AN INVESTIGATION INTO THE CONCEPTUAL CHALLENGES FACED BY ZJC PUPILS IN DEALING WITH THE DIRECTED NUMBERS. A CASE STUDY OF A SECONDARY SCHOOL IN MARONDONA DISTRICT OF MASHONALAND EAST

BY
ZIKI ALICIA

B1337578

SUPERVISOR: MR MAPUWEI

A THESIS SUBMITTED TO BINDURA UNIVERSITY OF SCIENCE EDUCATION IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE BACHELOR OF SCIENCE EDUCATION DEGREE HONORS-MATHEMATICS.

SEPTEMBER 2017
RELEASE FORM

NAME OF AUTHOR: ZIKI ALICIA

TITLE OF THESIS: AN INVESTIGATION INTO THE CONCEPTUAL CHALLENGES FACED BY ZJC PUPILS IN DEALING WITH THE DIRECTED NUMBERS. A CASE STUDY OF A SECONDARY SCHOOL IN MARONDERA DISTRICT OF MASHONALAND EAST

DEGREE PROGRAMME: Bachelor of Science Education Degree Honors- Mathematics

YEAR THIS DEGREE WAS GRANTED: 2017

Permission is hereby granted to Bindura University of Science Education library, to produce single copies of this project and lend or sell such copies for private, scholarly or scientific research purposes only. The author does not reserve other publication rights and neither the project nor extensive extracts from it may be printed or otherwise produced without the author’s permission.

Signed by: ……………………………………………………
Date: ………………………………………………………....
APPROVAL FORM
BINDURA UNIVERSITY OF SCIENCE EDUCATION

The undersigned certify that they have read and recommended to Bindura University of Science Education for acceptance research project entitled “an investigation into the conceptual challenges faced by ZJC pupils in dealing with the directed numbers. A case study of a secondary school in Marondera district of Mashonaland east” submitted by Ziki Alicia in partial fulfillment of the requirements for a degree of Bachelor of Science Education Honors-Mathematics.

SUPERVISOR

PROGRAMME COORDINATOR

EXTERNAL EXAMINER

DATE…………………….. DATE……………………..
DECLARATION

I, Ziki Alicia declare that this project is my original work and affirm that it has been submitted to this or any other University in support of any application for a degree or any other qualifications.

Signed…………………………… Date…………………………………………
Witness…………………………. Date…………………………………………

Supervisor
I, ............................................Declare that I have supervised this thesis and am satisfied that it can be submitted to the Faculty of Science Education of Bindura University of Science Education.

Date…………………………………………
Signature……………………………………
DEDICATION

This work is dedicated to my beloved husband, Ambrose Sledge Chinyoka, my pillar and source of inspiration, to my two boys, Watidaishe and Anotidaishe and to my mother, Melania Ziki who gave me all the support and encouragement throughout my studies. God bless you all.
ACKNOWLEDGEMENTS

Carrying out this research project left me with an experience of a lifetime. I would like to render my great appreciation to the following persons:

Firstly, my gratitude goes to God the Almighty for His providence throughout the experience, for giving me the opportunity to do this program and the knowledge and wisdom in conducting the research.

To my husband Ambrose Sledge Chinyoka, my two sons Watidashe and Anotidashe, who had to endure gruesome sacrifices without a mother for most of the time during the course of carrying out the study.
I would like to acknowledge the assistance, patience and wisdom given by my supervisor, Mr Mapuwei. He provided the much-needed direction, scholarly and academic advice for me to be able to complete the project. I salute you sir.

A great deal of gratitude goes to Mr Dera for editing the project and for giving his tireless effort, time and expertise on Statistics.

Special thanks go to the Form one learners and head, Mr Ndowora, of the school at which this project was carried out at in Marondera, I owe them my sincere gratitude for their time and making this study a success.

I also appreciate the love, support and prayers of my family and friends. May the dear Lord keep on blessing you.
ABSTRACT

ZJC learners’ progress from their junior level to Ordinary level, irrespective of their competence in dealing with directed numbers. During the first two terms as form ones’, learners are bound to meet challenges. This study set out to investigate conceptual challenges met by ZJC pupils’ in dealing with directed numbers. A random sampling was done for both form 1 A an B to select those who represented all the ZJC pupils at the school. A sample of 12 research participants was used in the study. Mathematics education researchers have argued about whether to teach negative numbers through models or to wait until students are ready to copy with intra-mathematical justification (Galbraith, 2004; Linchevski & Williams, 1999). When a student is ready for that? The researcher made use of interviews, pre and post test to collect relevant data. A paired t-test was the statistical method used to give meaning and interpretation to encoded data. The researcher made use of tables, frequency polygon and column table to present the unstructured data. The findings revealed that some of the pupils do not understand operations of directed numbers and the number line. Table 4.3 shows that the majority of the ZJC pupils, detected from the sample, are facing conceptual challenges in dealing with directed numbers. However, table 4.4 indicated that there was a great improvement after the intervention lessons were conducted, despite the challenges of not having enough resources to vary teaching methodologies and learning media. The study, therefore, recommends the use of various teaching aids. Kasambira (1993) says lessons that are not accommodated with learning aids and models results in de-motivation and boredom of pupils and thus progress of learning is reduced. Additional to that, for teachers to use these learning aids, they should have them. Therefore, the study recommends the schools to purchase enough and adequate resources such as manila, mighty markers, glue and some other required resources for teachers to use for better learning of pupils.
TABLE OF CONTENTS

CHAPTER ONE
INTRODUCTION

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 Background of the study</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Statement of the problem</td>
<td>3</td>
</tr>
<tr>
<td>1.3 Research questions/ Hypothesis</td>
<td>4</td>
</tr>
<tr>
<td>1.4 Objectives of the study</td>
<td>4</td>
</tr>
<tr>
<td>1.5 Assumptions of the study</td>
<td>5</td>
</tr>
<tr>
<td>1.6 Significance of the study</td>
<td>5</td>
</tr>
<tr>
<td>1.7 Delimitations of the study</td>
<td>6</td>
</tr>
<tr>
<td>1.8 Limitations of the study</td>
<td>7</td>
</tr>
<tr>
<td>1.9 Definitions of terms</td>
<td>7</td>
</tr>
<tr>
<td>1.10 Summary</td>
<td>9</td>
</tr>
</tbody>
</table>

CHAPTER TWO
REVIEW OF RELATED LITERATURE

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0 Introduction</td>
<td>10</td>
</tr>
<tr>
<td>2.1 Concepts of formation</td>
<td>10</td>
</tr>
<tr>
<td>2.2 Misconception of directed number</td>
<td>11</td>
</tr>
<tr>
<td>2.3 Contributing factors</td>
<td>13</td>
</tr>
<tr>
<td>2.4 Conceptualizing negative numbers and zero</td>
<td>14</td>
</tr>
<tr>
<td>2.5 Making a simple problem complicated</td>
<td>14</td>
</tr>
<tr>
<td>2.6 Empirical evidence</td>
<td>18</td>
</tr>
<tr>
<td>2.7 Summary</td>
<td>22</td>
</tr>
</tbody>
</table>
**CHAPTER THREE**  
**RESEARCH METHODOLOGY**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0 Introduction</td>
<td>23</td>
</tr>
<tr>
<td>3.1 Research design</td>
<td>23</td>
</tr>
<tr>
<td>3.2 Population, sample and sampling technique</td>
<td>25</td>
</tr>
<tr>
<td>3.3 Data collection instruments</td>
<td>26</td>
</tr>
<tr>
<td>3.4 Intervention strategies</td>
<td>27</td>
</tr>
<tr>
<td>3.5 Procedures for data collection</td>
<td>28</td>
</tr>
<tr>
<td>3.6 Procedures for data presentation and analysis</td>
<td>28</td>
</tr>
<tr>
<td>3.7 Ethical consideration</td>
<td>28</td>
</tr>
<tr>
<td>3.8 Summary</td>
<td>28</td>
</tr>
</tbody>
</table>

**CHAPTER FOUR**  
**DATA PRESENTATION, ANALYSIS AND INTERPRETATION**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0 Introduction</td>
<td>29</td>
</tr>
<tr>
<td>4.1 Presentation of data and analysis</td>
<td>29</td>
</tr>
<tr>
<td>4.2 Interview Results</td>
<td>31</td>
</tr>
<tr>
<td>4.3 Pre- test</td>
<td>38</td>
</tr>
<tr>
<td>4.4 Post- test</td>
<td>43</td>
</tr>
<tr>
<td>4.5 Hypothesis test</td>
<td>48</td>
</tr>
<tr>
<td>4.6 Discussion of findings</td>
<td>50</td>
</tr>
<tr>
<td>4.7 Summary</td>
<td>53</td>
</tr>
</tbody>
</table>

**CHAPTER FIVE**  
**SUMMARY, CONCLUSIONS AND RECOMMENDATIONS**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0 Introduction</td>
<td>49</td>
</tr>
<tr>
<td>5.1 Summary</td>
<td>49</td>
</tr>
<tr>
<td>5.2 Findings and conclusions of the study</td>
<td>50</td>
</tr>
<tr>
<td>5.3 Recommendations</td>
<td>51</td>
</tr>
<tr>
<td>References</td>
<td>52</td>
</tr>
<tr>
<td>Appendix 1</td>
<td>59</td>
</tr>
<tr>
<td>Appendix 2</td>
<td>63</td>
</tr>
<tr>
<td>Appendix 3</td>
<td>64</td>
</tr>
</tbody>
</table>
LIST OF TABLES

Table 4.3: Pre-test results

Table 4.4 Post-test results

Table 4.5 Comparison between pre-test and post-test for individual marks.

Table 4.6 Calculations for test statistic.
LIST OF APPENDICES

1. References 59
2. Pre- test/Post-test 63
3. Interview for teachers
4. Interview for pupils
CHAPTER 1: INTRODUCTION

1.0 Introduction

Mathematics has been taught formally in schools for the past centuries and is known as one of the gatekeepers for success in all fields of life. It is a common saying that mathematics is mother of all subjects. In Zimbabwe, progress and growth has been observed in teaching mathematics in a broader and general sense and this is why the country enjoys high levels of literacy. One of the main objectives of teaching and learning mathematics is to prepare students for practical life. Students can develop their knowledge; skills; logical and analytical thinking while learning mathematics and also to solve problems in almost all fields of life. Mathematics has, however, experienced challenges that have compounded to lower pass rates across the country, Marondera and Mashonaland East, as a whole have not been spared either. Algebra is a generalized form of Arithmetic and for the purpose of generalization of arithmetic; letters of the alphabet and signs are used. No doubt, the use of letters and signs make it an abstract topic and that’s why Algebra is considered to be a difficult area of mathematics.

The Research will have an in-depth study into the conceptual challenges faced by ZJC pupils in dealing with directed numbers, as a noble contribution to the existing body of knowledge aimed at improving teaching and learning of mathematics as a useful subject, in this chapter, several aspects of the research will be highlighted and described. The chapter consists of the background of the study which will be an overview of why the problem observed is a problem. The statement of the problem; objectives of the study with some significance of the research will be discussed. The justification and the assumptions; delimitations and Summary will also be included in this chapter.

1.1 Background of the study

Despite the gains of growth in the educational sector inters of teaching and learning in the past decade, subjects such as mathematics, have continued to be poorly failed as compared to other subjects. Mathematics has given both teachers and students difficult moments in teaching and learning respectively. This has led to students having challenges appreciating and grasping concepts in the long run. Algebra gives students a torrid time to understand though major experiences have been observed when dealing with directed numbers by ZJC pupils’. These challenges manifest themselves fully especially simplifying directed numbers that involve negative numbers and this has remained a grey area for considerable period of time.

In mathematics education, there are many studies dealing with the development of specific content areas (i.e. algebra; fractions and functions) and other studies analyzing learners’ perspectives. Many of this empirical work focus on the outcomes of students’ learning but not on the learning processes themselves. Furthermore, the misconceptions and difficulties in individual
learning process are analysed with respect to an ideal learning trajectory, whereas the productive potential of these individual processes is not taken into consideration. In our contribution, the aim is to structure and understand students’ learning processes in mathematics, thereby we use a theoretical approach which enables us to reveal key points of misunderstandings as well as main influencing factors. These constitute the basis for an adaptive structuring of the mathematical content and hence of the development of instructional designs.

Negative integers constitute a significant topic in mathematics teaching and learning. The concept of negative integer is relevant for both the handling of inner mathematical situations such as solving the equation $x + 3 = 1$ as well as the handling of real world situations, for instance, comparing temperatures below zero or situations with debts and assets. Negative integers are nowadays considered as a rather simple subject that may be taught even in elementary schools. It apparently contains no difficulties, except for a multiplication of a negative number by a negative.”...but several studies show low success rates of students solving computations involving negative numbers, Human and Murray (1987); Murray (1985) which indicate deeply –rooted and widely- held misconceptions.” These difficulties are not restricted to the above mentioned multiplication –which does indeed constitute a big challenge for students but they also concern other contents like addition and subtraction of integers …….but these findings about teaching and learning of negative numbers are rare, Bruno and Martinon (1996) and still not sufficiently clarified., Kishimoto(2005).

Directed numbers cannot be ignored and deserve a special interest as we sustain growth of mathematics as a subject. The importance of studying directed numbers cannot be overemphasized as they are the backbone of mathematics as most topics revolve around and involve directed numbers( Stacey and McGregor ,1997) assert that when directed numbers are misconstrued by pupils ,this has cumulative effects as topics such as equations are also affected as well. This consequently leads to holistic and sound understanding by students of mathematics eluding them. It is important to study directed numbers so as to find ways to reduce the misconception and improve the teaching and learning of directed numbers.

Most pupils had problems in perfoming the algorithm which involved adding directed numbers with different signs as well as those with two or more negative signs were involved. Challenges in comprehension have been noted with problems that involved directed numbers without algebraic expressions. Worrying trends have been echoed by researchers where pupils have some misconceptions that were persistent and sometimes they repeatedly made the same error despite frantic efforts to assist on a one on one basis. Also, through discussions with fellow teachers, the researcher realized that their explanations of these types of behaviors were surprisingly consistent with hers. However, one thing was clear that, these challenges were neither inborn nor were they instantaneous. She further argues that, rather pupils have acquired those challenges during their learning process for yet unknown reasons, which this study endeavors to look into.
1.2 Statement of the problem.

Teaching of directed numbers involving both positive and negative numbers has largely been a difficult task for teachers. It has been observed that ZJC pupils at a secondary school are facing challenges of adding, subtracting, multiplying and dividing directed numbers hence the need to investigate their conceptual challenges they face in their learning of directed numbers. Furthermore pupils have difficulties, more often, face conceptual challenges in handling directed numbers when learning this topic. Linchevski and Williams (2005) asset that mathematics has fared badly especially when handling directed numbers whereas these from the foundation and core of mathematics as a subject. Zimbabwe has not been spared and less research has been done in this area and this has led to some pupils developing resistance to learning the topic and the subject thus indirectly contributing to the low pass rate in mathematics as a subject.

1.3 Objectives of the study

The main objectives of the study are to unpack various conceptual challenges ZJC pupils face when dealing with directed numbers.

The study seeks to investigate conceptual challenges faced by ZJC pupils when:

- Adding and subtracting numerical expressions involving directed numbers.
- Multiplication and division of numerical expressions involving directed numbers.
- Simplifying and evaluating directed numbers that involve three or all the four basic mathematical operations in one expression.

1.4 Research Questions:

- What are the conceptual challenges that pupils face when dealing with directed numbers?
- To investigate how do ZJC pupils add and subtract expressions involving directed numbers?
- To investigate how do ZJC pupils multiply and divide expressions involving directed numbers.
- What strategies can be implemented to mitigate the conceptual challenges that these pupils face?

1.5 Assumptions of the study.

It is assumed that mathematics is done on a daily basis at schools and directed numbers form the basis of many mathematics topics, and the teachers have the expertise in teaching of directed
numbers. Participants (students’ and teachers) will give relevant, honest and unbiased information for the study.

1.6 Significance of the study.

The study envisages inputting into the body of knowledge of mathematics teaching and seeks to see teachers improve in their endeavour to impact mathematical skills and technique to the pupils. With the help of the study, teachers will device suitable methodologies and technique to counter challenges faced by ZJC pupils in dealing with directed numbers. Research has been done in challenges encountered by pupils in various subjects though limited research has been done in the root cause of challenges experienced by ZJC pupils in comprehending proper and correct handling of directed numbers involving all the four mathematical operations, especially on expressions involving algebra.

Examiners will also benefit from the study as they will or may make use of the findings to examine pupils on problems involving directed numbers which tend to be a challenge to most pupils especially in their O’ level paper 1 where they are not allowed to use calculators. The major beneficiaries of the study would be the ZJC pupils themselves as all that will be done will in turn be implemented for them and make a difference in their learning and mastery of the concepts involved in directed numbers.

1.7 Limitations of the study.

The study could have incorporated the whole set of Form 1 pupils at the school but because of the shortage of resources at the school, only randomly selected pupils were used for the study. We could not vary lessons by using various methodologies like power-point presentations because the school has no projector. Due to some commitments by the researcher’s other mathematics colleague at the school, the researcher was also the teacher for the study group, she could not engage a teacher to teach the pupils addition, subtraction, multiplication and division of directed numbers. The study is also limited to one topic Directed numbers. Some other topics that involve directed numbers like Equations would have been used.

1.7 Delimitations of the study.

The study was carried out at a secondary school in Marondera district of Mashonaland East. Random selection of participants from Form 1A and 1B was done and make up the boundary of the study and it is restricted to the topic Directed numbers from the ZJC national syllabus. The
period from which the research was conducted is from the 1st of January 2017 to the 30th of September 2017.

1.8 Definitions of key terms.

❖ Directed Numbers:
These are sets of whole numbers that consists of both negatives and positives. They stretch from minus infinity to infinity and always preceded by a plus or a minus. Merriam Webster (2003).

❖ Conceptual challenges:
Instigation intended to convince a person to perform an action they otherwise would not relate to concepts abstractor a conceptual discussion that antedated development of the new product, Houghton Mifflin (2016)

1.9 Summary.

In this chapter the researcher dealt with the background of the study, statement of the problem, sub-problems and objectives of the study. The significance of the study, delimitations and limitations were also discussed in this chapter. The researcher also defined keys terms that are important in clarifying issues with the study. In the next section, the researcher will focus on the Literature review that is related to the research study.
CHAPTER 2: LITERATURE REVIEW

2.0 Introduction

This chapter shall focus on the literature review where a link to this topic shall be made with what other authorities have to say about the conceptual challenges faced by ZJC pupils when dealing with directed numbers. Reference shall also be made of case studies that have been carried out elsewhere.

Ball (2003) introduced negative numbers as subject matter in teaching experiment in an endeavour to engage learners in intellectual and practical forays and help to extend their ways of thinking mathematically by letting the learners explore different representations and models. Hamilton in Beery etal(2004) justified that the product of two negatives as positive in the following assuming a forward starting position, a product of two negatives represent exactly two reversal of direction. Rudin (2006) defines negative numbers in terms of ordered pairs of natural numbers and all operations in terms of additions made the negative numbers fit into a mathematical structure already well established. Thompson and Martinsson (1991) claimed that the established view of mathematics is "the science of numbers, of space and of the many generalizations of these concepts that have been created by the human intellect".

Different authors had come up with how directed numbers were invented. Schering (2005) also stated that the critical point to the theory of epistemological obstacles described by Glasier is that the historical developments of a concept are not necessarily linear. He also critics the anachronistic view of concept development visible when an obstacles presented as having always self-evident. It is only in the hand sight of the development of the concept of negative numbers that, for example, unifying the number line could be seen as an obstacle. Schering (2005) claims that the study of the concept development of the rules of signs, but to the more general one of the existence of negative numbers, the separation between the concept of numbers from that of magnitudes or quantities will prove to have been desicive.

Mumford (2010) describes the evolution of negative numbers as culturally different, claiming that China and India both seemed ready to extend the numbers domain to include zero and negative numbers, whereas European mathematics resisted the extensions. He blamed Euclidean mathematics for this stating that in Euclidean mathematics numbers only appear in three forms of which none can be negative or zero.

Beery et al (2004) pointed out that the use of symbols + and - for subtraction was invented in the 15th century. A surplus in measure was to be denoted by the sign + and the deficiency with the sign -. At this time the idea of a negative quantity became accepted even if many mathematicians still did not accept negative solutions.
In Europe during the 18th century negative numbers were usually regarded as being less than nothing metaphorically linked to debt as opposed to possession. Fontanella deepened the concept of opposite by differentiating between quantities and qualitative aspects… "Every positive or negative magnitude does not have numerical being by which it is a certain thing opposite to another…. Fontenella (1927)".

Mumford (2010) distinguishes between quantities that are naturally positive and signed quantities. Naturally positive quantities are all measures of weight, length, volume and area as well as number of objects and proportions. Kaplan (1999) gives the idea that it seems as if the idea of opposite quantities, although a very old notion and the root of what was to become a negative number has constraints as well as affordances. Symbolizing a problem of quantities is one thing but thinking of numbers as quantities limits the possible interpretations.

### 2.1 Concepts of formation

Kajori (1991) stated that multiplication of two negative appear in problems of the type (a-b) (c-d), not proper negative numbers in the modern meaning. Diaphantos (1991) also stated that a number multiplied by a number to be subtracted gives a number to be added. When negative numbers stated to appear as isolated objects it was noted that the rule of signs was justified geometrically as shown in the given example. So it is how the teacher applies his/her methodology that will determine the understanding of the pupils in the learning of directed numbers. Mathematics education researchers have argued about whether to teach negative numbers through models or to wait until students are ready to copy with intra-mathematical justification (Galbraith, 2004; Linchevski & Williams, 1999). When is a student ready for that? If we do not see the development of mathematical thinking as qualitative changes in biological modes of functioning but rather s increasingly sophisticated ways of reasoning about mathematics, the question should perhaps be: How do we make students ready for intra-mathematical justification?

### 2.2 Misconception of directed numbers

The problem of elaborating a coherent mathematical status for negative numbers developed over long periods of time. It challenged the first traditional understanding of mathematics. Its first paradigm in Kuhn’s terms is the understanding of it being a science of quantities that while being abstracted to attain autonomy from objects of the real world, continued at the same time to be epistemology legitimized by the latter. The various cultures succeeded over a long time in finding various auxiliary constructions that permitted them to remain within the existing paradigm, Schering (2005).

The real development of the negative numbers as mathematical objects came with the introduction of Algebra. It was a result of an increasing symbolization of mathematics and the acceptance of zero as a number. An overview of the concept of negative numbers in Western
mathematics from the 16th Century on is given in stages of development suggested by Arcavi and Bruckheimen (1983).

In the study of historical texts Gleasar (1981) identified about twenty different obstacles for understanding negative numbers. He highlighted particularly six authors and the obstacles identified in their text. These obstacles can be described as follows:

- Inability to manipulate isolated negative quantities.
- Difficult to make sense of isolated negative quantities.
- Difficult to unify the number line, that is to see it as one line, one axis, instead of semi lines opposite one another with difficult symbols or understanding the positive and negative quantities as having different quality.
- Difficult to accept two different concepts of zero: Zero as an absolute, where zero is understood as the bottom, below which there is nothing, and zero as origin where zero is arbitrary point on an axis of orientation from which there are two directions.
- Stagnation in the phase of concrete operations and not entering the phase of that is seeing numbers as representing something substantial concrete.
- The wish for a unified model that will cover addition and multiplication.

The study deals with the difficulty factor assessment given before and after instruction to tap hurdles in negative number learning. Vlassis (2002) found out that many errors made when solving equations are caused by the presence of negative numbers and concludes that it is the degree of abstraction created by the negative that creates these difficulties rather than the presence of variables or the structure of the equation. This conclusion supports earlier findings suggesting that success in Algebra may depend on part of structural understanding of the relationship of addition and subtraction of directed numbers, Shiu (1978).

Prather and Alibali (2008) pose the question of how people acquire knowledge of principles of arithmetic with negative numbers. Is it a process of detecting and extracting regularities through repeated exposure to operations on negative number or do they transfer known principles from operations with positive numbers?. Exposure to operations on negative numbers is fairly scarce. For many problems in schools or in everyday context, it is often possible to find solution without including negative numbers. Prather and Alibali (2008) used the following task in a study concerning knowledge and principles of arithmetic, “Jane’s checking account is overdrawn by $378. This week she deposited her pay check of $263 and writes a check for heating account. If her checking account is now overdrawn by $178, how much was her heating bill?”

This problem was represented by one student as -378 +263 –x = -178 and by another as 378 (-263) + x = 178. These representations are mathematically correct but only the first one involves negative numbers. As the problem was posed there is no mentioning of negative numbers. In many solutions people avoid negative numbers if they can, for example, when Celsius constructed the thermometer, he originally placed zero at boiling point and 100 at freezing point.
so that for almost everyday situations we would not have to deal with negative numbers and on Freinheit scale normal temperatures are positive. Carlson, Hke and Oberg (2002) originally in Swedish took an exercise from the section about negative numbers which reads “Emperior Augustious was born the years 63 BC…” that would be written as -63. He died in the year 14 AC, how old was he when he died?”

This problem was easily solved by relating to zero and just adding 63 years before then 14 years after zero. 63+14=77. The task could be used to illustrate the fact that 14-(-63) = 14+63 but that is not self evident and would need a lot of explicit reasoning. The important thing to ask about the exercise is whether the goal is to solve the problem or to develop reasoning with negative numbers. If students work by themselves with kinds of exercises they will probably focus on solving the problem, in doing so, they might choose not to involve negative numbers. It cannot be easy to motivate students to solve problems using negative numbers which are new to them when they can easily solve problems which are comfortable. The main interest of this research is to explore how students make sense of negative numbers when they appear as part of the school mathematics.

Many textbooks are visual representation of number line, scale, a time line and everyday life presentations such as temperatures or money to explain subtraction with negative numbers. Most commonly such representations are referred to as models. Thomaidis (1993) claims that “… the various concrete models employed are not convincing enough for the necessity of these numbers. Students know quite well that they can work out the difference between two temperatures or determine the position of a moving point on an axis without having to resort to the operations between negative numbers. To discover more about that role and what mathematically develop these models afford or constrains is an important research interest.

2.3 Contributing factors

As a result of much discussion, the following is a list of some of the problems which may contribute to a student’s lack of understanding of the concepts involved in manipulating directed numbers.

- Students have very few real life experiences of negative values. For example, they do not always experience temperature of -10˚c, in winter season where temperatures may drop, although they may know how cold the inside of the refrigerator is… Prather & Alibabi (2008).
- We rarely write fifteen as +15, this is a new concept which students must become familiar with when dealing with directed numbers by Stacey (2001).
- Stacey & McGregor (1997) also noted that, notations cause problems, that is, when we have a situation of – (-) which means we must add or -1-2 where students are required to add the numbers and put the common sign.
Best & Khan (1993) postulate that students experience problems ordering negative numbers and usually they are not in a good position to judge which one is greater than the other, suppose they are given the following numbers -1;-5;-4 and -8.

Hamilton in Beery et al (2004), Students experience problems handling addition and subtraction of positive and negative of directed numbers, multiplication and division generally causes less confusion.

Students often confuses the rules, and will approach a question such as (-3)+(-5) as two negatives which makes a positive and give the answer as +8 of which the correct answer is +2, Human & Murray (1987).

2.4 Conceptualizing negative numbers and zero

According to Resnick (1989) interpretation of the magnitude and direction of negative numbers in the mind of students is the most important stage in learning the concept of negative numbers. When existence of negative numbers have been accepted, many question arise about the properties of these numbers, one of the common misinterpretations which was discovered by Stacey et al (2001) was that sometimes decimals associated with negative numbers were conceived as smaller than zero. The confusion between decimals, fractions and negative numbers proved to be so common and proposed as an explanation that the natural numbers are the primary elements from which the concept of other numbers are constructed. Stacey et al (2001) also observed that the confusion is as a result of a student merging the different targets of the same feature of the mirror metaphor under different analogical mappings. A mental number line is an important feature in the conceptualization of negative numbers. Fischer (2003) argued that the effect has been taken as evidence for an existing mental number line where numbers are ordered left to right and increasing to the right. While other researchers have investigated if the mental number line extends to the left of zero then the extended number line results have been somewhat contradictory. According to Fischer and Ratmann (2005) negative numbers differ from the positive numbers in that they are not automatically associated with space.

There are two forms of number line representation in the historical development described by Glaesar (1981). He claimed that one of the big obstacles that have a need to overcome is the unification of the number line, that is, to see it as one line, one axis, instead of two semi lines opposite one another with different symbols, understanding positive and negative quantities as having different quality. These two versions of number lines were also found by Peter et al (1989) among pupils in grades 1; 3; 5; 7 and form 1 and 2.

For positive numbers these size properties coincide, but for negative numbers they diverge. Ball (1993) also writes that simultaneously understanding that -5 is in one sense, more than -1 and in another sense, less than -1 is at the heart of understanding negative numbers. In ordinary use of language, we have different words to distinguish direction. We say, for example, temperature decreased or increased we do not normally say temperature increased by -5°. Using a double
language is by Bruno and Martinon (1999) taken as an important test in developing knowledge. They concluded that the meaning assigned to these operations is controlled by students’ prior knowledge for positive numbers.

Subtraction situations with natural numbers are characterized in terms of take away, combine and compare Fuson (1992). Often take away situations dominate in early Algebra education and Fuson (1992) writes considerations of the full range of addition and subtraction situations which requires an extension to the integer. Vergnauds (1982) analysis indicates that integers need to enter into the classroom mathematical discourse at an earlier stage to facilitate working with compare and combine situations. In a learning study where teachers collaborated in the planning and revising of a lesson on negative number they found out that switching from the take away situations to compare situations was an aspect of understanding negative numbers Kullberg (2010).

2.5 Making a simple problem complicated

Symbolic representations should help students solve problems they would otherwise fail to solve. People tend to avoid negative numbers in their daily lives if they can. When students are presented with a problem to solve it would therefore not come as a surprise if they avoid negative numbers if they can, this was given by Apartimark and Ozdoan (2010).

Bruno and Martinon (1999) found out that students often first solve a problem on the number line and then look for a calculation when the results match the results previously obtained on the number line. Galbraith (1974) stressed that the point that most models only partially represent is the phenomenon to be illustrated, so collection of such models must be used to illustrate different aspects of the problem. I see no reason why signed numbers cannot be introduced at an early stage. However the operations of subtraction, multiplication and division of integers are best approached not trying to extend our models to embody the operations. Also Lincheviski and William (1999 p 143) supported the idea that at least subtraction with negative numbers can be understood through models not one single model but a multiplicity of models, even if they do “acknowledge that multiplication and division may require a purely Algebraic approach”. For example, Kuchemann (1981) and Gallardo (1995) recommended that the number line should be abandoned in favor of discrete models where whole numbers represents objects of an opposing nature. While in contrary, Bruno and Martina (1999) found out that the number line became an indispensable tool when solving problems for the students in their experimental working with additive where the identification of addition with subtraction was a primary focus.

2.6 Empirical evidence

The current Algebra reform movement, Kaput (1995) stresses the importance of building upon students’ language learning abilities and their natural generalizing and abstracting powers in order to deepen and extend their reasoning about quantity number and space and uncertainty. Instead, in Algebra education we generally emphasize on procedures for symbol manipulation
and solving for the unknown. Our habits have been associated with weaknesses in students’ abilities to connect algebraic representations with life situations Mcoy (1994), Monk (1989) and Wolman (1983). These inabilitys may be behind difficulties that the American Citizenship in general was with relating high school algebra to everyday problem situations.

Issues regarding beneficial aspects of contextualized problems are of particular problem to this study. Carrahear and Schliemann (1987) found out that enhanced problem solving performances among problem contexts evoked ‘street know –how” rather than school learned algorithms. Lave (1988) postulates similar results with home makers. However, little is known about the particular aspects of some contextualized situations that facilitate the problem, likewise, we have little evidence that an approach that capitalizes on constructive experiences with contextualized experiences would result in learning to transfer to abstract and/or novel contextualized situations.

The results of the instructional intervention indicated positive learning gains as much as possible as consequences of the relatively short simulation analysis and simulation building project. These gains were limited however to story problems and to the best students. The effect on story problems only however is consistent with cognitive theory, for example, Singley and Anderson (1989) predict that transfer will be limited to fairly specific overlaps between instruction and target performance particularly contextualized instruction will improve decontextualised problem solving.

These results suggest caution in the implementation of mathematics reforms which sometimes have eliminated basic skills practice in the over-zealous pursuit of more meaningful real world problem solving. Greater emphasis on application is a welcome change for many classrooms that have traditionally –over emphasized on basic skills, however, finding the appropriate balance appears to be right and most difficult approach.

Gallardo (1995) pointed out that teachers should employ interesting techniques that motivate pupils to learn. He added that teachers should also provide pupils with numerous opportunities to work together to practice learning such as group work and pair work so as to develop concepts, discuss ideas and produce quality products.

2.7 Summary.

This chapter reviewed literature on what other authorities are saying about the research problem, the literature was divided into subections that were derived from the research topic. The research is intended to close the gap left by other researchers on the research problem by reviewing their reviews about the problem.
CHAPTER 3: RESEARCH METHODOLOGY

3.0 Introduction

This chapter focuses on the research design, the population and sample, sampling, research instruments, data collection methods and analysis that the researcher intends to use in this research. It describes the research methodology used by the researcher in carrying out the research and in collecting data.

3.1 Research Design

The research design is the general plan that outlines the procedures of the research. The theme of the research is to investigate the understanding of pupils in the teaching and learning of directed numbers. A case study at a secondary school for the research to obtain primary data, the action research is seen most appropriate. Best (1993) postulates that Research design is the study of social situation with the view of improving the quality of education. Hence, action research is carried out by practitioners in order to enhance effective classroom management skills and solving learning problems.

To investigate the conceptual challenges faced by ZJC pupils in dealing with directed numbers, the researcher used the action research design. An action research is a “systematic collection of information that is designed to bring about social change” (Bicklen, 1992). It aims at providing an immediate solution to problems at hand. The researcher chose action research as it enhances the development of new skills, concepts and solve problem with direct application to the classroom situation for the purpose of the study, the researcher will use teacher tailored objective tests.

The researcher also made use of case studies in investigating the conceptual challenges faced by ZJC pupils when dealing with directed numbers. Case studies often employ documents, artifacts, interviews and observations during the course of research. A case study is a research method which allows for in-depth examination of events, phenomena or other observation within real life context for purposes of investigating, theory development and testing, or simply as a tool for learning.

3.2.0 Population, Sample and Sampling Technique

3.2.1 Population

Best and Kahn (1993) view population as any group of individuals that have one or more characteristics in common that is of interest to the researcher, thus an indication that there should be some form of similarities among members of the population. The population for the study was the set of ZJC level form ones at a secondary school. A random selection of 12 pupils was chosen to represent 30% of the whole stream in the research.
3.2.2 Sample

A sample is a portion of the whole population that is actually selected for the study. Since the researcher cannot accurately enumerate characteristics of the whole population, Haralambos (1984) agrees that a sample is indeed a portion that is selected for the study or testing. It also implies that sampling has various functions such as enhancing accuracy, saving time and easy to manage.

The researcher used random sampling on form ones in selecting the sample group to partake in the study. According to Stimpson (2002) random sampling ensures each pupil has an equal chance of being selected for the sample. Random selection reduces bias so that results obtained gave a near-perfect reflection of the whole stream at the school.

3.3.0 Data Collection Instruments

These are tools the researcher used to gather information and the examples of these are test exercises and interviews. Since the researcher was exposed to, and interacted with the population throughout the whole research process, the researcher used pre-tests, post-tests and interviews to identify the various conceptual challenges that the ZJC form ones pupils face when dealing with directed numbers.

3.3.1 Tests

Deghton (1971:168) define a test as a “methodology for looking and recording teaching and student behavior in the classroom precise, relatively unbiased manner.” This gives an impression that tests solicit understanding in behavior after learning a certain concept. The instruments are basically research tools used in gathering data or collecting required information. The researcher made use of two instruments the pre-test and post-tests and interviews.

3.3.2 Pre-test

It is a test given before the pupils have been given lessons on a particular topic. Kasambira (1997) suggests that “pre-test enables teachers to get an extensive picture of depth and scope of pupil’s under this study before any intervention strategy was conducted.

3.3.3 Post-test

It is a test given after pupils had been given lessons or after revising the pre-test. It requires pupils to apply what they have learnt. Kasambira (1997) went on to say that post-test are intended to find out how much well a pupil can perform a specific skill after instruments. After the pre-test the researcher used pupil’s marks to identify their problems and take necessary strategies to assist pupils to improve on their performance.

3.3.4 Advantages of pre-test and post-test
1. They assist the researcher to analyze the effectiveness of strategies implemented
2. Most pupils are not aware that they are being observed therefore they perform naturally. Kasambira (1997)
3. Marks are well recorded and the researcher will also be able to analyze pupils’ marks to interpret the effectiveness of teaching and learning media and other intervention strategies

3.3.5 Disadvantages of pre-test and post-test

1. Some pupils may lack interest when it comes to writing
2. Pupils can also copy from each other when writing the test

3.3.6 Interviews

It may be verbal or written questions and answers between interviewer and interviewee to get or acquire more information regarding the phenomenon under study. Written interview shall be a data collection technique for this study. It leaves evidence of field work for future references.

Haralambos (1984) maintains that written interviews are a cheaper and easier way of gathering data than oral interview. The researcher also interviewed pupils.

3.3.7 Advantages of Interviews

1. It leaves evidence of fieldwork for future references. Bans and Gail (1983) states that interview can explain more on the purpose of the study
2. It affords a high degree of flexibility to the researcher since the interviewer can clarify questions and answers not clearly understood. Farrant (1990:13) asserts that “…………flexibility of interviews allows the researcher to rephrase or explain his question….” Interviews are flexible.
3. It permits even exchange of ideas and information given
4. It is a faster way of obtaining data

3.3.8 Disadvantages of Interviews

1. Interviews may give a false information due to some pupils which may fail to understand the demand of the question
2. Some of the pupils and teachers may have positive attitudes towards the interviews and give wrong answers to the questions and it affects the research negatively

3.4 Intervention strategies
3.4.1 Lessons

The pupils were given a pre-test which helped the researcher to check their level of understanding of the content regarded as assumed knowledge when teaching the concept of addition, subtraction, multiplication and division of directed numbers. The pre-test acted as an indicator on pupils’ problematic areas, where they needed help and clarification. It also acted as a compass to the researcher on which direction to take to help the pupils in understanding the concept of simplifying directed numbers expressions.

3.4.1.2 Intervention lesson one:

Addition and Subtraction of directed numbers

For the pupils to be able to simplify or evaluate problems involving addition and subtraction of directed numbers they are to be familiar with the number line. The objective of the lesson was to familiarizing the pupils with addition and subtraction of directed numbers using the number line and be able to use it when simplifying or evaluate directed numbers especially those involving negatives. Pupils were put in groups of three threes to solve tasks on addition and subtraction of directed numbers using the number line. The objective of this task was achieved in this encounter.

The researcher went on to explain the addition and subtraction rules of directed numbers. The same tasks were given again to the pupils in their respective groups and were asked to use addition and subtraction rules of directed numbers. The objective of the lesson was also achieved in this encounter.

Most pupils were failing to use the rules to simplify tasks given, though able to manipulate directed numbers and use of the number line very well, hence the researcher had to embark on another strategy to make these pupils understand the use of rules when dealing with addition and subtraction of directed numbers.

3.4.1.3 Intervention lessons two:

Multiplication and division of directed numbers

The main objective of the lesson was to help pupils to be able to multiply or divide expressions involving directed numbers. As an introduction, the researcher gave pupils some tasks involving simple multiplication and division ……for example (-2 X 3) and (-9 ÷ -3). This was used to help pupils to be able to manipulate multiplication and division of directed numbers. The researcher went on to give and explain multiplication and division rules of directed numbers.

3.4.1.4 Extra work and Remediation
As some pupils still had some challenges on the concepts, the researcher had to give them extra work for more practice of the addition, subtraction, multiplication and division of directed numbers. Remedial work was given to two pupils who needed one on one approach.

Pupils were given post-test to review how the lessons helped them on acquisition of concepts.

3.5 Data collection procedures

The researcher use pre, post tests, Interviews and the t-test to carry out the proposed research process.

3.5.1 Tests

Pupils were given tests (pre and post) to identify the problem area that are challenges in dealing with directed numbers. Deghton (1971) define a test as a” methodology for looking and recording teaching and student behavior in the classroom precise, relatively unbiased manner”. This gives an impression that tests solicit understanding in behavior after learning a certain concept. The instruments are basically research tools used in gathering data or collecting required information. The researcher made use of two instruments pre-test and post-test and interviews.

3.5.2 Interviews

It may be verbal or written questions and answers between interviewer and interviewee to get or acquire more information regarding the phenomenon under study. Written interview shall be a data collection technique for this study. It leaves evidence of field work for future references.

Haralambos (1984) maintains that written interviews are a cheaper and easier way of gathering data than oral interview. The researcher interviewed pupils at a Secondary school questions on addition, subtraction, multiplication and division of directed numbers.

3.6 Data presentation and analysis procedure

The researcher used Microsoft word and Excel as software packages used in presentation and analysis of collected data during the research. The collected data is presented in form of tables, frequency polygon, a column table and a t-test as the researcher inputted all marks obtained by pupils in all the two research tests administered then using commands calculated percentages, and also used the software package to produce frequency polygon and a column table.

3.7 Ethical Considerations

The results of the personal data collected were to be held on high confidentiality by the researcher in order to protect the child respondent. All the necessary procedures were followed
by the researcher as he pursued the research project. The names of all persons who participated in the research do not appear anywhere in the project.

3.8 Summary

The chapter focused on research design, the population samples, the sampling procedure used by the researcher, also the choice of instrument used, data gathering used and statistical treatment that the gathered statistic shall be put through.
CHAPTER 4 : DATA PRESENTATION, ANALYSIS AND DISCUSSION

4.0 Introduction

In the previous chapter, the researcher looked at population, sampling and instruments of collecting data. The researcher also highlighted how she used her samples and instruments to come up with unbiased data and its trends. This chapter will focus on data presentation, analysis and interpretation of the data collected in chapter three.

4.1 Presentation of data and analysis

Data is analyzable using Best and Kahn’s (1993) strategy of developing explanation of events so that theories, reasons and processes of any behavior are assessed. It is an analytical way of perceiving given information in order to obtain a certain meaning.

4.2 Interview Results

The researcher had an opportunity to interview seven pupils and obtained the following information.

One student from the seven interviewees said she fails to understand operations of directed numbers and the number line. When asked by the researcher what really is your challenge on the use of the number line and directed numbers and whether she understands the functions of the number line?, she had to reply…”I don’t even know how to number the number line on negatives and the direction to take when adding or subtracting…”The researcher had to note that as well, most pupils they have challenges in understanding the interpretations and functions of the number line. This was in support of Ball (1993) said….he writes that simultaneously understanding that -5 is in one sense more than -1 and less than -1 is at the heart of understanding negative numbers and the number line.

A mental number line is an important feature in the conceptualization of negative numbers as argued by Fisher that, the effect has been taken as evidence for an existing mental number line where numbers are ordered left to right and increasing to the right. From this argument, the researcher can safely conclude that -5 are less than -1.

Three of the seven pupils interviewed had to give the following information when asked by the researcher if they encounter any challenge in handling directed numbers and whether they understand the operations of directed numbers when simplifying or evaluating expressions involving directed numbers. One of the three had to say….” I don’t understand why the positive numbers have to be read with their sign….that confuses me the most….“ His argument was that “if I am asked 2 + 3 I am able to give the correct answer which is 5, but if I am to be asked the same question with sign involved , thus, +2 + (+3)…..I take time to answer or even give a wrong answer because of the signs involved.” Galbraith (1974) postulates that “I see no reason why signed numbers cannot be introduced at an early stage so as to avoid this confusion with
learners.” The researcher also supports this point as it will reduce the confusion with learners as numbers with signs are only introduced at form 1 when pupils are covering the topic directed numbers. One of the pupils interviewed had to say that.” what confuses me is adding a negative number to a negative number…and also adding a positive number to a negative number….I don’t understand that.” Apartimark and Ozdoan (2010) concluded that people tend to avoid negative numbers in their daily lives if they can. When students are presented with a problem to solve, it would therefore not come as a surprise if they avoid negative numbers if they can.

The other three pupils from the seven interviewed had to say they face difficulties in multiplying a negative number and another negative. One of them had to say “to me if given two negatives and be asked to multiply the result will be a negative….why does the answer became a positive when I am only dealing with negatives?

The researcher also managed to interview one mathematics teacher. The teacher had to say that the major challenge faced by students at ZJC pupils is to add the notion that there exist negative values. Thus, the introduction of negative numbers is a challenge as pupils initially fail to write down negative numbers in descending order. Furthermore, pupils tend to think that if two different natural numbers are given negative sign, the value of the bigger natural number will become the smaller number. The multiplication and division of negative numbers has proved to be a challenge.

The teacher in his interview went on to say that, one strategy that can be used to introduce negative numbers is the use of visual aids. A chart can be drawn with both negative and positive numbers and pinned in front of the classroom just up the chalk board, where the pupils can see and use on a daily basis. He continued to say that these pupils who fail to write directed numbers in ascending order, the teacher can instruct the pupils before writing any exercise to start by writing numbers \(-10 \leq x \leq +10\).

The teacher in his interview also highlighted that real life examples can also be given to pupils, for example, that of money, where the money one owes being the negative the one received being positive. The pupils are then asked to add the money they owe to the money they received. He added that charts can also be used for pupils to see what happens when negatives are multiplied by another negative or positive number.

4.3 Pre-test

Following the discovery of the problem faced by pupils of the same level of understanding as the chosen population, the researcher administered a pre-test to twelve pupils who were randomly sampled from 1A and B on addition, subtraction, multiplication and division with directed numbers. The results are summarized by the frequency table 4.3.

Table 4.3: Frequency distribution for the pre-test
The table 4.3 shows the score in percentages obtained from the pre-test by the pupils as well as the number of pupils in each interval. Three pupils got marks between 0 and 24, seven pupils got marks between 25 and 49, two pupils got marks between 50 and 74 and none of them got marks between 75 and 100. The results from the pre test are summarized on a Frequency polygon

**Frequency polygon 4.3 showing results from the pre-test:**

Using the class centre, the average score/mark for the group was:

\[
\mu = \frac{\sum f(x)}{\sum f} = \frac{12(3) + 37(7) + 62(2) + 87.5(0)}{12} = \frac{36 + 259 + 124}{12} = \frac{419}{12} = 34.91666667 \\
= 35\%
\]

The results from the pre test indicated that indeed these learners were encountering conceptual challenges in handling directed numbers. The first challenge that the researcher had noted was the fact that most of these pupils failed to arrange directed numbers in ascending order. The first question of the test was to arrange directed numbers in their ascending order and the numbers were: a) -6; -2; -7; -1; -4

b) 0; -2; 1; -5; 3. One student had to react as follows: “a) -1; -2; -4; -6; -7
b) 3; 1; 0; -2; -5”. This really indicates that this pupil does not know or understand the positioning of numbers on the number line; to him signs are not important but the numbers on his answer on part a). But on part b) he went on to confuse himself more as he indicated that negative numbers are greater than positive numbers. Best and Khan (1993) also argued that some students experience problems ordering negative numbers and usually they are not in a good position to judge which one is greater than the other.

Secondly, on addition and subtraction, most pupils are still facing challenges even after the intervention lessons. One pupil after being asked to simplify $-5 - 6$, she gave the following answer…” $-5 - 6 = -11”$, This is in support of what has been said by Vlassis (2002) that many errors made when simplifying directed numbers are caused by the presence of negative numbers and concludes that it is the degree of abstraction created by the negative that creates these difficulties rather than the presence of positive numbers or the structure of the expression. This conclusion supports earlier findings suggesting that success in dealing with directed numbers may depend on part of structural understanding of the relationship of addition and subtraction of directed numbers.

Thirdly, on questions requiring multiplication and division of directed numbers, one pupil had to give the following answers, I quote, “a)… (-7) $\times$ (-3) = - 21………b) 33 $\div$ -3 = 11”. This indicates that this pupil had not understood the rules of multiplying and diving directed numbers. Kaput (1995) argued that the importance of building upon students’ learning abilities and their natural generalization of concepts is to deepen knowledge about directed numbers and extend their reasoning about quantity number, space and uncertainty.

Due to the fact that most students scored less than half and that the average score for the group was also below half mark, the researcher conducted two intervention lessons and one remedial so as to try to mitigate these conceptual challenges that these learners face in dealing with addition, subtraction, multiplication and division of directed numbers. The intervention lessons were done during spare time after normal school periods. The researcher used the following approaches:

I) Directed numbers and the number line

ii) Addition and Subtraction rules with directed numbers

iii) Multiplication and division rules with directed numbers

After the intervention lessons the researcher gave a post test and the results are shown and summarized in table 4.4.

**4.4 Post- test**

The researcher administered the post-test and the results are shown on frequency table 4.4 and the column table 4.4.
Table 4.4: Post-test results

<table>
<thead>
<tr>
<th>Score (%)</th>
<th>0 - 24</th>
<th>25 - 49</th>
<th>50 - 74</th>
<th>75 - 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>0</td>
<td>0</td>
<td>11</td>
<td>1</td>
</tr>
<tr>
<td>Percentage</td>
<td>0</td>
<td>0</td>
<td>92</td>
<td>8</td>
</tr>
</tbody>
</table>

The table shows the results after the intervention lessons on directed numbers were conducted. The table shows scores percentages and the frequencies. None of the pupils got marks between 0 and 49, eleven fell between 50 and 74 and only one got marks between 75 and 100. Results of the post test were also summarized on the column table 4.4. There are no pupils with marks between 0 and 24 and also between 25 and 49. There are 11 pupils who scored marks between 50 and 74 and only one pupil scored a mark between 75 and 100. Only those with their marks in the intervals 50-74 and 75-100 will be able to be shown on the column chart and their percentages are shown as well.

The mean mark for the post-test is as follows:

\[ \mu = \frac{\sum f(x)}{f(x)} \]

\[ = \frac{12(0) + 37(0) + 62(11) + 87.5(1)}{12} \]

\[ = \frac{682 + 87.5}{12} \]

\[ = \frac{769.5}{12} \]

\[ = 64.125 \]
4.5 Hypothesis test.

Table 4.5: Comparison between pre-test and post-test for individual marks.

<table>
<thead>
<tr>
<th>Student number</th>
<th>Pre-test</th>
<th>Post-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>12</td>
<td>8</td>
<td>11</td>
</tr>
</tbody>
</table>

The table 4.5 shows comparison between the pre-test and the post-test of each individual in the sample. All the pupils showed improvement from the pre-test to the post test on the comparison table 4.5.

A t-test is going to be done to test if there are significance difference the two tests. The researcher decided to conduct a t-test because the sample size was too small; it consisted of only 12 pupils and also that each pupil has a score from the pre-test and from the post-test after they have been taught by the researcher for two lessons and a remedial, which the researcher termed the intervention lessons.

**Calculations for \( t_{crit} \)**

\( H_0 \) : There was no significance change in performance after the intervention lessons.

\( H_1 \) : There was a significance change in performance after the intervention lessons.

\( \alpha = 5\% = 0.05 \)

\( \frac{\alpha}{2} = \frac{0.05}{2} = 0.025 \) (two tailed)

\( d.f = n - 1 = 12 - 1 = 11 \)

\( t_{crit} = 2.201 = 2.20 \)
Decision:

Reject $H_0$ when $t_{\text{stat}} > 2.20$

Table 4.6: Calculations for $t_{\text{stat}}$

<table>
<thead>
<tr>
<th>Student number</th>
<th>Pre-test mark</th>
<th>Post-test mark</th>
<th>$d$</th>
<th>$d^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>13</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>10</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>9</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>9</td>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>9</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>9</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>10</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>11</td>
<td>9</td>
<td>81</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>9</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>9</td>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
<td>10</td>
<td>7</td>
<td>49</td>
</tr>
<tr>
<td>12</td>
<td>8</td>
<td>11</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>55</strong></td>
<td><strong>297</strong></td>
</tr>
</tbody>
</table>

NB: ‘$d$ ‘in the table stands for the difference between the scores before and after intervention lessons were conducted by the researcher.

$$\bar{d} = \frac{\sum d}{n}$$

$$= \frac{55}{12}$$

$$= 4.583$$

$$t_{\text{stat}} = \frac{\bar{d}}{\sqrt{\frac{\sum d^2 - (\sum d)^2}{n(n-1)}}}$$

$$= 4.583 \div \sqrt{\frac{297 - 55^2}{12}}$$

$$= 4.583 \div \sqrt{\frac{297 - 252.083}{132}}$$

$$= 4.583 \div \sqrt{\frac{44.917}{132}}$$
\[ \frac{4.583}{0.583} = 7.86 \]

**Conclusion:**

Since \( t_{stat} > 2.20 (t_{crit}) \), the researcher reject \( H_0 \) and conclude that at 5% significance level, there was a significant change in the performance of the pupils after the intervention lessons were administered to the pupils.

### 4.6 Discussion of findings

Results from table 4.2 reflect that some of the pupils do not understand operations of directed numbers and the number line. Out of the seven interviewed only one indicated that she/he is still finding it difficult to understand mathematical operations of directed numbers and the number line. This was supported by Bruno & Martinon (1999) as they found out that most students often first solve a problem on number line, for instance \(-5 + 2\), using addition and subtraction rules and then try to use the number line lastly to check if the answers match. Human & Murray (1987) had to also say, students often confuse the rules and the number line and will approach a question such as \((-3) - (-5)\) as two negatives and moves 5 steps to the left from -3 and gives the answer as -8 of which the correct answer is +2.

The majority of the pupils are still finding it difficult to deal with addition, subtraction, multiplication and division of directed numbers as indicated by those who were interviewed by the researcher. This was supported by Hamilton in Beery et al (2004) as he postulates that, students experience problems handling addition and subtraction of positive and negative directed numbers, multiplication and division generally causes less confusion. From the pre-test, one student had to give the following answer when asked to simplify \(-5 + 16\), “\(-5 - 16 = 1\)”. This really shows that the student did not understand that \(- - = +\), then the expression will be \(-5 + 16 = 11\).

Table 4.3 shows that majority of the ZJC pupils, detected from the sample are facing conceptual challenges in dealing with directed numbers. From Table 4.3, ten out of twelve pupils selected for the study, had their pre-test marks less than 50%. This was a great indicator of how difficult it was for these juniors to evaluate or simplify expressions with directed numbers that involve addition, subtraction, multiplication and division. The researcher also noted that these ZJC minors are still failing to understand simplification and evaluation of simple mathematical expressions combining two or more mathematical operations. The average mark of the sample group for the pre-test was 35%. Resnick (1998) argued that the interpretation of the magnitude and direction of negative numbers in the mind of students is the most important stage in learning the concept of negative numbers. Stacey (2001) also observed that the confusion is as a result of
a student merging the different targets of the same feature of the mirror metaphor under different analogical mappings. A mental number line is an important feature in the conceptualization of negative number.

The results shown by Table 4.4 indicated that there was a great improvement after the intervention lessons conducted as this may have led to pupils scoring very high marks in the post test given. The conclusions were reached at as the researcher conducted a hypothesis test from the Table 4.5, for comparison of the pre-test and the post-test, to check whether there was a significant change in results from the pre and the post test and indeed the results from the test indicated that there was a great improvement in the tests given after the intervention lessons were conducted to help pupils mitigate the challenges they face when dealing with directed numbers. Despite the challenges of not having enough resources to vary teaching methodologies and learning media, the pupils reflected that they have understood the concepts well and indeed managed to curb some of the challenges they were facing. All the pupils in the sample class managed to score marks above 49%. This translated the average mark to 64% which was way above the expected mark 50%. The researcher also managed to interview one mathematics teacher. The teacher had to say that the major challenge faced by students at ZJC pupils is to add the notion that there exist negative values. Thus, the introduction of negative numbers is a challenge as pupils initially fail to write down negative numbers in descending order. Furthermore, pupils tend to think that if two different natural numbers are given negative sign, the value of the bigger natural number will become the smaller number. The multiplication and division of negative numbers has proved to be a challenge.

The teacher in his interview went on to say that, one strategy that can be used to introduce negative numbers is the use of visual aids. A chart can be drawn with both negative and positive numbers and pinned in front of the classroom just up the chalk board, where the pupils can see and use on a daily basis. He continued to say that these pupils who fail to write directed numbers in ascending order, the teacher can instruct the pupils before writing any exercise to start by writing numbers \(-10 \leq x \leq +10\).

The teacher in his interview also highlighted that real life examples can also be given to pupils, for example, that of money, where the money one owes being the negative the one received being positive. The pupils are then asked to add the money they owe to the money they received. He added that charts can also be used for pupils to see what happens when negatives are multiplied by another negative or positive number.
4.6 Summary

This chapter focused on presenting data collected from the previous chapter. Analysis and discussion on the results was carried out and the data was presented on tables and pie chart. In the next chapter, the researcher will mainly focus on the overall reflection, findings of the study and as well as recommendations.
CHAPTER 5: SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.0 Introduction

The previous chapter focused on data presentation and analysis. There was also discussion and interpretation of themes and patterns arising from the data. This chapter also seeks to find a solution and answers to the statement of the problem. It is also going to look at what can be done by the teachers and pupils to reduce the conceptual challenges regarding directed numbers.

5.1 Summary

This chapter focused on the reflections on the findings of the study and recommendations made by the researcher for those who might wish to further study this line study on the conceptual challenges faced by ZJC pupils in dealing with directed numbers. The constraints that the researcher faced was to only manage to interview one mathematics teacher and that the sample was too small such that the researcher had to use a hypothesis test (paired t-test) to validate the results of the comparison between the pre-test and the post test.

5.2 Conclusions on findings

From this research, the researcher noted that, with ZJC pupils at a secondary school, there were challenges on grasping the concept and rules of operations with directed numbers. Pupils did not grasp well the rules of addition and subtraction with directed numbers and also multiplication and division with directed numbers. The researcher also depicted that there is still lack of order of operations with directed numbers at Form One since the pupils will be doing it for the first time meaning more time is needed when introducing this concept at form one but due to less time allocated to Mathematics, it becomes difficult to exhaust such topics.

Fig 4.4 reflects a great improvement after the intervention lessons which means that, for these minors to grasp concepts well, there is need for variation of teaching and learning aids. Lecture method is frequently used and learning aids are not usually used by Mathematics teachers, thus pupils might have lost interest leading to poor performance on the pre-test and low participation. For pupils to avoid facing conceptual challenges in dealing with directed numbers there is need to introduce student centered teaching methods, for example, the guided discovery method which allows pupils to discover concepts on their own.

The researcher managed to tackle the conceptual challenges pupils face through the use of live teaching methods such as group work, demonstration and discovery methods, and also through the use of effective use of learning aids such as group work cards and wall charts to explain concepts clearly so as to impact the interest among the pupils.
This indicates that the use of different teaching approaches and learning aids are best ways in avoiding conceptual challenges faced by ZJC pupils in dealing with directed numbers.

5.3 Recommendations

Best on the findings of this research, the researcher recommends classroom practitioners to check for the pre-requisite skills before teaching a topic and then incorporate a wide variety of teaching strategies that address different learning styles as well as incorporating more learning aids. This will help pupils to make connections from prior learning experiences to new learning across disciplines and be able to cater for their different learning needs. Individual differences are supposed to be catered for as well.

The researcher recommends that classroom practitioners should find out what pupils know and what information do they have about the concept they will be teaching at that particular time and build the structure from there. Where no structures exist, a teacher should create a good environment for the new information to avoid conceptual challenges. This will help pupils to learn from the known to the unknown as recommended by Piaget. Understanding should not be compromised at the expense of completing the syllabus.

The use of various teaching aids is recommended and highly encouraged. Kasambira (1993) says lessons that are not accommodated with learning aids and models results in de-motivation and boredom of pupils and thus progress of learning is reduced.

Teachers should also employ interesting techniques that motivate pupils to learn more. The researcher encourages teachers to provide pupils with numerous opportunities to work together to practice learning such as group work and pair work so as to develop concepts, discuss ideas and produce quality products such as was the case on her intervention. This means that practitioners should encourage pupils to discover a solution to a given problem on their own than being spoon-fed on how to solve the problem. Pupils should be directly involved rather than being passive observers in the learning process so as to increase motivation and enhance active participation of the pupils especially when simplifying directed numbers. At all times, a supportive and nurturing classroom is important.

For teachers to use these learning aids, they should be having them. Therefore, the schools are encouraged to purchase enough and adequate resources such as manila, mighty markers, glue and some other required resources for teachers to use for better learning of pupils. The researcher also recommends the teachers to employ the strategies acquired from the teacher who was interviewed. The teacher in his interview went on to say that, one strategy that can be used to introduce negative numbers is the use of visual aids. A chart can be drawn with both negative and positive numbers and pinned in front of the classroom just up the chalk board, where the pupils can see and use on a daily basis. He continued to say that these pupils who fail to write directed numbers in ascending order, the teacher can instruct the pupils before writing any exercise to start by writing numbers $-10 \leq x \leq +10$.
The teacher in his interview also highlighted that real life examples can also be given to pupils, for example, that of money, where the money one owes being the negative the one received being positive. The pupils are then asked to add the money they owe to the money they received. He added that charts can also be used for pupils to see what happens when negatives are multiplied by another negative or positive number.

Finally, the researcher encourages all stakeholders in Mathematics fraternity to collaborate on the problems, give enough support and guidance to solve those problems. Education officers for the subject, Mathematics, should organize workshops for subject teachers regularly to staff develop them on how to approach various mathematics topics, especially directed numbers in simplifying and evaluating directed numbers. This can be done at provincial, district and/or at cluster level.

The researcher recommends that teachers should use Bloom’s taxonomy levels when teaching mathematics and use Piaget’s cognitive theory of learning. This means that teachers should start from knowledge questions, comprehension, application, synthesis, analysis and trial to questions of higher order that need evaluation to study deeply.
APPENDICES

Appendix 1

References

1. Arcavi, A & Bruchheimer, M (1983); the negative numbers, Rehovat: Israel, the Weizmann institute of science.
11. Fischer M and Rottmann J (2005); Do negative numbers have a place on the number line? Psychology science, 47(I), 22-32: New York.
12. Fischer, M (2003); Cognitive presentation of negative numbers, Psychological science, 14(3), 278-282.
17. Linchevski, M and Stacey K (1999); Educational studies in Mathematics, 39, 173-147
Appendix 2

Pre-test/ Post-test (All questions carry 1 mark each)

1. Write the following numbers in their ascending order:
   a) -6; -2; -7; -1; -4.
   b) 0; -2; 1; -5; 3.

2. Evaluate the following expressions as far as possible:
   a) -6 + 5
   b) -1 -1
   c) -2 -13
   d) -5 - -16
   e) 3 - 7
   f) -181 + 97
   g) 2 + (-36) + (-43)

3. Evaluate the following expressions as far as possible:
   a) (-5) X 3
   b) (-7) X (-3)
   c) 33 ÷ (-3)
   d) (-16) ÷ (-4)
   e) \[\frac{-100}{4}\]
   f) \[\frac{-40}{-2}\]
Appendix 3

Interview for teachers

Interview

1: What is your name sir?

2: For how long have you been teaching mathematics?

3: Do you teach ZJC pupils?

4: What is your comment on conceptual challenges faced by ZJC pupils’ in dealing with directed numbers?

5: What strategies can be implemented to mitigate these challenges pupils’ face?
Appendix 4

Interview for the pupils

Interview

Introduction

I am a final year in Bachelor of Science Education student at Bindura University of science education carrying out a study on the conceptual challenges faced by ZJC pupils in dealing with directed numbers.

1. What is your name?
2. What form are you doing and how old are you?
3. Do you have a Mathematics teacher in your form?
4. If yes, did you understand the teacher when he/she was teaching directed numbers?
5. Did you encounter any challenge in handling directed numbers?
6. What was your real challenge in that area?
7. How best can we help you in that area?
8. Have you ever used the number line? Do you understand how it functions?
9. How do you understand the operations of directed numbers when evaluating expressions involving directed numbers?

Thank you for your cooperation and time.