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A RESEARCH PROJECT SUBMITTED TO THE DEPARTMENT OF SCIENCE AND MATHEMATICS

By

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**TOPIC: AN INVESTIGATION INTO THE ERRORS MADE BY ORDINARY SCHOOL LEVEL
LEARNERS IN SOLVING MATHEMATICAL PROBLEMS INVOLVING LAWS OF INDICES.**

**BERNARD MIZEKI
COLLEGE.**

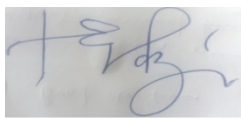
**A DISSERTATION SUBMITTED IN PARTIAL FULFILMENT OF THE
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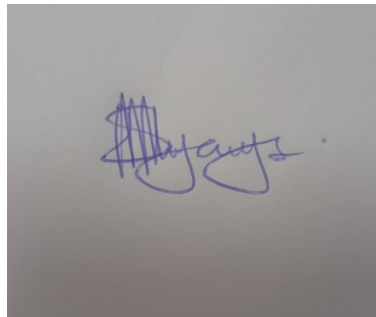
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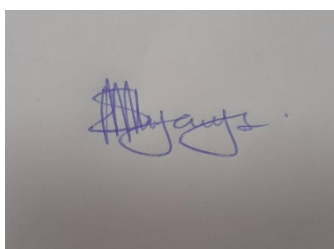
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DEDICATION

To my family, who have been my pillar of strength and inspiration throughout this research journey.

Your unwavering support, encouragement and patience have meant the world to me.

To my friends and colleagues, who have provided valuable feedback, guidance and motivation. Your contributions have enriched this research and helped shape its outcome.

To the educators and learners at Bernard Mizeki College who have participated in this study , thank you for sharing your experiences , insights and perspectives. Your contributions have brought depth and meaning to the research.

This research is dedicated to all those who strive to mitigate the errors made by Ordinary school level learners in solving mathematical problems involving the laws of indices.

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ABSTRACT

Title: An investigation into the challenges faced by Ordinary school learners in solving problems involving the laws of indices. A case study at Bernard Mizeki College in Marondera District in Mashonaland East Province.

This study investigated the errors made in solving mathematical problems involving the laws of indices among Ordinary Level learners. A mixed methods approach was employed, combining both quantitative and qualitative data collection and analysis methods. A survey questionnaire was administered to two of the Mathematics teachers and also another questionnaire was administered to the learners.

The study recommends the use of different pedagogical techniques to try and make sure that every learner is involved in the lesson. It also highlights the need for teacher training and support, infrastructure improvement and continuous evaluation and improvements in the teaching methodologies.

Key words: Errors, laws of indices , Mathematics education, Ordinary School Level learners.

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CHAPTER 1: INTRODUCTION AND BACKGROUND

1.1 INTRODUCTION

Mathematics is universally recognized as a foundational subject, providing essential tools for logical reasoning, problem-solving, and advancement in science, technology, engineering, and mathematics (STEM) fields. At the Ordinary Level (O-Level) in the Zimbabwean education system, indices (also known as exponents) form a critical component of the algebraic curriculum. Mastery of indices is prerequisite for understanding more complex topics like logarithms, surds, and calculus at higher levels. Problems involving indices require not only numerical manipulation but also a solid conceptual grasp of the laws governing powers and roots.

Despite the fundamental importance of indices, anecdotal evidence from educators and recurring trends in examination results suggest that O-Level learners consistently face significant difficulties in solving problems related to this topic. These difficulties often manifest as systematic errors, procedural breakdowns, and a failure to apply the correct laws in varied problem-solving contexts. This failure to master the basic concepts of indices creates a substantial barrier to further academic progress in mathematics and related disciplines.

The study will use a mixed-methods approach, including quantitative and qualitative data collection methods. The quantitative data will be collected through a survey administered to Form 4 learners, and the qualitative data will be collected through interviews with learners and teachers. The data will be analysed to identify the most common errors made by learners, and to explore the factors that contribute to these errors. The findings of this study will have implications for mathematics education, and they will help to inform the development of instructional materials and teaching practices to improve learners' understanding of the laws of indices.

This chapter sets out the foundational framework for an investigation into the challenges faced by Ordinary school level learners at Bernard Mizeki College in Marondera when solving problems involving indices. It establishes the background, articulates the specific problem, outlines the study's objectives and questions, and defines the scope and significance of the research.

1.2 BACKGROUND OF THE STUDY

Bernard Mizeki College is a boarding school located in Marondera district in Mashonaland East Province. The researcher is a teacher at the aforementioned school, was prompted to carry out a research study on the form 4 class upon observing gross incompetence in Mathematics. The researcher's interest in conducting the study was stirred up by such anomalies as the drop out of learners at O' level. In most schools the pass rate for Mathematics is always low as compared to other subjects. High failure rate at schools had pushed the researcher to find out why learners fail to solve mathematical problems involving indices. Many learners are dropping out Mathematics at Ordinary Level, the dropout rate has forced the researcher to explore every avenue in search of reasons why learners drop out Mathematics. Young learners developed a negative attitude towards Mathematics whilst it is very important in the world.

This left the researcher with no option other than to investigate why? The study outcome is going to be of great importance to Mathematics teachers if analysed and implemented properly. The Zimbabwean mathematics curriculum emphasizes the importance of understanding and applying mathematical concepts, including indices. However, many learners struggle to grasp these concepts, leading to poor performance in mathematics examinations.

1.3 STATEMENT OF THE PROBLEM

Performance analysis in O-Level mathematics across various Zimbabwean schools, including Bernard Mizeki College, frequently highlights indices as an area of high error frequency and low learner attainment. While the existence of low scores is known, the specific nature, causes, and underlying pedagogical challenges contributing to this poor performance remain insufficiently documented and understood within the context of this specific institution. The lack of detailed insight into why learners struggle whether it is due to conceptual misunderstanding, failure to recall rules, procedural errors, poor application, or instructional methods, prevents the implementation of targeted and effective intervention strategies. Therefore, the problem this study addresses is the lack of empirical evidence and specific understanding of the core challenges O-Level learners at Bernard Mizeki College encounter

when solving problems involving indices, resulting in persistent learning gaps and suboptimal performance.

1.4 RESEARCH QUESTIONS

Based on the objectives outlined above, this study seeks to answer the following research questions:

1. What are the properties of indices covered at ordinary school level in the ZIMSEC syllabus 4576.
2. What are the common errors made by Ordinary school level learners at Bernard Mizeki College when solving problems involving indices?
3. What are the major factors that contribute to the challenges learners face in mastering indices?
4. How can the mistakes made by Form 4 learners at Bernard Mizeki College be minimised and improve their performance in solving problems involving indices?’

1.5 PURPOSE AND OBJECTIVES OF THE STUDY

The main purpose of this study is to investigate and identify the specific challenges faced by Ordinary school level learners in solving problems involving indices at Bernard Mizeki College.

The specific objectives of the study are:

1. To identify and categorise the common mistakes made by Form 4 learners when solving problems involving the laws of indices.
2. To determine the contributing factors, pedagogical, conceptual, or procedural that lead to these challenges.
3. To ascertain the learners perceived levels of difficulty and conceptual understanding of the topic of indices.
4. To find some possible remedies to the challenges faced by Form 4 learners in solving problems involving the laws of indices.

1.6 SIGNIFICANCE OF THE STUDY

This investigation holds significance for several stakeholders within the education sector:

- **Learners:** By identifying specific error patterns and conceptual roadblocks, the study will pave the way for targeted teaching interventions that directly address learners' weaknesses, ultimately leading to improved understanding and performance in mathematics.
- **Mathematics Teachers:** The findings will provide teachers at Bernard Mizeki College and other similar institutions with empirical data on the specific difficulties encountered by their students. This evidence can inform their teaching practices, allowing them to adjust instructional methods and place greater emphasis on known areas of conceptual struggle.
- **Curriculum Developers:** The results may highlight gaps or ambiguities in the O-Level Mathematics curriculum and textbooks regarding the topic of indices, providing valuable feedback for future curriculum review and development.
- **Educational Researchers:** The study contributes to the body of knowledge on mathematics education, specifically within the context of algebraic difficulty in Zimbabwean secondary schools, and can serve as a foundation for comparative or larger-scale research.

1.7 DELIMITATIONS AND SCOPE OF THE STUDY

The study is delimited by the following parameters:

- **Geographical Scope:** The investigation is strictly confined to a single educational institution, Bernard Mizeki College, located in Marondera, Zimbabwe.
- **Target Population:** The participants will be limited to Ordinary school level (O-Level) mathematics learners at Bernard Mizeki College.

- **Content Focus:** The research focuses exclusively on the challenges faced by learners in solving problems related to the mathematical topic of indices. Other algebraic or mathematical topics will not be covered.

1.8 LIMITATIONS OF THE STUDY

The following potential limitations should be noted:

- **Generalisability:** Due to the study being a specific case study confined to a single, potentially high-performing, boarding school (Bernard Mizeki College), the findings may not be directly generalizable to all O-Level learners in other schools or regions of Zimbabwe.
- **Participant Bias:** Learners might provide socially desirable responses during interviews or questionnaires regarding their study habits or perceived difficulties, potentially masking their true challenges.
- **Sample Size:** The number of participants selected from the O-Level cohort may not fully capture the diversity of challenges across the entire population, potentially impacting the comprehensiveness of the error analysis.

1.9 DEFINITION OF KEY TERMS

Indices (Exponents): In this study, indices refer to the mathematical notation used to indicate repeated multiplication of a number by itself. The notation involves a base and a power (or exponent), such as in a^n , where a is the base and n is the index.

Ordinary School Level Learners (O-Level Learners): Refers to secondary school students typically in Forms 3 and 4 (or equivalent) undertaking the prescribed syllabus for the Zimbabwe General Certificate of Education Ordinary Level Examination in Mathematics.

Challenges: In the context of this study, challenges refer to the systematic conceptual, procedural, and application difficulties, as well as the contributing factors (for example teaching methods, anxiety), that hinder the ability of O-Level learners to correctly solve problems involving indices.

Bernard Mizeki College: The specific secondary school located in Marondera, Zimbabwe, serving as the case study site for this investigation.

1.10 CONCLUSION

This chapter provides an introduction to the study, including the background, statement of the problem, research questions, objectives, significance, and delimitations. The study aims to investigate the challenges faced by Ordinary Level learners at Bernard Mizeki College in solving problems involving indices and suggest strategies that teachers can use to address these challenges. The next chapter will critically discuss literature related to the study.

CHAPTER 2: REVIEW OF RELATED LITERATURE

2.1 INTRODUCTION

This chapter reviews relevant theoretical and empirical literature related to the teaching and learning of mathematics, specifically focusing on algebraic concepts and the topic of indices. The review aims to establish a conceptual framework for the study, identify common student errors and instructional challenges documented globally and regionally, and highlight the existing research gap that this investigation seeks to address. The literature is structured around the theoretical underpinnings of mathematical understanding, the role of indices in the curriculum, and a detailed examination of common challenges faced by learners.

2.2 THEORETICAL FRAMEWORK

The current study is informed by two primary theoretical perspectives: Richard Skemp's theory of relational versus instrumental understanding, and Newman's Error Analysis framework.

2.2.1 SKEMP'S THEORY OF RELATIONAL AND INSTRUMENTAL UNDERSTANDING

Richard Skemp (1976) defined two types of mathematical understanding: instrumental understanding (knowing what to do, but not why) and relational understanding (knowing what to do and why).

- **Instrumental understanding** in indices involves rote memorization of the laws (e.g., $a^m \text{ times } a^n$) and applying them mechanically. While this allows learners to solve basic, routine problems, it often leads to failure when problems require modification, combination, or application of the laws in non-standard contexts, which is common in O-Level examinations.

- **Relational understanding** involves grasping the underlying structure and principles of the laws (e.g., understanding why $a^0 = 1$ *and not* 0). Literature suggests that learners who possess relational understanding are better equipped to tackle novel problems and correct their own errors, a critical requirement for success in advanced algebra. This study will use this framework to explore whether learner challenges stem from a pervasive over-reliance on instrumental rules.

. Conceptual Error: Exponentiation as Multiplication

Many learners suffer from a "detachment" of the index from the base, treating the exponent as a multiplier. This is often attributed to a lack of pre-conceptual thinking, the ability to see a mathematical symbol as both a process and a concept (Gray & Tall, 1994). This usually happens with the form four learners at Bernard Mizeki College where they are not able to separate the index from the base when dealing with problems involving the laws of indices. Most of them multiply the index as if they are multiplying the base number.

The Error: $3^2 = 6$ *or* $x^3 = 3x$. Learners often reduce exponentiation to repeated addition or simple multiplication because it is the most familiar schema available to them (Brodie & Berger, 2010).

The "Law Over-Generalization" Error

Learners often apply laws of indices to operations where they do not belong, particularly addition and subtraction.

The Error: $a^n + a^m = a^{(n + m)}$. This is known as structural error. Learners perceive the "pattern" of adding exponents and apply it indiscriminately to any expression containing indices, failing to recognize that the laws are operation-specific (Kaye, 1989).

Misconceptions of the Zero and Negative Index

The zero and negative indices are counter-intuitive because they cannot be easily explained through the "repeated multiplication" model.

Zero Index ($a^0 = 0$): Learners struggle to accept that "nothing" happening to a base results in a 1. The correct working and answer would be $a^0 = 1$. It is the learners' perception that anything to do with a zero especially when there is some multiplication taking place, should result in a zero. They take their experience with a term zero from their junior stages of education where the answer was going to be a zero most often.

Negative Index ($a^{-n} = -a^n$): Many learners view the negative sign in the exponent as a signal of the "sign" of the entire number. Instead of $a^{-n} = \frac{1}{a^n}$

These are "definition-based" errors. Learners rely on an informal definition of exponents (multiplying a number by itself, n, times), which fails when n is a zero or is a negative index. When the model fails, they revert to intuitive "rules of thumb" like "negative means negative" (Weber, 2002).

Coefficient-Base Confusion (The Distributive Error)

A common error involves failing to distinguish between a coefficient and a base when applying a power.

The Error: $(2x)^3 = 2x^3$

This is often linked to parsing errors, the inability to correctly group terms within an algebraic string. Learners may correctly apply the index law to the variable but "ignore" the coefficient due to a lack of understanding of the distributive property over multiplication (Cangelosi et al., 2013).

Transition Errors in Rational Indices

When moving from integer to rational (fractional) indices, learners often treat the fraction as a coefficient rather than a root.

The Error: $x^{\frac{1}{2}} = \frac{1}{2}x$

This is classified as an interference error. Previous knowledge of fractions as multipliers "interferes" with the new rule where the fraction represents a radical (Mustafa et al., 2017).

2.2.2 NEWMAN'S ERROR ANALYSIS (NEA)

Newman's Error Analysis (Newman, 1977) provides a diagnostic framework for categorizing student errors in written mathematical problems. Newman's Error Analysis is a framework for identifying the specific stages where students make mistakes in solving math word problems. By analysing errors at each stage, teachers can better understand a student's difficulties and provide targeted instruction. Developed by Anne Newman, it breaks down problem-solving into five stages: reading, comprehension, transformation, process skills, and encoding.

Reading Error: Inability to read the problem (for example difficulty reading symbols like the root sign or power fractions).

1. **Comprehension Error:** Inability to understand the overall meaning of the problem.
2. **Transformation Error:** Inability to select the appropriate mathematical operation or law (for example deciding which law of indices to apply).
3. **Process Skill Error:** Error in performing the chosen procedure (for example an error in arithmetic or procedural execution).
4. **Encoding Error:** Inability to write down the final answer in an acceptable form.

This framework is highly relevant as it provides a tool for Objective 1: to identify and categorise the common errors made by O-Level learners, moving beyond simply marking an answer as wrong to determining where the breakdown occurred.

2.3.0 INDICES IN THE SECONDARY SCHOOL CURRICULUM

2.3.1 THE ROLE OF INDICES IN ALGEBRA

Indices are a foundational concept in secondary school algebra, serving as the gateway to logarithms, surds, exponential functions, and polynomial manipulation. The O-Level curriculum typically introduces fundamental laws, including multiplication and division of powers with the same base, the power of a power, zero index, negative indices, and fractional indices (roots). Failure to internalize these concepts systematically creates cognitive gaps that hinder progress in subsequent mathematical topics.

2.3.2 PEDAGOGICAL CONTEXT IN ZIMBABWE

The Zimbabwean O-Level mathematics curriculum emphasizes problem-solving and application. However, studies (e.g., Jaji, 2017) often point out that teaching practice in many Zimbabwean secondary schools remains largely traditional, focusing on teacher-led instruction and procedural drills. This instrumental approach may fail to build the necessary conceptual depth for a topic like indices, where understanding the relationship between the base, exponent, and result is vital.

2.4.1 MISAPPLICATION OF THE DISTRIBUTIVE LAW TO EXPONENTS

One of the most frequent conceptual errors is the incorrect application of the distributive law. Students often confuse the laws of indices with the laws of addition, leading to errors such as:

- $(a + b)^n = a^n + b^n$ (Incorrectly distributing the exponent over addition).
- $a^m + a^n = a^{(m + n)}$ (Incorrectly adding exponents for terms that are being added, not multiplied).

2.4.2 DIFFICULTY WITH ZERO AND NEGATIVE INDICES

Learners frequently struggle with the conceptual meaning of non-positive exponents. Common procedural errors include:

- **Zero Index:** Mistakenly stating that $a^0 = 0$ instead of the correct relational understanding that $a^0 = 1$.
- **Negative Index:** Confusion regarding the reciprocal, leading to errors like $-a^n$ or simply introducing a negative sign, for example, $a^{-n} = -a^n$ instead of $a^{-n} = \frac{1}{a^n}$.

2.4.3 FRACTIONAL INDICES AND ROOTS

The transition from fractional notation to radical notation is a significant hurdle. Learners often struggle with which part of the fraction represents the root and which represents the power, leading to misinterpretations such as:

$$\bullet a^{\frac{m}{n}} \text{ is confused with } \frac{a^m}{n} \text{ or } \frac{m}{a^n}$$

2.5 TEACHER-RELATED AND INSTRUCTIONAL CHALLENGES

The literature suggests that teaching methodologies are a critical factor in mitigating these student challenges (Hiebert & Carpenter, 1992).

- **Over-Emphasis on Procedures:** When teachers prioritize speed and procedural correctness over conceptual depth, learners do not develop the relational understanding needed to transfer knowledge to new problem types.
- **Lack of Connection to Prior Knowledge:** Many difficulties with indices stem from weak foundations in basic arithmetic operations, especially involving negative numbers and fractions. Effective instruction requires teachers to explicitly link the laws of indices back to the principles of repeated multiplication.
- **Conceptual Obstacles:** Teachers themselves may sometimes lack the pedagogical content knowledge necessary to explain the ‘why’ behind the index laws, especially the non-integer exponents (zero, negative, and fractional), relying instead on instrumental explanations.

2.6 SUMMARY OF THE LITERATURE REVIEW

The reviewed literature establishes that difficulties in solving problems involving indices are widespread and systematic. These challenges can be categorized as conceptual (misunderstanding the meaning of the laws, especially non-integer exponents), procedural (misapplying or forgetting the steps of the laws), or pedagogical (instructional methods promoting instrumental over relational understanding).

While global and regional studies have identified these common difficulties, there is a distinct gap in the literature providing a specific, in-depth diagnostic analysis of the challenges within the context of Bernard Mizeki College. This study will fill this gap by applying the diagnostic power of the Newman's Error Analysis framework and the relational/instrumental

understanding theory to a specific O-Level cohort to provide evidence-based recommendations for the institution.

CHAPTER THREE: RESEARCH METHODOLOGY

3.0 INTRODUCTION

This chapter outlines the research procedures that were put in order to collect, present, analyse and interpret data. Thus, the research design, instrument, and methods used to gather relevant information about the study are spelt out. The researcher also justifies the selection of some data collection methods that were used during the study. Consideration is also put on the demerits of these methods and ways of reducing the negative effects of using such research as were employed in this study.

The research methodology employed to investigate the challenges Form 4 learners face when solving problems involving indices are meticulously outlined in this chapter. It functions as the blueprint of the study, providing justification for the chosen research design, detailing the composition of the sample, describing the instruments utilized for data collection, and specifying the statistical and thematic techniques used for data analysis. The procedures detailed herein were rigorously applied to ensure the study's internal and external validity, thereby generating objective and reliable findings to address the core research objectives.

3.1 RESEARCH DESIGN

This is the overall plan of how the research will be conducted. In the opinion of Kumar (2019), research design refers to the overall strategy or plan implemented to address a research question or hypothesis. It outlines the systematic approach and methods that researchers employ to collect analyse and interpret data in order to answer their research questions. A well-designed research study ensures that the data collected is reliable, valid and relevant to the research objectives.

3.2 RESEARCH PHILOSOPHY

The research is based on two philosophies namely positivism and interpretivism. According to Denzin (2018) interpretivism emphasizes understanding social phenomena through subjective meanings and the context in which they occur. It recognizes that individuals construct their own interpretations of reality and that the interpretations shape their behaviour

and experience. With this philosophy the researcher would use interviews as qualitative method of data collection.

Positivism emphasizes the use of objective, empirical methods to uncover causal relationship and explain phenomena. It assures that the social world can be studied using approaches similar to those used in natural sciences. The researcher will use test scores on student academic achievement.

3.3 RESEARCH METHODOLOGY

The research design chosen for this study is a mixed-methods approach, combining quantitative and qualitative research methods. This approach allows for a comprehensive examination of the factors contributing to errors made by form 4 learners at Bernard Mizeki College in Marondera. The quantitative component involves the analysis of standardized test scores, while the qualitative component includes interviews and focus on group discussions with learners.

3.3.1 QUALITATIVE METHODS

Qualitative research is a scientific method of inquiry that focuses on understanding the why and how of human behaviour, experiences, and social phenomena. At its core, it is an interpretive and naturalistic approach, meaning researchers study things in their natural settings to make sense of the world based on the meanings people bring to them.

Education at Bernard Mizeki College extends far beyond the mastery of textbooks; it is centred on the development of the whole man spirit, mind, and body. While numbers can measure academic performance, they often fail to capture the lived reality of the students. Qualitative research addresses this gap by providing an in-depth understanding of the complex social and personal dynamics that define life within the college's gates.

The primary strength of the qualitative approach lies in its flexibility. Unlike rigid statistical models, qualitative methods like open-ended interviews or focus groups allow a researcher to adapt to the flow of conversation. In the context of Bernard Mizeki College, the researcher can uncover the unspoken needs and motivations of his peers, such as the underlying reasons for a house's particular traditions or the specific emotional challenges of being away from

home. By focusing on the why and how, this methodology provides rich, contextual insights that numbers alone cannot convey.

Furthermore, qualitative research is a powerful tool for fostering empathy and community cohesion. By listening to the diverse experiences of boys from different backgrounds, researchers can identify the root causes of issues, rather than just their symptoms. This process is essential for informing innovation and policy, whether it involves a prefect proposing a new mentorship program or the administration refining school culture. It generates new theories that can later be tested, ensuring that the college remains a dynamic and evolving institution.

Qualitative research in this study serves as the heartbeat of academic inquiry at Bernard Mizeki College. It ensures that the human element remains at the forefront of every investigation. By complementing quantitative data with a deeper narrative, it provides a complete picture of the college environment, empowering students to lead with both intelligence and compassion.

3.3.2 QUANTITATIVE METHODS

At Bernard Mizeki College, the pursuit of academic excellence and leadership is grounded in discipline and a commitment to truth. For a learner at this institution, quantitative research serves as a vital tool for discovery, offering a structured and objective way to understand the world. By focusing on numerical data and statistical analysis, this methodology aligns perfectly with the rigorous standards expected of a Bernard Mizeki College gentleman.

One of the primary strengths of quantitative research is its objectivity. In a community that values integrity, this approach minimizes personal bias, ensuring that conclusions are based on hard evidence rather than mere opinion. Furthermore, its efficiency is indispensable for the busy Bernard Mizeki College learner. By using surveys or structured observations, a learner can gather data from a large portion of the learner body, across different houses and forms, providing results that are generalizable to the entire college.

3.3.3 MIXED METHODS RESEARCH METHODOLOGY

In the pursuit of comprehensive research, the researcher used mixed methods research methodology which is a combination of the two research methodologies. A scholar often finds that neither numbers alone nor stories alone can capture the full truth. Mixed methods research is the strategic integration of both quantitative and qualitative approaches within a single study. For a Bernard Mizeki College learner, this represents the ultimate intellectual toolbox, combining the precision of the mathematician with the insight of the historian. By blending these two traditions, mixed methods research allows you to "count the votes" while also hearing the voices.

3.4 AREA OF STUDY

The research was conducted at Bernard Mizeki College in Marondera District in Mashonaland East Province. This geographical area was purposively chosen to leverage the inherent diversity in school types and historical academic performance levels, ranging from high-achieving institutions to those with lower mean scores in Mathematics. This variation was considered essential to capture a wide spectrum of the challenges faced by learners under different academic and infrastructural conditions, thereby increasing the ecological validity of the findings. Bernard Mizeki College was a key institution included in the sampling frame.

Justification: This area was chosen due to its accessibility and the presence of a diverse range of school performance categories, allowing the researcher to sample learners from different academic environments for a richer dataset on the variability of challenges.

3.5 POPULATION OF THE STUDY

This is the entire set of members under investigation. Cresswell (2018) said population refers to any group of individuals that have one or more characteristics in common that are of interest to the researcher. In this view a population consisted of 80 form 4 learners at Bernard Mizeki College of which a stratified sampling was carried out to come up with 12 learners.

Their mean age was 16 years. The researcher chose to carry out the study on the Form 4s upon noticing that the problem in hand was more pronounced in this class. The teachers were used as source of secondary data. Only the subject teachers were used to acquire facts about the subject under study seeing time would have little to offer in a Mathematics oriented research study. This group was targeted because Form 4 is the final stage of secondary

education where mastery of indices is essential for success in final examinations and higher education. Thus the population had 80 members.

3.6 THE SAMPLE

According to Cresswell (2018), sampling is the process of choosing a number of individuals from a given group to serve as a representative sample of that community. According to Smith (2022), sampling is a process by which the researcher deduces features of some things (the population) based on experience with fewer than all potential parts of that set of objects (the sample). Thus, a sample is a subset of a specific population that is used for variable measurement. Twelve participants, all boys, made up the study's sample.

3.6.1 SAMPLING PROCEDURES

The sampling method which was a stratified sample where pupils were arranged in order of their performance. The sample frame did not offer equal chances to all pupils. However, according to Neuman (2020), samples should be carried out fairly to avoid bias and also to the risk of obtaining wrong results.

3.6.2 REASONS FOR SAMPLING

By reducing the number of students to a manageable amount, sampling enhances the quality of the data that is collected. A small sample size saves time and lowers data collection costs. As a result, the study's findings were discovered earlier. Since the researcher does not deliberately choose the persons to be researched, sampling is also done to prevent individual bias.

3.7 DATA COLLECTION PROCEDURES:

Quantitative data will be collected through the administration of standardized mathematics tests to the selected sample of form four students. These tests will assess various mathematical concepts and skills in solving problems involving the laws of indices. Additionally, demographic information such as gender, socioeconomic background, and academic performance will be obtained from school records.

Qualitative data will be collected through semi-structured interviews with students after writing the tests. The interviews will explore their perceptions, experiences, and attitudes towards the laws of indices. Focus group discussions will also be conducted to encourage in-depth conversations and generate a broader range of perspectives.

3.8 DATA ANALYSIS TECHNIQUES:

Descriptive statistics will be used in quantitative data analysis to look at the distribution of math performance and mean scores for all the selected learners. To identify significant gender differences, inferential statistics like t-tests and chi-square tests will be used. Regression analysis and other multivariate analytic approaches can be used to investigate how different factors affect arithmetic achievement. Thematic analysis will be used to analyse qualitative data. The focus groups and interviews will be coded, transcribed, and divided into themes and sub-themes. To better understand the factors causing gender differences in arithmetic performance, the emergent themes will be examined and interpreted.

3.9 RESEARCH INSTRUMENTS

Even though there were just two instruments utilized in the study, there were many more, such as written and oral interviews, exams, records and documentation, field excursions, questionnaires, and observation. Tests and interviews were chosen as data collection methods for the study. In research, an interview is a qualitative data gathering technique that entails a thorough discussion between a participant and a researcher (Fontana and Frey 2019). In order to learn more about the participant's experiences, beliefs, ideas, and perspectives on a certain subject, the researcher guides the conversation and poses open-ended questions. According to Denzin and Lincoln (2020), "it is in a sense an oral questionnaire, where one gives the information orally and on a face-to-face basis instead of writing the response." An interview offers prompt feedback and is usually much reliable because one has no adequate time to think out of topic in question for example thinking of his personal relationship with the interviewer and give answers that suits the relationship.

According to Fontana and Frey (2019), an educational test is a systematic procedure for measuring a sample behaviour considered to be representative of important educational goals. He further says that, a test is constructed, administered and scored according to prescribed rules. Denzin and Lincoln (2020) say, "A test is an assessment which is carried out to check

how much is known about something supposedly learnt”. The researcher imposed tests on the learners to check on their understanding after using the suggested ways of teaching during practical lessons.

3.9.1 LEARNER ACHIEVEMENT TEST

- **Objective:** To quantitatively assess the learners’ mastery of the various laws of indices and identify specific items where performance is weakest.
- **Content:** The test comprised structured questions covering all core concepts of indices, including negative, fractional, and zero exponents, and mixed application problems.

3.9.2 LEARNER QUESTIONNAIRE

- **Objective:** To gather data on learners’ attitudes, self-reported difficulties, and perceptions of instructional methods.
- **Format:** It included closed-ended items using a 5-point Likert Scale (to quantify perceptions) and open-ended questions (to gather qualitative explanations for their challenges).

3.9.3 TEACHER INTERVIEW SCHEDULE (TIS)

- **Objective:** To collect in-depth qualitative data from teachers on perceived learner errors, causes of challenges, and current instructional strategies.
- **Format:** A semi-structured interview format was used, allowing the researcher flexibility to probe further into critical issues raised by the teacher while ensuring all core themes were covered.

3.10 VALIDITY AND RELIABILITY OF INSTRUMENTS

According to the American Psychological Association (2020), validity is the degree to which a measure or research tool accurately reflects the variable that it is meant to measure. According to the American Psychological Association (2020), study findings' consistency and stability are referred to as dependability. It shows the degree to which a study tool or technique yields consistent outcomes when used repeatedly under comparable circumstances. The researcher precisely outlined the characteristics or attributes he intended to assess in order to guarantee the highest level of validity and reliability of the measuring tools.

Additionally, the study established conditions designed to reduce the impact of each measurement instrument's drawbacks. Each questionnaire's validity and reliability were pertinent to the study's goals and the research questions were asked.

3.11 DATA COLLECTION PROCEDURES

Permission to undertake a research study on the Form 4 classrooms and conduct interviews as part of data collecting was requested in a letter to the school head. The request was approved, and the research was started after authorization was formally granted.

1. **Authorization:** Formal ethical approval was obtained from Bindura University ethics board. Permission to conduct the research was then sought from the relevant educational authorities and school principals.
2. **Administration:** The researcher personally administered the Learners Assessment Tests and Learner Questionnaire to the sampled learners during a scheduled class period to ensure a consistent testing environment.
3. **Interviewing:** Each sampling teacher at the school was interviewed in private at a time that did not interfere with their teaching schedule. With the teachers' express consent, all interviews were audio recorded and then verbatim transcribed.

3.12 CONCLUSION

The techniques utilized to collect the essential factual data for the study were described in this chapter. It was discussed how to employ research methods like concentrated observation, questionnaires, and pre and post testing. In light of the current study, the benefits and drawbacks, validity, and reliability of various approaches were also taken into account.

CHAPTER FOUR

DATA PRESENTATION, ANALYSIS AND DISCUSSION

4.0 INTRODUCTION

The information gathered from the study, which examined the difficulties Form 4 students at Bernard Mizeki College encountered when attempting to solve problems requiring the laws of indices, is presented in this chapter. The information obtained using a variety of research tools such as tests and questionnaires is represented by the use tables, graphs, and bar charts, among other data display techniques. The numerical literal facts were combined to provide a meaning that satisfies the predetermined goals of the study.

4.1 BIO-DATA SUMMARY

This study was carried out with the Form four learners, typically consisting of adolescents in their final year of lower secondary education, representing a pivotal transition point in gtheir academic careers. The group consist of boys only with ages ranging from 15 to 17 years old. Actually the average age for these learners is 16years old. These learners are characterized by their focus on national examinations and career pathing, as they are currently selecting subjects or preparing for certifications like the Ordinary Level. Geographically, they often reside within the school's catchment area, though their socio-economic backgrounds may vary significantly, influencing their access to digital learning tools and extracurricular resources. Their linguistic profile often includes a primary instructional language (such as English) alongside local or native languages spoken at home.

4.2 PROPERTIES OF INDICES COVERED AT ORDINARY LEVEL

In Ordinary Level (O-Level) mathematics, the properties of indices (also known as powers or exponents) are fundamental rules used to simplify expressions. These laws allow you to perform operations on numbers or variables that share the same base. They are about 6 of them which learners need to grasp when they are solving problems involving the laws of indices. The following are the primary properties covered in the syllabus:

4.2.1 MULTIPLICATION LAW (Product Rule)

When multiplying terms with the same base, you add the indices.

For example, multiplying $2x^{-4} \times 3x^7$ to get $6x^3$.

4.2.2 DIVISION LAW (**Quotient Rule**)

When dividing terms with the same base, you subtract the index of the denominator from the index of the numerator.

For example, dividing $6x^{-4} \div 3x^7$ to get $2x^{-11}$

4.2.3 POWER OF A POWER LAW

When a term with a power is raised to another power, you multiply the indices.

For example, multiplying $(x^2)^3$ we get x^6 .

4.2.4 ZERO INDEX LAW

Any non-zero base raised to the power of zero is always equal to 1.

Example $a^0 = 1$.

4.2.5 NEGATIVE INDEX LAW

A negative index indicates a reciprocal. To make the index positive, move the base to the denominator (or numerator if it's already a fraction). The negative index can be removed as follows: $3a^{-2} = \frac{3}{a^2}$.

4.2.6 FRACTIONAL INDEX LAW (Roots)

Fractional indices represent roots. The denominator of the fraction tells you the root, and the numerator tells you the power.

For example, simplifying: $8^{\frac{2}{3}} = \sqrt[3]{8^2} = 4$

4.3 COMMON ERRORS

The study of indices is often the done for the first time in learners' transition from "arithmetic thinking" to "algebraic thinking." While the rules themselves appear simple, the errors learners make are rarely random, they are usually the result of conceptual interference. This happens when a learner applies a rule that works in one area of math (like basic multiplication) to a new area where it no longer applies. Learners often make these mistakes until they are taught using the well planned and structured approaches so that their thinking is corrected and they know the right procedure of solving the problems. When studying indices, learners often fall into predictable traps. These errors usually stem from confusing index laws

with basic arithmetic or misinterpreting the notation. Recognizing these common mistakes is the first step toward achieving accuracy in their exams.

4.3.1 MULTIPLYING THE BASE BY THE INDEX

The error of multiplying the base by the index (for example, saying $4^2 = 8$) is the most persistent mistake in introductory algebra. It is not usually a "careless" error, but rather a conceptual replacement where a student uses a familiar operation (multiplication) to solve an unfamiliar one (exponentiation). This is the most frequent error. Learners often treat the index as a multiplier for the base.

By the time learners reach Form four, they have spent nearly a decade mastering multiplication. When they see two numbers positioned together, such as a large base and a small superscript, their brain instinctively seeks a relationship between them. Since multiplication is the strongest operation they know for combining numbers, they default to base times Index.

4.3.2 CONFUSING ADDITION AND MULTIPLICATION LAWS

Confusing the addition and multiplication laws of indices is a common hurdle for Form four learners because it involves a cross-over of operations. The operation being performed on the bases is different from the operation performed on the indices. Learners often see the multiplication sign between the bases in $x^2 \times x^3$ and think, Multiplication means I should multiply the powers, leading to the incorrect answer x^6 . Conversely, they see brackets in $(x^2)^3$ and think they should add the numbers to get x^5 . Learners often mix up when to add the powers and when to multiply them.

$$\begin{aligned}
 &1) \quad 2x^4 \times 3x^{-7} \\
 &= 2(3) x^4 \times x^{-7} \\
 &= 6x^{4+(-7)} \\
 &= 6x^{11}
 \end{aligned}$$

On learner C, the learner had problems in simplifying the directed numbers in indices, like adding the indices $4 + (-7)$ as shown by the above attachment. This might be due to fact that the topic of laws of indices comes after directed numbers which must be taught first and that will be the assumed knowledge. Most probably, the learner did not master the concept of directed numbers which resulted in these challenges on our topic now (indices).

On why he made these errors, the learner said this was due to rushing since time was almost up and as a result, some silly mistakes are made leading to very low grades in examinations. On how to mitigate these errors the learner said teachers need to encourage learners to write fast and also cross check their answers all the time.

4.3.3 HANDLING DIFFERENT BASES INCORRECTLY

When learners encounter mathematical problems involving the laws of indices with different bases, they often feel a need to combine them. This is where most errors occur, as students incorrectly apply laws that only work for identical bases. If the bases are not the same, then the law should not be applied. Handling different bases requires recognizing when you can simplify and when you cannot simplify. The core rule is that different bases cannot be combined. If the bases are different and the powers are different, the index laws for addition or subtraction do not apply. You must evaluate each term individually. A common misconception is that index laws apply even if the bases are different.

4.3.4 ZERO AND NEGATIVE INDEX MISCONCEPTIONS

The widespread occurrence of errors in indices among Form four learners is rarely a reflection of low ability, but rather a byproduct of the cognitive transition from concrete arithmetic to abstract algebraic logic. This conceptual interference occurs because learners often attempt to force new, complex index laws into the familiar templates of basic multiplication and addition they mastered in primary school. For instance, the most pervasive error is simplifying the indices like $(a^0 = 1)$, most of the learners give the answer 0 because they confuse the index zero to be a multiplier of a. Similarly, the confusion between the product rule $(a^m \times a^n)$ and the power-of-a-power rule $(a^n)^m$ arises from a breakdown in "structural sense, where students struggle to distinguish between the accumulation of identical bases and the iterative scaling of an existing power.

Furthermore, the counter-intuitive nature of the zero and negative index laws often triggers a logical rejection, learners find it difficult to accept that a zero exponent yields a value of 1 rather than 0, or that a negative exponent dictates a reciprocal position (a fraction) rather than a negative value. These errors are often reinforced by a reliance on rote memorization over the visual understanding of expanded notation, where x^3 is seen as $x \times x \times x$, which remains the most effective diagnostic tool for correcting these deeply ingrained mathematical misconceptions.

In addition, learners fail to tackle questions with negative indices like $a^{-2} \times (2a)^{-3}$. They usually confuse the negative index as to where to multiply the index and to which terms

should it multiply. Bellow is one example from a learner (learner A) who did not manage to simplify the negative index well with regards to the terms to be affected.

$$\begin{aligned} 2) \quad & (4x^2)^{-2} \times 2x^{-2} \\ = & \frac{1}{(4x^2)^2} \times \frac{1}{2x^2} \times \frac{2}{x^2} \\ = & \frac{1}{16x^4} \times \frac{1}{2x^2} \\ = & \frac{1}{32x^6} \end{aligned}$$

On learner A, the error was on failure to simplify the negative index. The learner did not manage to separate the terms that were to be affected by the negative power from the whole expression considering the brackets that were used there as shown in the attachment above. In response to why this error was made, the learner said he did not grasp the concept of brackets well, when it was taught. On how to mitigate these errors the learner said there is need for teachers to give more time to the topic of indices with a negative index. He also said there is need to give more work on the laws of indices for learners to practice.

4.3.5 APPLYING POWERS TO COEFFICIENTS

When a term has a number in front (a coefficient), students often forget to apply the power to that number. One of the most subtle yet frequent errors for Form four learners make when they apply an index to a term that includes both a coefficient (the number in front) and a variable. The mistake lies in overlooking the coefficient and only applying the power to the variable, usually because the index is physically written closer to the letter. In mathematics, brackets act as a container. Anything inside the container is subject to the operation outside. Without brackets, the index only claims the item immediately to its left.

$$\begin{aligned}
 & 2x^{-3} \times 3x^{-7} \\
 &= 2(3)x^{-3} \times x^{-7} \\
 &= 6x^{-3+(-7)} \\
 &= 6x^{-4}
 \end{aligned}$$

4.4 MAIN FACTORS CONTRIBUTING TO THE CHALLENGES LEARNERS FACE IN MASTERING INDICES

The challenges Form 4 learners face with indices are rarely due to a lack of effort, but rather a complex intersection of cognitive, linguistic, and instructional factors. As students transition from the concrete world of basic arithmetic to the abstract nature of algebraic manipulation, they encounter a clash of systems where their prior mathematical intuition often works against them. This transition requires a significant mental shift one where the familiar rules of addition and multiplication must be applied in entirely new, and often counter-intuitive, ways.

4.4.1 PEDAGOGICAL

Pedagogical factors refer to the methods, strategies, and instructional choices made by teachers that can either facilitate or unintentionally hinder learning. In the case of indices, several pedagogical roadblocks contribute to student confusion.

The challenges faced by Form four learners in mastering indices are deeply rooted in pedagogical approaches that often prioritize rule-following over mathematical reasoning. One primary factor is the premature transition to symbolic manipulation, when educators introduce the laws of indices as a set of isolated shortcuts to be memorized, they inadvertently bypass the essential phase of expanded notation, leaving students without a logical safety net when their memory fails. This focus on procedural fluency rather than conceptual depth creates a black box effect where students apply rules mechanically, leading to the common confusion between the addition and multiplication of exponents.

Furthermore, the lack of diagnostic or refutational teaching, where common misconceptions are explicitly brought to the surface and debated allows intuitive but incorrect arithmetic baggage to persist. For instance, if a teacher does not explicitly bridge the gap between repeated addition (multiplication) and repeated multiplication (indices), the learner continues to treat the index as a multiplier.

Additionally, the rapid pace of the syllabus often prevents students from internalizing the operator shift required in algebra, where the operation performed on the base must be demoted by one level when applied to the index. Consequently, these pedagogical gaps result in a fragile understanding where learners can solve standard textbook problems but struggle to apply index laws in unfamiliar or multi-step algebraic contexts.

4.4.2 CONCEPTUAL

Conceptual factors contributing to the challenges in indices centre on the difficult mental shift from additive/multiplicative thinking to exponential thinking, which requires a higher level of abstraction than most previous topics. Learners often struggle with conceptual interference, where they mistakenly apply the properties of lower-level operations to exponents, such as assuming that because $2 \times 3 = 6$, then 2^3 must also equal 6. This is compounded by the reification problem, the difficulty of seeing an expression like x^n not just as a command to perform a calculation, but as a single mathematical object with its own properties.

Furthermore, concepts such as the zero index ($a^0=1$) and negative indices ($a^{-n} = \frac{1}{a^n}$) create significant cognitive dissonance because they contradict a student's numerical intuition. In the physical world, zero usually implies nothingness and negative signs imply less than zero, making the idea of a zero power resulting in one, or a negative power resulting in a fraction, feel logically inconsistent. Without a solid conceptual grasp of repeated multiplication as the foundation, students view the laws of indices as arbitrary magic rather than a logical extension of number patterns, leading to a fragile understanding that collapses under the pressure of complex algebraic problem-solving.

4.4.3 PROCEDURAL

Procedural factors contributing to the challenges in indices revolve around the rigid and often narrow application of mathematical steps without a flexible understanding of their underlying structure. Many learners treat index laws as a fixed recipe, which leads to failure when a problem deviates from a standard format, such as when coefficients are introduced or when multiple laws must be applied in a specific sequence. This is most evident in the misapplication of order, where students might incorrectly prioritize the multiplication of bases before addressing the powers, or fail to distribute a power across all terms within a bracket. A procedural oversight that stems from a lack of mastery over the hierarchy of operations.

Furthermore, the procedural burden of moving between different representations, such as converting a radical into a fractional index or a negative power into a reciprocal, often leads to notation fatigue where small clerical errors occur. Because these procedures are frequently learned in isolation, students struggle with multi-step integration, often successfully applying one rule only to abandon the logic of the next, ultimately resulting in a correct process applied to an incorrect stage of the problem.

4.5 WAYS OF MINIMISING THE MISTAKES MADE BY FORM FOUR LEARNERS IN SOLVING THE LAWS OF INDICES.

To minimize errors in indices, the focus must shift from memorizing rules to developing a structural awareness of how powers function. By implementing specific mental checks and visual strategies, learners can bridge the gap between intuition and accuracy.

4.5.1 LEARNERS' VIEWS

Based on common feedback from Form four learners who have successfully navigated these challenges, their views on minimizing mistakes often centre on practical, mental-safety strategies. The following are some perspectives representing how different learners approach the problem.

1. One learner suggests that the best way to stop making silly mistakes is to stop trying to be fast. They believe that writing out the expression in expanded form is the only way to be 100% sure. He says, whenever I see 4^3 , I don't just write 12 or 64. I write 4 times 4 times 4 on my rough paper. It takes five seconds longer, but it stops me from accidentally multiplying the big number by the small number."

2. Another learner focuses on the mistakes made with coefficients and brackets. Their view is that errors happen because some numbers look like they don't have powers. "I used to forget to square the 3 in $(3x^3)^2$. Now, my trick is to write a small '1' on every number that doesn't have a power. If I write $(2x^2)^3$, it's much easier to remember to multiply both the 1 and the 2 by the 3 outside the bracket."

3. A third learner suggests that when you get stuck or confused (especially with zero and negative indices), you should recreate the division pattern. They believe that understanding the why prevents the 0 and negative value mistakes. "I used to think 5^0 was 0. Now, I just quickly do $5^2 = 25$, $5^1 = 5$, so 5^0 must be 1 because I'm dividing by 5 each time. Seeing the pattern makes me trust the rule more than my instinct."

4. The fourth learner emphasizes the importance of family groups. They argue that most mistakes happen when students try to combine different bases like 2 and 4.

"I always look at the big numbers first. If I see 2, 4, and 8, I know they are all 'Base 2' family. I rewrite everything as 2 to the power of something before I even touch the index laws. It keeps the problem clean and stops me from multiplying bases that shouldn't be multiplied."

4.5.2 TEACHERS' VIEWS

The transition from basic arithmetic to the Laws of Indices represents a significant cognitive leap for students. Mathematics educators often view this topic as a gateway to higher algebra; if the foundation is shaky, subsequent topics like logarithms, calculus, and scientific notation become inaccessible. To minimize the frequent mistakes made by learners, teachers advocate

for a pedagogical shift away from blind substitution toward a strategy of conceptual transparency and procedural rigor.

4.5.3 WHAT WAS OBSERVED

During the observation, the researcher noted that:

- Learners were having difficulties on adding the powers when multiplying by the same base (Law of Multiplication) such as $2x^{-4} \times 3x^7$.
- They also faced a challenge in removing a negative power on the brackets to simplify the index (Law of a Negative index) such as $(2x)^{-2}$
- Learners failed to simplify a term with a fraction on the power. They were confusing the fraction upon simplification.

4.5.4 RESULTS OF THE TEST

Table 4.1 below shows the results of the performance of learners in the pre-test written during the course of the study. The test was marked out of 20 and converted to percentages.

TABLE 4.1 SHOWING SCORES ON THE PRE-TEST

LEARNERS	A	B	C	D	E	F	G	H	I	J	K	L
MARK/20	12	7	1	7	2	2	3	2	0	3	0	5
PERCENTAGES	60	35	5	35	10	10	15	10	0	15	0	25

Out of the 20 learners who wrote the test, it shows from this table that only one learner was above half with 12 marks out of the total possible 20, resulting to a 60%. The next in the order got 7 marks out of the total possible, resulting to a 35%. This is a very big gap and many of the learners failed the test implying that there was really a problem somewhere somehow which needed some intervention to try and rescue the situation. Out of the 20 learners who were observed in the test, only one passed the test and 19 failed the test resulting in a 5% pass rate. This is not acceptable and some measures must be put in place to try and find out what exactly might be the cause and try to find some solutions to the problems learners are facing and hence improve the pass rate, which is the main reason why

the parents are paying fees for their children to go to school. These results can be well interpreted on a frequency table.

Below is a frequency table for the results from the pre-test scores.

Table 4.2

MARK RANGE (%)	0-10	11- 20	21- 30	31-40	41-50	51-60	61-70	71-80	81- 90	91-100
NUMBER OF LEARNERS	6	2	1	2	0	0	1	0	0	0

High failure rate was observed since the highest frequency occurred in the low range of marks which is disheartening. As a teacher, the pride comes when the learners are getting what the teacher is teaching and are passing the tests, but if this is the case, then something must be done. It might not be wise to just fold hands and leave the learners with their mistakes and errors but rather one should look closely into the situation and try to come out with some remedies.

The bar graph bellow will further illustrate the data.

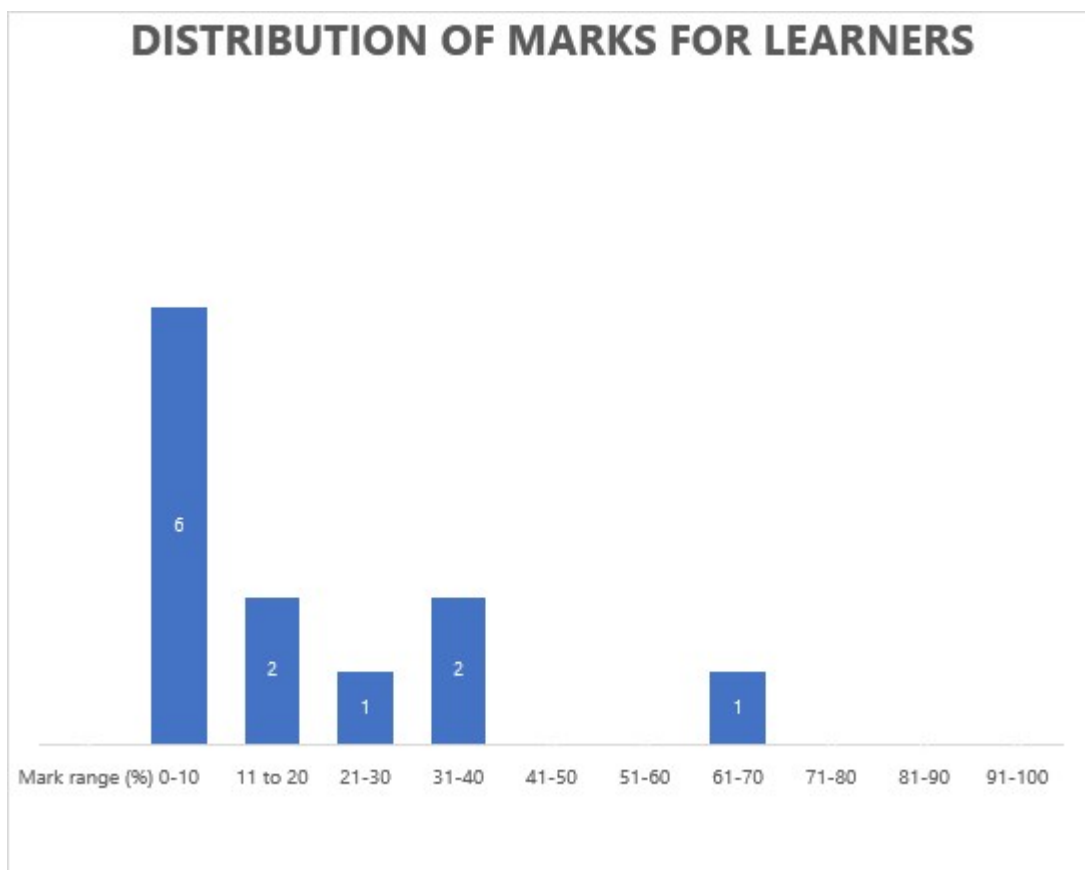


FIG 4.3 PRE-TEST RESULTS

The bar graph above also helps to illustrate the distribution of marks that learners got from the pre-test that was written. It shows that out of the 12 learners only one passed the pre-test with the highest mark of 12 out of 20 which is 60%. High failure rate was observed since the highest frequency occurred in the low range. The illustration is very clear from the bar graph that 6 learners are within the range of marks 0-10, two learners fall in the range 11-20, one learner in the range 21-30, two learners in the range 31-40 and only one learner passed the test with a mark in the range 61-70. This shows that the results were not pleasing, learners had failed the test.

4.5.5 OBSERVATION FROM THE PRE-TEST

- Learners were having difficulties on adding the powers when multiplying by the same base. Multiplication and division of positive and negative powers proved to be a major challenge.
- Learners failed to simplify terms with a fractional index. They were confusing the fractions upon simplification.

- They also faced challenges in removing a negative power on the brackets to simplify the index.

4.5.6 RESPONSES FROM TEACHERS' INTERVIEWS

The H.O.D mathematics and one other mathematics teacher were interviewed. The following are the findings from the researcher.

4.5.7 TEACHING EXPERIENCE

All the teachers indicated that they had more than 10 years in the teaching field. This implies that all of them are more experienced in the teaching field. Also all the teachers indicated that they like the topic of Laws of indices.

4.5.8 TOPICS CONSIDERED BEFORE TEACHING THE TOPIC

Teachers indicated that they have got a list of topics considered before the teaching of the Laws of indices. Combining their topics, they indicated that they considered the following concepts:

- Algebraic expressions
- Like and unlike terms
- Directed Numbers

4.5.9 SOURCES USED BY THE TEACHERS

- New General Mathematics Book 4
- Focus On Mathematics Book 4
- Mathematics 'O' Syllabus

4.6 TEACHER IDENTIFICATION OF LEARNER DIFFICULTIES

Both teachers identified key areas of difficulty for their Form 4 learners:

Teacher 1 who happened to be the H.O.D mathematics argued that "the biggest problem is with the negative and fractional indices. They often forget that $a^{-n} = \frac{1}{a^n}$, and they confuse $a^{\frac{1}{n}}$

with $a \times \frac{1}{n}$. They also struggle with factorising bases to have a common prime base, especially when a number needs to be simplified to a lower base.

Teacher 2 who was a general mathematics teacher put forward the fact that "many learners rely on rote memorization and when a question is set differently from the textbook examples, they are lost. They apply the wrong law because they fail to identify the structure of the problem. Also, algebraic manipulation skills are sometimes weak, which makes simplifying the final expression difficult even if the laws are correctly applied."

SYNTHESIS:

The teachers' views corroborate the learners' data, highlighting the struggle with negative and fractional indices. They further introduce the challenge of weak conceptual understanding (rote learning) and prior knowledge gaps in algebraic manipulation and factorisation, which are prerequisite skills for solving complex index problems.

4.7 CAUSES OF POOR PERFORMANCE IN MATHEMATICS

In trying to explain the causes of poor performance of learners in Mathematics specifically in the laws of indices in the school, the teachers suggested that the failure might be due to poor teaching methods employed by teachers on different concepts. Teachers also pointed out that some learners lack interest in solving the problems involving the laws of indices. They also suggested lack of basic mathematics skills from primary and junior levels at secondary school that is form 1 and 2. This implies that poor performance of learners in Mathematics could be because of poor teaching methods, careless tackling of questions by learners and logical deductions lead to production of wrong answers.

POST TEST RESULTS

Table 4.3 below indicates the results of the performance of learners in the post test written after effective teaching of the topic. The test was marked out of 20 and converted into percentages.

Table 4.3

LEARNERS	A	B	C	D	E	F	G	H	I	J	K	L
MARK/20	13	16	12	16	10	16	9	20	09	4	5	11
PERCENTAGES	65	80	60	80	50	80	45	100	45	20	25	55

Out of the 12 learners who were under the study with the researcher, 8 of them have now scored a mark that is at least 50% out of the total possible 100%. 8 out of 12 learners passed the test, and only 4 learners did not make it although their marks were a bit close to the pass mark for the test (Learner G with 45%, learner I with 45%, learner K with 25% and learner J with 20%) which shows that there is an improvement and the marks are now pleasing. Consistency in this direction can yield a very good quality grades at the end of their final examinations. Once these learners grasp the concept, it is very rare that they can totally forget the concepts they have passed in the previous tests. This will help guarantee a very good pass rate at the end which will make all the stakeholders happy and rejoicing.

Below is a frequency table for the results from the post-test scores.

Table 4.4

MARK RANGE (%)	0-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
NUMBER OF LEARNERS	0	1	1	0	3	2	1	3	0	1

Table 4.4 above illustrate that among the 12 learners, only four failed the post test, the lowest score is 20% and the average mark of the test is 58.75%. Among the group, 8 learners passed the test, 4 learners passed with the scores above 70% and 1 of them managed to pass with 100%, which is the highest possible score. This implies that a greater number of learners performed very well in the post test. Comparing these results to those of pre -test, it might be shown that effective teaching of the laws of indices may enable learners to bring good results than that of pre-test.

Below is a bar graph to illustrate the distribution of the marks that learners got from the post-test they wrote.

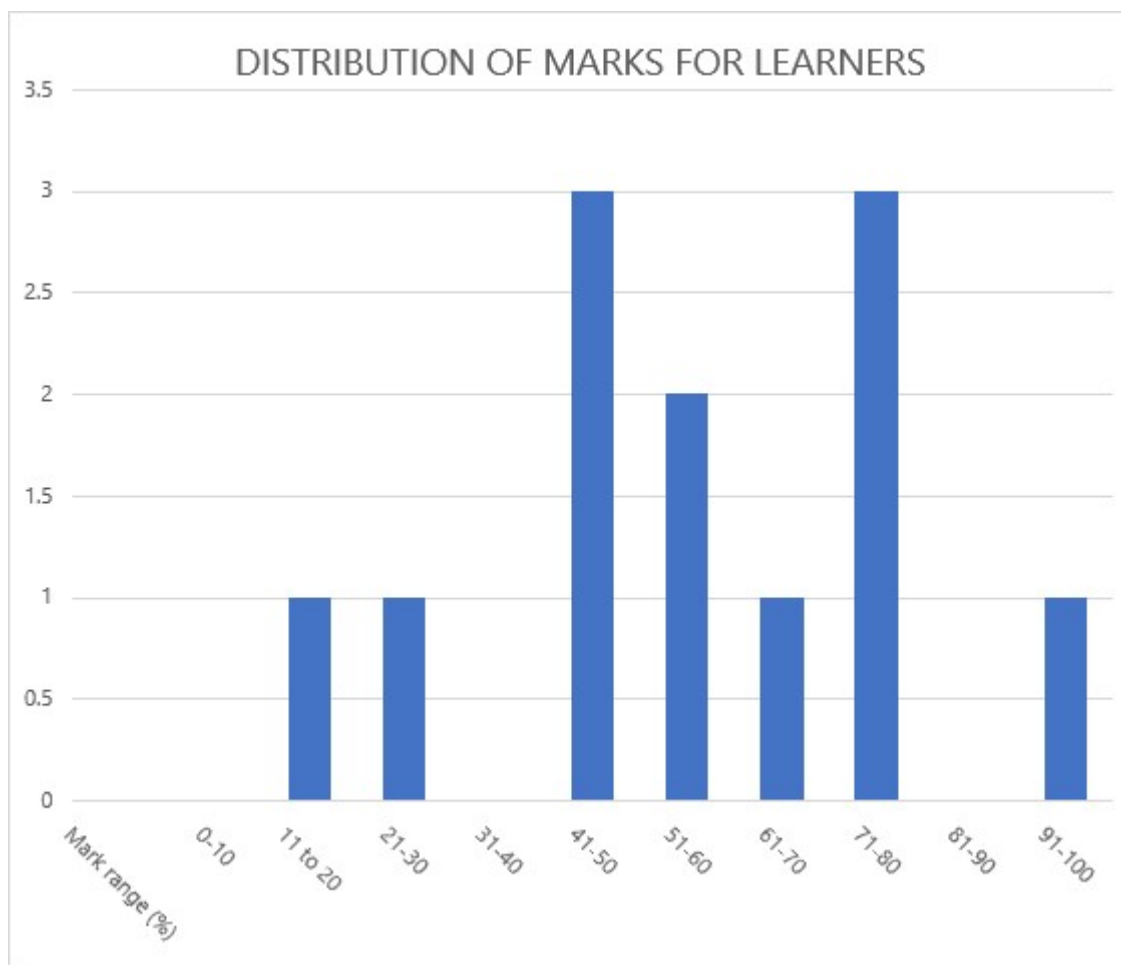


FIG 4.5 DISTRIBUTION OF MARKS FOR POST-TEST

The bar graph in fig 4.5 above also helps to illustrate the distribution of marks that learners got from the post-test that was written. It shows that out of the 12 learners, 8 of them passed the post-test with the highest mark of 20 out of 20 which is 100%. High pass rate was observed since the highest frequency occurred in the upper range.

The illustration is very clear from the bar graph that one learner was within the range of marks 11-20, one learner again fell in the range 21-30, three learners in the range 41-50, one learner in the range 61-70 and three learners in the range 71-80 and one learner in the range 90-100 marks. This shows that the results were now pleasing, learners had passed the test.

4.8 PRE-TEST AND POST TEST ANALYSIS

The table below shows a comparison of the pre-test and the post-test results gathered during the course of the study.

Table 4.6

MARK RANGE (%)	0-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
PRE-TEST FREQUENCY	6	2	1	2	0	0	1	0	0	0
POST-TEST FREQUENCY	0	1	1	0	3	2	1	3	0	1

Out of the 12 learners who were under the study with the researcher, 8 of them have now scored a mark that is at least 50% out of the total possible 100%. 8 out of 12 learners passed the test, and only 4 learners did not make it although their marks were a bit close to the pass mark for the test which shows that there is an improvement and the marks are now pleasing. Consistency in this direction can yield a very good quality grades at the end of their final examinations. Once these learners grasp the concept, it is very rare that they can totally forget the concepts they have passed in the previous tests. This will help guarantee a very good pass rate at the end which will make all the stakeholders happy and rejoicing.

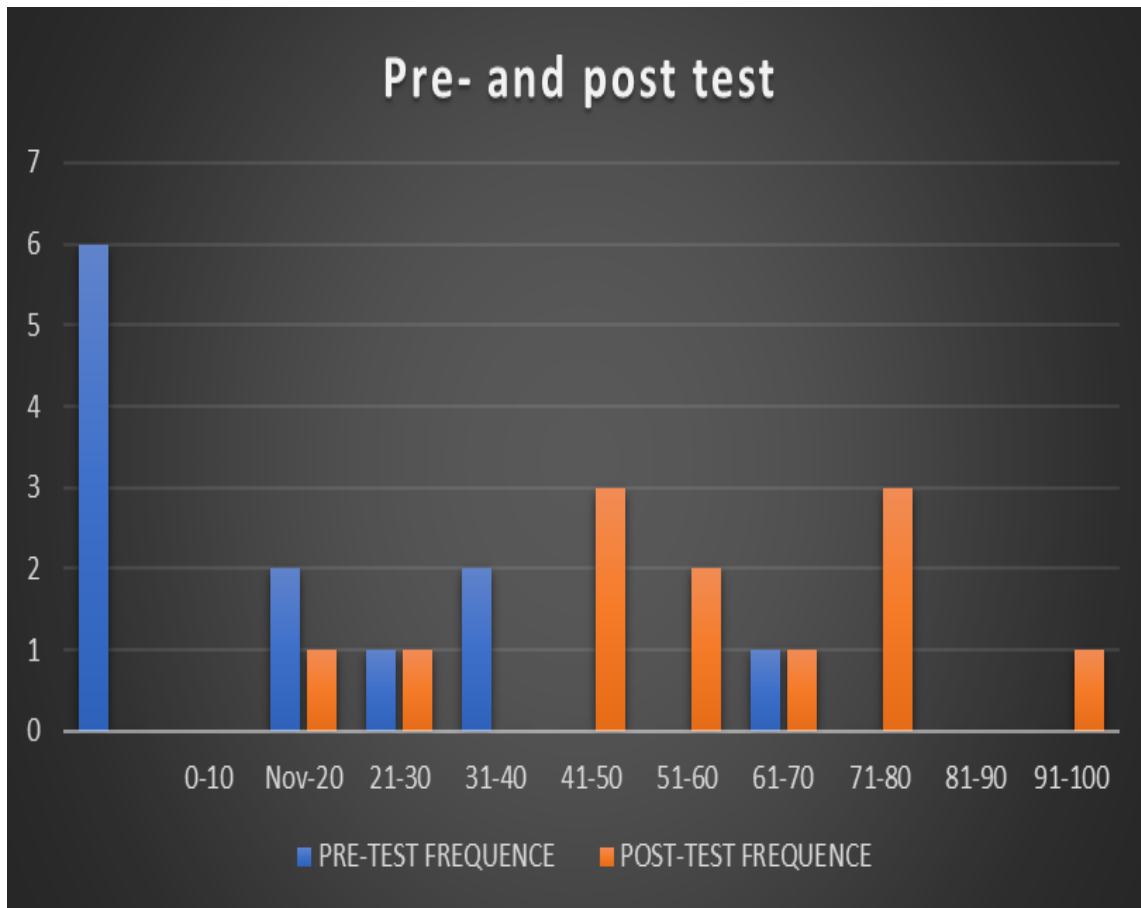


FIG 4.7 PRE AND POST TEST RESULTS

SHOWING A COMPARISON OF PRETEST AND POSTTEST IN THE STUDY.

Fig 4.7 above illustrates that among the learners who wrote the test only 4 failed the post test, their marks show a great sign of improvement. Some even managed to score a 100% mark in the post-test. In the pre-test, almost all learners failed the test and the highest mark recorded was in the range 51-60, which was 60% from only one learner to be specific. This implies that the moment intervention was applied by the researcher in the teaching of the topic, there was a great and notable change. The greatest frequency was in lowest range of marks on the pre-test whilst on the post test results, the greatest frequency was now on largest range implying most of the learners who were making several mistakes and errors in solving problems involving the laws of indices have now mastered the concept and are scoring better than what they used to do before the intervention was made.

Learners were failing in the tests because they did not actually know the correct procedure to solve the problems involving the laws of indices. They were making conceptual errors

because they were using their junior school knowledge where some of the concepts are not yet well explained. This huge improvement in performance marks the turn around in the way the concepts are taught and also in addressing the errors that learners make and gradually eradicate them. This also shows that if the learners are taught well using the methods that makes them feel involved and allow them to ask questions about the concepts they are struggling in, they can improve in their performance.

$$\begin{aligned}
 2) \quad & (4x^2)^{-2} \times 2x^{-2} \\
 = & \frac{1}{(4x^2)^2} \times \frac{1}{2x^2} \times \frac{2}{x^2} \\
 = & \frac{1}{16x^4} \times \frac{1}{2x^2} \\
 = & \frac{1}{32x^6}
 \end{aligned}$$

On learner B, the error was on failure to simplify the negative index. The learner did not manage to separate the terms that were to be affected by the negative power from the whole expression considering the brackets that were used there as shown in the attachment above. In response to why this error was made, the learner said he did not grasp the concept of brackets well, when it was taught. On how to mitigate these errors the learner said there is need for teachers to give more time to the topic of indices with a negative index. He also said there is need to give more work on the laws of indices for learners to practice.

$$\begin{aligned}
 1) & \quad 2x^4 \times 3x^{-7} \\
 &= 2(3) x^4 \times x^{-7} \\
 &= 6x^{4+(-7)} \\
 &= 6x^{-3}
 \end{aligned}$$

4.9 CONCLUSION

This chapter has presented that data collected in different methods. The majority of the findings was presented, analysed and interpreted in narration form especially from finding of the tests were presented in tabular, graphical and narration form. The summary, conclusion and recommendations on this study were done in the next chapter.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.0 INTRODUCTION

In this chapter, the researcher is going to summarize the whole research, concludes the research by interpretation of the results. The researcher is also going to make some recommendations based on the findings of the study.

5.1 SUMMARY

This study or research was carried out at Bernard Mizeki College in Marondera in Mashonaland East Province. The research mainly focused on errors made by Form 4 learners in solving problems involving the laws of indices and their suggested remedies. There was a noticeable improvement on the learners' performance as noticeable in the post test they were given. The research also proved that if learners are going to learn concepts connecting in everyday life, they learn better than when faced with situations they do not know. The use of media is very important as it enhances understanding in learners.

However, there were also some constraints the researcher faced during the period of the research such as lack of resources. Resources were very scarce and it was very difficult for the researcher to move to all the places that needed to be reached and also to get to the people that were relevant in the study. There was also another challenge that the study was time consuming. The researcher needed to ask some questions to the teachers but the teachers were very busy with their work and could not have enough time to attend to the questionnaire.

5.2 CONCLUSION

It was found from the study that the challenges learners are facing when solving problems involving the laws of indices include but not limited to, multiplying the base by the index, confusing addition and multiplication laws, handling different bases incorrectly, zero and negative index misconceptions and applying powers to coefficients

From the investigation carried out by the researcher, it can be concluded that errors made by learners when solving problems involving the laws of indices can be better minimised and eradicated through the use of a wide variety of teaching methods. No learners can be said to be totally incapable of understanding mathematics. Different learners have different levels of

grasping concepts involved in Mathematics. The learners should also be given enough time to practice on their own after a topic is covered so that they get used to answering questions which is basically what they are going to do in their examinations. The learners on their own are not tabula rasas but they know. However, there is need for the teacher's assistance and the use of different media. It is always important to identify the most appropriate method for a given set of learners.

5.3 RECOMMENDATIONS

The researcher recommends that:

- Errors made by learners can be overcome by giving learners enough time to practice on their own.
- Mathematics teachers must be encouraged to identify different errors made by learners, different cognitive abilities of learners by using various testing and once this has been done, they must employ different methods of teaching to different groups.
- In this regard, mathematics teachers must be discouraged from condemning some learners as poor in mathematics before exhausting all possible teaching methods.

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APENDICES

Interview questions for learners

1. What were the specific errors you made in solving problems involving the laws of indices?
2. Can you explain why you made these errors?
3. Did you have any difficulty understanding the concept of indices?
4. Where there any particular areas you felt needed more support or clarification?
5. Is there anything else you would need to add about your experience in solving problems involving the laws of indices?

Interview questions for the teachers

1. Can you recall any specific errors that you have observed in Form 4 mathematics classes that learners were making when solving problems involving the laws of indices?
2. What do you think are the reasons behind these errors?
3. How do you think these errors could be addressed or prevented in future teaching?
4. What strategies do you use to help learners understand the concepts involved in solving problems involving the laws of indices?