were explained.
CHAPTER 3

3.0 Introduction

In this chapter, the methodological structure of the study is presented. The chapter is divided into six sections. The first section outlines the justification of the research design, followed by a description of the participants, the description of the schools, the instruments used, procedure for data collection and lastly a description of how the data was analysed.

3.1 Research Paradigm

There are two approaches to research, namely quantitative and qualitative. The difference between the two approaches is based on methodology. According to Denzin and Lincoln (1998), qualitative research emphasises the process of discovering how the social meaning is constructed. On the other hand quantitative research is based on the measurement and the analysis of causal relationship between variables. Bryman (1998) feels that the decision to choose a specific methodology should be based on its suitability to answer the research questions. The researcher will use qualitative approach since this research study will focus on identifying errors and misconceptions that are held by pupils in algebra.

3.2 Research design

A research design is “the plan and structure of investigations so conceived as to obtain answers to research.” Copper and Schindler (2003:148). The research design is therefore a blueprint that enables the investigator to come up with solutions to the problems and guides in the various stages of the research. This concurs with Kothari (1980:31) when he points out that a research design is conceptual structure within which a research is concluded. It constitutes the blueprint for the collection, measurements and analysis of data. Copper and Schindler (2003) further
argues that research design is an activity and time based plan always based on the research question and guides the selection of sources and types of information.

The investigator adopted the use of descriptive survey. Descriptive survey is a means of describing what one is seeing (Chiromo, 2006). It entails a study of limited number of cases with a view to draw up conclusion that covers the generality of the whole group under review. In this study, descriptive survey was used to study a case of a school in Mashonaland East province in Zimbabwe. Descriptive survey is appropriate since the research focuses on identifying and describing the misconceptions that are held by pupils in learning algebra on a sample.

3.2.1 CASE STUDY

According to Goldin (2008) in Gunawardena (2011) case study procedure is one of the main research methodologies in a qualitative research design. Wiersma and Jurs (2005) defined a case study as a detailed examination of a specific event, an organization, or a school system. Much of the data of case studies come from observations, documents, and interviews. Case study research can be used to address exploratory, descriptive, and explanatory research questions (Yin, 2003; Johnson & Christensen, 2008 in Gunawardena 2011). Robert (1994) said that case studies employ multiple sources of information to represent the case but not the world.

3.3 Research population

The focus of this subsection is to identify the general group under review. The research population includes form three and four pupils. A total of 45 form three and 47 three form 4 pupils are going to be used as the sample. Form three pupils are to be selected in order to know some of the misconceptions which might have originated from form one and two. Form four
pupils are to be selected so that the researcher could identify some misconceptions which are held by pupils from form 1, 2 and 3. Pupils at the selected school are streamed, so those in alpha classes are considered to be the best. The researcher feels that it is easier to identify a misconception from a best class than from the other class. The assumption being that intelligent students will attempt to reason in a meaningful way unlike mathematically challenged students they may write irrelevant work.

3.4 Sampling Frame

The researcher used a sampling frame of an up-to-date class attendance register which would give the names of all the pupils in the class. A sample frame was accurate and care was taken to ensure that it is free from omissions or duplications.

3.5 Sampling techniques

In this research non-probability sampling method was used which is purposive sampling method. Thus the mathematically gifted students are our target hence three alphas and four alphas were used. The researcher will further select students who showed errors and misconceptions per question from each class for interview.

3.6 Research instruments

Test instrument

The researcher used a test as research instrument. A prepared test was given to the participants in order to find out their misconceptions in algebra. The tests included algebraic expressions, equations, variables and word problems. Pupils were asked to show all their working on the paper to enable the researcher to identify procedural errors and conceptual errors.
3.6 Data collection procedures

3.6.1 Test

The researcher prepared some test questions on variables, algebraic expressions, equations and word problems. Using tests to conduct assessments is advantageous for several reasons. Tests yield quantifiable information and the results can be used in screening program, for example identifying pupils in need of further assessment. Tests results provide information regarding an examinee’s areas of strength and weaknesses. Test results allow a pupil to be compared to age or grade peers. Results from tests can be documented and empirically verified. They allow for the results to be interpreted and ideas about an individual’s skills generalized.in this research the test paper was structured and pupils answered individually. The test was marked and errors and misconceptions were noted. Depending on the number of misconceptions made on each question, selected individuals were invited for an oral interview to explain their answers. This enabled the researcher to note possible sources of procedural and conceptual errors and misconceptions. The order of the interview questions asked was determined by the answers which were written in the test and response from the previous question.

3.6.2 Interviews

After the test, the researcher interviewed selected individuals who got different questions wrong. Kwesu (2001) defines the interview as a questionnaire run down or answered orally. Instead of writing the responses, the subject or interviewee gives the needed information face to face. This concurs with Choga and Njaya (2011) who defines an interview as a conversation in which one person (the interviewer) elicits information from another (the interviewee). He further defines a standardized open-ended interview as the type of interview in which the same open-ended questions are asked to all interviewees. Thus the researcher used this approach as it facilitates fast interviewees that can be more easily be analysed. Open ended interviews approach is appropriate since there is a room for probing and clarification. This means that the
interviewer can note specific reactions or responses and then eliminate misunderstandings about the questions asked and in the same vein the interviewer facilitates the asking of more complicated questions (Wimmer and Domminick, 1997). The presence of the interviewer ensures that nobody else contributes to answering the questionnaire other than the particular respondent. None-verbal responses can be observed and noted and simultaneously has more control of the situation. Personal interview can also considered as flexible and visual materials can be utilized effectively. However interviews are time consuming. More to the above interviewer bias is common for example facial expression or comment by the interviewer can influence or affect the response obtained.

3.7 Triangulation
The adopted instruments were interviews and tests. The instruments were adopted in order to make them relevant to the purpose of the study. Triangulation involves corroborating evidence from different sources to shed light on a particular issue. Triangulation is necessary to validity issues such as checking the truthfulness of the information collected (Black 199). This combination of several data collection methods is called triangulation (Creswell, 1998).

3.8 Pilot Testing
A pilot study is a small study conducted prior to larger piece of research to determine whether the methodology, sampling, the instruments and analysis are adequate and appropriate (Crewell 2003). This mini-research is intended to expose deficiencies of the measuring instruments or the procedure to be followed in the actual. Twenty students from form three and four students involved in piloting of the test. Necessary corrections for example wording of certain items and space left for working were made due to feedback from the pilot study. According to Crewell (2003) a pilot study is important since it makes researcher aware of any possible unforeseen problems that may emerge during the main investigation.
3.9 ETHICAL CONSIDERATIONS

Canon (1992) define that ethics define how individuals choose to interact with one another. Bulger (2002) said that code of ethics for educators always address issues such as fairness and confidentiality. According to National Education Association, teachers may not discriminate against students for any reason and may not share information about the students with anyone other than school professionals who need the information to assist the student.

According to Carry (2007) there are many converting communication barriers between participants and the researcher that lead to misunderstandings. Bulger (2002) said that it is ethical for researchers to account and correct misunderstanding in informed consent process.

Bulger (2002) said there are two standards that are applied in order to help protecting privacy to the researcher participants which are confidential and anonymity.

Confidentially, pupils are assured that identifying information will not be made available to anyone who is not directly involved in the study. The information and results from the study are to be kept secretly by the researcher. Canon (1992) said that the striker standard is the principle of anonymity which essentially means that the participants will remain anonymous throughout the study. The anonymity standard is guarantee of privacy but it is difficult sometimes to accomplish.

3.10 Data analysis procedure

The information obtained from the tests was analysed qualitatively. Statistics was used to describe the total number of misconceptions exposed in the pupils’ answers. To simplify the interpretation of the information, the data in this research are transformed into visual presentations which are more readily comprehended. The researcher described the pupils’ responses from the interviews and tried to link their answers in the test and their explanations.
**3.11 Summary**

This chapter examined the research methodology that was employed in this study. The descriptive survey was used to gather data. The justification of methods used was also discussed. The site for data collection, procedure for selection of participants and collection of data was described. The next chapter will focus on the presentation, analysis and discussion of the results obtained.
Chapter 4

DATA PRESENTATION, ANALYSIS AND INTERPRETATION

4.0 INTRODUCTION

In this chapter the main findings are presented and analysed. This will involve description on the number of pupils who exhibit misconceptions on the following conception areas; variables, expression, equations and word problems. The data was presented and analysed in the form of tables, pie charts and bar graphs. Also the qualitative analysis of the results from the interviews will be given.

4.1 Form three test results on algebraic expressions

4.1.1 Incorrect cross multiplication

When an algebraic fraction has to be multiplied, students used cross multiplication. From the interview it seems that they are confused with the arrangement of literal terms.

Q1a) \( y \times \frac{x}{y} \)

Mary was confident with her answer she said that “since there is no denominator for the other expression we cross multiply”. From the interview it was shown that students lack the
experience of making one as the denominator in such situations. However some students were able to answer the question correctly.

4.1.2 Oversimplification

From the results of the test some students oversimplify algebraic expressions by illegal cancellations. Q1a) \( \frac{x}{y} \)

One of the interviewee, Samantha said “I got 1 because 1x is equal to 1”, she misused cancellation procedures. Samantha cancelled x as a way of simplifying the answer. The error has happened since she used an illegal cancellation procedure. Thus students attempt to simplify the problems using wrong procedure.

4.1.3 Incorrect simplification

Some students solved the problem however they terminate the process without arriving at the final answer. Q1 a \( \frac{x}{y} \) = \( \frac{xy}{y} \)

from one of the interviewee, Anita

Interviewer: Can you simplify your final answer

Anita: I am not sure of how to go about it because we do not have like terms

Interviewer: What about y?

Anita: the other one is squared so they are unlike terms
Anita showed misconception on like terms which led her to fail to simplify the expression. Some of the interviewed students thought that they had actually reached the final answer just like Anita. For these incomplete answers, further simplification was possible to reach the final answer. This error is actually the opposite of oversimplification.

### 4.1.4 Wrong denominator

From the test results students were multiplying denominators to get the common denominator instead of taking their lowest common multiple.

\[
Q1b \quad \frac{mn}{xy} + \frac{1}{y}
\]

Below are some of the quotes of the interview with Kelly

Interviewer: How do you find the common denominator when multiplying fractions?

Kelly: We find the lowest common multiple

Interviewer: So how did you get your lowest common multiple?

Kelly: I multiplied x and \(y^2\) because I want to get a number which can be divisible by both denominators.

Interview: what if we have 2 and 3 as our denominators, how would you get your common denominator?

Kelly: It is just the same, I will say 2 x 3 to get 6
From the above answers it showed that Kelly knows that to get common denominator one has to find the lowest common multiple. Kelly however does not know how to find the lowest common multiple of algebraic expressions. This misconception is from the topic fractions and factors and multiples.

Figure 4.1.1 Distribution of errors committed in algebraic expression

From the above graph it is shown that incorrect cross multiplication with 67% is more popular among students followed by over simplification with 32%, incomplete simplification with 22% and wrong denominator with 18%.

4.2 Form four results on algebraic expressions

4.2.1 Incomplete simplification

Some students solved the problem however they terminate the process without arriving at the final answer. Loretta said “I think this is the final answer”. She did not know how to proceed. For these incomplete answers, further simplification was possible to reach the final answer. This error is actually the opposite of oversimplification.
4.2.2 Oversimplification

From the results of the test some students oversimplify algebraic expressions by illegal cancellations and divisions of terms. One of the interviewee misused cancellation procedures.

Susan said that we cancel the like terms on the numerator and denominator to reduce it in lowest terms”. The error has happened since she used an illegal cancellation procedure of b. She used the correct application of the rule (Matz, 1980) but incorrect cancellation of terms in the algebraic expression made the cancellation incorrect.

4.2.3 Incorrect cross multiplication

Some students showed conceptual error. From the interview it seems that they are confused with the arrangement of literal terms.
One of the interviewee, Amanda, was confident with her answer she said that “a is multiplying both the numerator and denominator”. From the interview it was shown that students lack the experience of making 1 as the denominator in such situations. However some students were able to answer the question correctly.

4.2.4 Wrong denominator

From the test results some students were multiplying denominators and some were multiplying denominators as the common denominator instead of taking their lowest common multiple.

An interview with Loreto showed that she could not get the lowest common multiple of given denominators.

Interviewer: How did you get the common denominator?

Loreto: I looked for common letters which are x and $y^3$.

Interviewer: Why $y^3$?

Loreto: So that I can easily divide y and $y^2$

Loreto was not aware of how the common denominator can be found. This misconception of common denominator led to conceptual error. Of the interviewed students no one showed knowledge of how to find a common denominator. The interviewer further asked a question to one
of the interviewee to find a common denominator of \( \frac{x}{5} \) and \( \frac{x}{7} \). She said “its 35, I got it by multiplying 5 and 7”. She justified why she got \( \frac{xy^3}{y} + \frac{1}{xy^2} \). This misconception is from the topic fractions.

**Figure 4.2.1 Distribution of form four errors committed in algebraic expressions**

![Chart showing distribution of errors]

From the above findings it has been shown that in form four incomplete simplification with 22% was common followed by wrong denominator with 18%, incorrect cross multiplication 17% then over simplification with 13%.

### 4.3 Form three test results on errors committed in variables

#### 4.3.1 Ignoring variables

Some students were not able to simplify some expressions because they did not consider the variables. Instead of operating like terms they ignored variables. From the interview it was seen that some pupils did not understand like terms.

**Q2.** \( 2n + 5 = 7 \)
One of the interviewee Grace said “I collected like terms which are 2 and 5 and then add them”. The interviewer further asked her what about the letter n she asked “Is it not just the same?” Thus Grace thought that letters can be considered or not, either way to her has the same meaning.

**4.3.2 Taking variables as labels**

Pupils were coming up with their own variables which were taken from initials of names and objects.

**Q3. A boy is x years old, his father is 12 years older. What is the fathers’ age?**

**Answer: F =12+b**

After being asked the meaning of the letters used, Grace said “F stands for father and b for boy”. He was then asked the meaning of x in the question; she said it is an unknown. Misinterpreting letters as labels is a basic misconception which will lead to many other errors in algebra. This was shown by the famous Student-Professor Problem (Clement, 1982; Clement, Lochhead, & Monk, 1981; Kaput, 1985) in Gunawardena (2011). This error should have not been high if the students would have known the meaning of the letters.

**4.3.3 Take variables as abbreviations**

Most students took variables as abbreviation. Of the interviewed students, it was shown that pupils were not aware of the concept.

**Q4. One shirt costs dollars s dollars and one pair of trousers cost b dollars. If I buy 5 shirts and 2 belts, explain what 5s + 2 b mean.**

Agatha said ‘it is obvious that s stands for shirts and b for belts”. He did not use mathematical reasoning rather she took the abbreviations for shirts and belts. This misconception according to Clement, Lochhead and Monk (1981) was prevalent also among students already in college.
Figure 4.3.1 Distribution of type of errors made by form threes in variables

From the pie chart it is shown that most students made an error of taking variables as abbreviations, 35% followed by taking variables as labels, 25% then lastly ignoring variables, 12%. 8% of the students who wrote the test did not answer the question.

4.4 Types of errors committed by form fours in variables

4.4.1 Ignoring variables
Some students were not able to simplify some expressions because they did not consider the variables. Most students did not group like terms before the answer. Instead of operating like terms they ignored variables. From the interview it was seen that some pupils did not understand the issue of like terms,

Q2b. Simplify  $2p + 4n + 7 + 2p$

$= 13$

One of the students who gave the above answer, Mercy said “I collected like terms which are 2; 4; 7 and 2 and then add them”. The interviewer further asked her what about the variables she said “I don’t think they are necessary”. Mercy viewed numbers as imported in this question she never considered letters as necessary to write.

4.4.2 Taking variables as labels

Pupils were coming up with their own variables which were taken from initials of names and objects.
Q3 David weighs 3kg more than Sarah, Sarah weighs \( w \)kg. What is Davids weight?

\[
D = 3 + S
\]

Anita said ‘\( D \) stands for David’s weight and \( S \) for Sarah’s weight’. The interviewer further asked her where she is supposed to use \( w \)kg. Anita said that it is already included on \( D \) and \( S \). Thus Anita gave labels to the given variables. Misinterpreting letters as labels is a basic misconception which will lead to many other errors in algebra.

### 4.4.3 Take variables as abbreviations

Most students took variables as abbreviation. The common answer was 4 pencils and 3 rulers.

All interviewed students gave the explanation that \( p \) stands for pen and \( r \) for ruler. One of the interviewee, Mercy said “I think \( p \) stands for pencil since one pencil cost 2p and one ruler cost 3r”. **Figure 4.4.1 Distribution of errors committed in variables by form fours**
From the findings the most common error was taking variables as abbreviations, 40% followed by taking variables as labels, 17% and lastly ignoring variables, 15%. 12% of the students did not answer the problem.

4.5 Test results of errors committed by form threees on equations

4.5.1 Inverse Error

From the interview of selected students who committed inverse error it showed that pupils were not aware of that as an error. Some realised that they have made an error in operation of directed numbers.

Q2a. 1-2x = 13

![Equation image]

One of the interviewee who wrote the above answer, after being asked why she got 12 she said the difference between 1 and 13 is 12, she only accepted that her answer was wrong after the interviewer asked her to draw a number line and use it to operate directed numbers. This error emanated from operation of directed numbers.
4.5.2 Omission error

From the selected students who committed omission error, it was evident that pupils were aware of the rule that “what you do to the left should be done to the right side”.

\[ b) \ 2x-5 = 10-3x \]

The problem emanated from operation of directed numbers “-5-5” most students assumed that the answer is zero. After being asked how she got third stage from first one of the interviewee, Tracy said “this is because -5-5 = 0”. Errors in operating directed numbers led to omission error.

4.5.3 Transposing Error

Interviewee who committed transposing error showed that they were not aware of the difference between \( m^2 \) and \( 2m \).

\[ b) \ m^2-16=0 \]

\[ b) \ m^2=16 \]

\[ 2m=16 \]

\[ m=8 \]
One of the interviewee who gave the answer above, Trish said “m squared and 2m are the same” then the interviewer asked her to solve “m x m”, she said its m squared, that’s when she realised that she had made an error. The error might have been from multiplication of algebraic expressions. Thus the misconception of adding algebraic expression led to misconception.

**Fig 4.5.1 Distribution of errors committed by form threes on equations**

From the pie chart, most form three students committed inverse error followed by omission then transposing error.
4.5 Form four test results on errors committed on equations

Figure 4.5.1 Distribution of types of errors committed on equations

From the bar chart it is shown that inverse error had the highest frequency, 35% followed by omission error, 28% lastly transposing error with 20%. 10% of the students did not answer the question.

4.5.1 Inverse Error
Like form three errors, inverse error is common among pupils of form four, though the percentage is lower than that of form threes. From the selected interviewee, most of them were not aware that they had made some mistakes.

\[ a) \quad 14x + 7 = 11 \]
One of the interviewee, Rudo said I forgot to put a negative sign, Some interviewees were confident with their working and could not see anything wrong with their working. Rudos error led to incorrect solution for word problems. According to Clement (1982), careless operation of directed numbers can lead to inverse error.

4.5.1 Omission Error

Of the interviewed pupils who made omission error, no one had realised the error in her working. Pupils were actually wondering why they were marked wrong. The aspect of operation of negative numbers “-15-15” pupils assumed the answer was zero.

One of the interviewee, who gave the above answer, Melinda said “isn’t it that we are subtracting 15 from 15”. It was after the interviewer asked her to draw a number line and use it to solve the problem so that she could identify their error.

4.5.2 Transposing error

On transposing error, pupils were not aware of their errors.
One of the interviewee, Gilian was very confident with her workings, she said “if we move one number from either side it changes sign”. She changed the sign of the number and subtracted instead of dividing. She knows a correct rule but she applied it on a wrong question.

Generally form threes had some misconceptions on equations they carried from ZJC and form fours also are still holding on some misconceptions from form one two and three. The most committed error is inverse error followed by omission then transposing error.

4.6 Form three test results on errors committed on word problems

<table>
<thead>
<tr>
<th>Type of error</th>
<th>% frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Syntactic</td>
<td>53</td>
</tr>
<tr>
<td>Semantic</td>
<td>34</td>
</tr>
<tr>
<td>Did not answer</td>
<td>30</td>
</tr>
</tbody>
</table>

The table shows that syntactic word order matching has the highest frequency as compared to semantic static comparison process. A greater percentage of pupils did not attempt the questions on word problems.
Figure 4.6.1 Distribution of sources of reversal error on word problem errors committed by form threes

It is shown on the graph that syntactic error has higher frequency as compared to semantic error.

4.7.1 Syntactic method

Syntactic word order matching process is when the student simply assumes that the order of key words in the problem statement will map directly into the order of symbols appearing in the equation. This assimilation can lead to reversal error Clement (1982). One of the interviewed student Samantha said “since 4 is for cars and 2 is for bicycle that why I got 4C = 2B. she was further asked why she has to have 4 for cars and 2 for bicycle. She said that in the given question 4 is near cars and 2 is near bicycle. This showed that she took the answer from the order it was given. Therefore according to literature the source of the reversal error is from syntactic word order matching.

4.7.2 Semantic static comparison process

From the findings one of the interviewee Grace said “the answer is 2C = 4B because it takes more time to make a car as compared to a bicycle”. As according to literature, Clement (1982)
gave the explanations referring to real life experiences, when arguments are based on real life situations they take variables as abbreviations instead of numerical quantities. From the findings it was shown that some reversal error committed were due to conceptual source of semantic static comparison process since most pupils could reason referring to real life situations.

4.7.3 Translating word problems into mathematical equations

Besides reversal error, pupils showed lack of understanding of English in given questions. After being asked why she did not answer the question, Theresa openly said “I do not understand what the question wants”. Most students who did not attempt the question gave the same reason.

According to Lesh (1987) students should be able to translate a word problem to an algebraic sentence in order to solve it. Pupils who could not translate the problem correctly gave wrong algebraic expressions.

**Q7. Starting with some number, if you multiply it by 5 and then add 30, you get 100. What number did you start with?**

Anna said that “I think there was a typing error”. When asked why she said that, Anna said “how can 30 + 5 add up to 100”.

Anna showed that she did not translate the given question correctly from word problem to algebraic sentence. Anna did not show any mathematical reasoning in her answer.
4.8 Form four test results on word problems

Table 4.8.1 Distribution of sources of reversal error committed on word problems

<table>
<thead>
<tr>
<th>Types of errors</th>
<th>% frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>syntactic</td>
<td>60</td>
</tr>
<tr>
<td>semantic</td>
<td>22</td>
</tr>
<tr>
<td>Did not answer</td>
<td>25</td>
</tr>
</tbody>
</table>

Syntactic process had more frequency as compared to semantic process. A number of students did not attempt problems involving word problems.

Figure 4.8.1 Distribution of errors committed in word problems

From the pie chart it is shown that syntactic word order matching process was more popular as compared to semantic static comparison process. A number of students did not answer word problems.

4.8.1 Syntactic word order matching process
Syntactic word order matching process is when the student simply assumes that the order of key words in the problem statement will map directly into the order of symbols appearing in the equation. This assimilation can lead to reversal error Clement (1982).

Q9 Write an expression in terms of S and B, For every 6 students 2 books are given.
One of the interviewed student Tecla said “since 6 is near students and 2 is near books the answer is 6S = 2b “. This showed that she took the answer from the order it was given. Therefore according to literature the source of this reversal error is from syntactic word order matching.4.8.2 Semantic static comparison process

Most pupils gave the explanations referring to real life experiences. Their argument was based on what they think rather than concepts. One of the interviewee, Nomsa said “the answer is 2S = 6B because 2 students can get 6 books other than 6 students getting two books. This was explained by Clement (1982) when he mentioned that students who relate the questions with the real life situation shows semantic instead of writing an expression which is being described in the questions.

4.8.3 Translating word problems into mathematical equations

Besides reversal error, pupils also showed lack of understanding of English in given questions. After being asked why the word problems were not answered, Alicia said “I found it difficult to understand what exactly is needed by the question”. She was then asked to say what she think about the question. She said the total inventory books were not given”. Thus Alicia failed to translate the word equation to an algebraic sentence. According to literature Lesh (1987) students should be able to translate a word problem to an algebraic sentence in order to solve it. Pupils who could not translate the problem correctly gave wrong algebraic expressions. Alicia was also asked to try and solve the problem .According to literature Van
Ameron (2002), students who encounter problems of translating they resort to other concept they believe will help them to solve the math problem. Anna resorted to use the given numbers in the question without mathematical reasoning.

**Figure 4.9 Overall distributions of errors in all conceptual areas**

From the graph it is shown that word problems have a high frequency of errors committed followed by equations, variables and algebraic expressions.

**4.8 Summary**

Pupils had some misconceptions in the four conceptual areas of algebra which are variables, algebraic expression, equations and word problems. Conceptual and procedural misconceptions were exposed through their test and interview. The highest number of errors was recorded in word problems which have 73.4%, followed by equations with 68.6%. The variables and algebraic expressions had 58.3% and 52% respectively. It was also observed from oral interviews that pupils have some conceptual misconceptions and procedural misconceptions.
Chapter 5

Summary, Conclusions and Recommendations

5.0 INTRODUCTION
This chapter summarizes the research findings. Briefings from all the previous chapters were given. Conclusions based on the research findings addressed the research problem and objectives. Special recommendations were outlined basing with the research results.

5.1 SUMMARY

The main purpose of this research was to identify errors and misconceptions made by pupils when solving algebraic problems. From the background of research problem the researcher came up with chapter 1. In this research study, answers to the four research questions were obtained which serve as solutions to research objectives. The research questions were

1. What are students’ categories of errors and misconceptions in solving problems related to variables?
2. What are students’ categories of errors and misconceptions in solving problems related to algebraic expressions?
3. What are students’ categories of errors and misconceptions in solving equations?
4. What are students’ categories of errors and misconceptions in solving word problems?
The related literature helped the researcher to come up with some solutions to the identified research questions.

In chapter 2 findings in current ZIMSEC mathematics syllabus showed that pupils are required to develop ability in mathematical problem solving. Some ZIMSEC’s mathematics reports proved a poor performance in algebraic problems. Some findings showed that pupils’ performance is haphazard. Errors in algebraic solving procedures provided valuable insights into pupils’ thinking. Found in literature review were types of errors and misconceptions in each concept of algebra. Some algebraic solving problems found were related to confusion and misconceptions.
In chapter 3, the researcher formulated research methodology. The descriptive survey design was used to gather credible data. In this design the researcher collected and analysed quantitative data followed by the collection and analysis of qualitative data. Data was drawn from a sample of 92 students, whose test was analysed and interpreted to give the research results. From the sample individuals were selected according to their performance for interview. Methods for data presentation analysis and interpretation were established. The methodology to this study helped the researcher to come up with research results.

In chapter 4, the test and open ended interview were used to come up with research results. Data collected from the research study was presented on graphs, pie charts and tables. These presented data gave answers to the research questions. The test was used to identify the categories of errors in each conceptual area. Interview was used to identify the major roots of errors and misconceptions. Errors identified were transposing, omission, inverse, semantic and syntactic errors, among others.

5.2 CONCLUSION
The main purpose of the study was to investigate and categorise the type of errors and misconceptions that are held by form three and form four pupils when solving algebraic problems. The study revealed categories of errors in each conceptual area is algebra. Through interview, it was clear that most of the misconceptions have their sources in other areas in mathematics. Thus poor background of students in arithmetic has led to poor performance in algebra. Word problems proved to be a difficult concept among others. It was noted that poor understanding of English language contributed a lot on misconceptions made in word problems. It can also be concluded pupils who find the basics of algebra difficult that is variables and algebraic expressions will also face problems in equations and word problems.

5.3 RECOMMENDATIONS
The following recommendations were made based on the findings of the study.

- Teachers and other stakeholders should make efforts to find possible solutions which may minimise these misconceptions. This can be done through carrying further research on the possible solutions.
- Teachers should eliminate some misconceptions during teaching or before new concepts are introduced.
• It was shown that some misconceptions emanated mainly from misconceptions of other topics for example directed numbers. Further research should be done on errors and misconceptions of other mathematical topics other than algebra.

• Teachers are advised to be critical when marking pupils’ work and this will allow them to identify pupils’ misconceptions. Since errors and misconceptions are found to be held up to form four, this will also help to eliminate errors and misconception at early stage of secondary school.
Appendix 1
Form three test

Topic: Algebra

Student Name:............................................................

This is a non-evaluative assessment. The assessment is designed to help you with algebra, by helping your teacher understand the errors you make, as well as why you make them.

Instructions:

1. Answer all questions

2. Show your working in the space provided

3. Use algebraic methods to solve all the problems.

4. Time: 45 minutes

1. Simplify the following expressions

   a) \( \frac{x}{y} \)
   b) \( \frac{mn}{xy} \frac{1}{y} \)

   a)............................  b)............................

2. Simplify the algebraic expressions

   a) \( 2n + 5 \)
   b) \( 4mn + 6n - 2mn \)

   a)............................  b)............................

3. A boy is \( x \) years old his father is 12 years older. What is the father’s age?

   .................................................................
4. One shirt costs $s$ dollars and one pair of trousers costs $t$ dollars. If I buy 5 shirts and 2 belts. Explain what $5s+2b$ represents.

5. Solve the following equations
   
   a) $1 - 2x = 13$  
   b) $2x - 5 = 10 - 3x$  
   c) $2(y + 5) = 6$

6. Solve the following quadratic equations
   
   a) $x^2 - 6x - 16 = 0$  
   b) $m^2 - 16 = 0$  
   c) $(m - 7)^2 = 0$

7. Starting with some number, if you multiply it by 5 and then add 30, you get 100. What number did you start with?

8. Write an equation using the variables and $w$ to represent the following statement: “At a bicycle manufacturing company, for every four red bicycles produced, there are five green bicycle produced”. Let $r$ represent the number of red cars and $g$ represent the number of green cars.
Appendix 2
Form four test

Topic: Algebra

Student Name: …………………………………………………………………

This is a non-evaluative assessment. The assessment is designed to help you with algebra, by helping your teacher understand the errors you make, as well as why you make them.

Instructions:

1. Answer all questions
2. Show your working in the space provided
3. Use algebraic methods to solve all the problems.
4. Time: 45 minutes

1. Simplify the following

a) \( \frac{xa+xb}{xb+xc} \)  
b) a \( \frac{x}{y} \)  
c) \( \frac{z}{xy} + \frac{z}{xy^2} \)

a)……………………………….b)……………………………….c)……………………………….

2. Simplify the following

a) 2p+4n+7+2p  
b) -3x+4y +2x+y

a)……………………………….  b)……………………………….

3) David weighs 3kg more than Sarah. Sarah weighs wkg. What is David’s weight?

…………………………………………..
4) A pencil cost 2p cents and a ruler costs 3r cents each. What is an explanation of 4p+3r.

5) Solve the following

a) 14x+7=11  
b) 5x-15=15-4x  
c) 4(8x+3)=8

a)……………………………  b)……………………………………………………c)……………………………………………………

5  Solve the following quadratic equations

7. A publisher published a book in the last year of which 1/8 of the total inventory were sold and 800 remained in the warehouse, how many books were published in total?

…………………………………………………………..

8. The result of taking 3 from y and multiplying the answer by 8 is the same as taking 5 from 12 times x.

………………………………………………………………

9. Write an expression in terms of S and B. For every 6 students, 2 books are given.

…………………………………………………………………………………………
REFERENCES


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