BINDURA UNIVERSITY OF SCIENCE EDUCATION



FACULTY OF SCIENCE EDUCATION

PROPOSAL: IMPACT OF CONSTRUCTIVIST INSTRUCTIONAL APPROACH ON LOWER 5
LEARNERS' UNDERSTANDING OF STATIONARY POINTS IN PURE MATHEMATICS:
A CASE OF LOWER 5 STUDENTS IN CHEGUTU DISTRICT.

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BINDURA UNIVERSITY OF SCIENCE EDUCATION



FACULTY OF SCIENCE EDUCATION

IMPACT OF CONSTRUCTIVIST INSTRUCTIONAL APPROA CALCULUS: A CASE OF L6 STUDENTS IN CHEGUTU DISTRICT.

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The undersigned certify that they have read and recommended to Bindura University of Science Education for acceptance research project entitled: "Impact of constructivist instructional approach on learners understanding of stationary points in differential calculus: a case of 16 students in Chegutu District", Submitted by CHITSURO MELODY in partial fulfillment of the requirements of the HONORS BACHELOR OF SCIENCE EDUCATION DEGREE IN MATHEMATICS

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DECLARATION

I, Chitsuro Melody.... declare that this research project "Impact Of Constructivist Instruction Approach On Learners' Understanding Of Stationary Points In Differential Calculus: A Case of L6 Students In Chegutu District", is my own work except as indicated in the references and acknowledgements. It is submitted in partial fulfilment of the requirements for the Honors Bachelor of Science Education Degree in Mathematics , Bindura University Zimbabwe. It has not been submitted before for any degree or examination in this or any other university.

Supervisor : Dr Z Ndemo

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1 .Declare that I have supervised this research project and satisfied that it can

be submitted to the Faculty of Science Education of Bindura University of Science Education.

Date :

Signature :

DEDICATION

To my husband, Chomusora F.

My sister Mrs Takundwa

To my children

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It would not have been possible to complete my HBScEdM without the grace Of God and the support of people around me, some of whom deserve to be acknowledged in a special way here.

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LIST OF ABBREVIATIONS AND ACRONYMS

ZIMSEC : Zimbabwe Schools Examination Council

GRP :Group

L6 :Lower Six

SOLO :Structure of Observed Learning Outcomes

MoPSE :Ministry of Primary and Secondary Education.

KIICJ :Key Informant Interview Guide.

ABSTRACT

Having realised that traditional instructional ppproach is not yielding pleasing results, although many educators are still resorting to it, a semi-experimental and descriptive research designs were employed to investigate whether the application of constructivist instructional approach in the learning of stationary points in differential calculus by Lower 6 learners in Zimbabwe would improve procedural fluency and conceptual learning in Pure Mathematics. Two high schools in Chegutu District with a total of 40 lower 6 learners constitute the research field; one served as the control group while the other representing the experimental group. Three teachers in the district were under the survey as they were requested to respond to specific questionnaires for information on their perceptions, attitudes and motivation in the area of study. Being a mixed-method research, quantitative data was gathered through pre-test and post-test while qualitative data was collected from classroom observations, interviews and questionnaires. Both inferential and descriptive statistical methods of data collection and analysis were used in this study.

Key Words :Impact,Consrtuctivism , Constructivist instruction , Differential Caculus, Stationary Points

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Abstract

Having realised that traditional instructional approach is not yielding pleasing results, although many educators are still resorting to it, an observational study and descriptive research designs will be employed to investigate whether the application of constructivist instructional approach in the learning of stationary points in Pure Mathematics by Lower 5 learners in Zimbabwe would improve procedural fluency and conceptual learning in Pure Mathematics. Two high schools in Chegutu District with a total of 52 Lower 6 learners will constitute the research field; one will serve as the control group while the other representing the experimental group. Six teachers in the district will also be under the survey as they will be requested to respond to specific questionnaires for information on their perceptions, attitudes and motivation in the area of study . Being a mixed-method research, quantitative data will be gathered through pre-test and post-test while qualitative data will be collected from classroom observations and questionnaires. Both inferential and descriptive statistical methods of data collection and analysis are intended to be used in this study.

Key Words**: , Constructivism, constructivist instruction, traditional instruction, differential calculus, stationary points, academic achievement, procedural fluency.**

CHAPTER ONE

INTRODUCTION

The teaching and learning of calculus can be challenging as it involves abstract and complex ideas. The purpose of this study is to investigate how the constructivist instructional approach will help learners and teachers to effectively solve problems in Pure Mathematics focusing on stationary points in particular. The participants of this study will be 52 Lower 5 students from 2 mixed ability classrooms of two core educational high schools in Chegutu District, Mashonaland West Province in Zimbabwe.

1.1 Background of the study

"Neglect of mathematics works injury to all knowledge, since he who is ignorant of it cannot know the other sciences or the things of the world." (Roger Bacon,1214-1294, cited by Fatima,2007 in www.ncert.nic). It is an unpardonable truth that mathematics plays a critical role in the development of every nation, hence it should be given the value it deserves by every citizen of every country. In spite of its value in human life, it is disheartening to note that most people express a serious negative emotion in the subject. Researches have shown that teaching methodologies have a significant contribution in regard to this matter.(Matthews (2000). Rudhumbu (2014) among other writers propounded that many teachers, although they agree that child centered methods are very critical to

generate interest and a deeper understanding of mathematical concepts, they still resort to traditional teaching methods which have a lot of limitations as far as creativity and innovativeness are concerned. The current adjustments in the mathematics curriculum of Zimbabwe reflect the nation's vision which is enveloped in the statement below:

> *"The curriculum review seeks to position Mathematics as a vehicle to acquire 21st century skills, provoke critical thinking and develop a well-rounded learners who can effectively contribute in Zimbabwe's development through improved numeracy skills and precision of thought," Dr Lazarus Dokora, Minister of Primary and Secondary Education.*
> *(Retrieved from worldbank.org, June 2016).*
>
> *''Mathematics should not be viewed as a difficult subject , but a gateway to opportunity , innovation and socio- economic transformation'', Permanent Secretary for Primary and Secondary Education Mr Moses Mhike speaking at the Zimbabwe Mathematics Olympiad Awards Ceremony in Harare .He reiterated the importance of Mathematics in shaping the future of learners , and that programs such as the Mathematics Olympiad align with the National Development Strategy 1 (NDS1) and the Heritage Based Curriculum .*

The statement above, which was made by the Minister is a clear indication on how the constructivist approach needs to be embraced in order to develop creative and critical thinkers who are badly needed to spearhead development in Zimbabwe through the learning of mathematics. Pure Mathematics being one of the mathematical component which have its application across scientific, social, business and economic fields need an earnest attention from educators, learners as well as curriculum planners if the nation is to fulfill its vision.

Research has indicated that the teaching and learning of Pure Mathematics can be challenging as it involves abstract and complex ideas (Gordon, 2004; Zachariades, Pamfilos, Christou, Maleev, & Jones, 2007). Most students taking advanced level would have done general mathematics at ordinary level and they will be introduced to Pure Mathematics for the first time. As a result, Pure Mathematics concepts, if unfriendly teaching methodologies are applied, become an area of threat to them. This means that students face difficulties in learning the key concepts of calculus (Artigue, Batanero, & Kent, 2007). It does not help that in Pure Mathematics traditional classes the emphasis is on computational procedures without understanding the concepts (Gordon, 2004).

The teaching strategies in Pure Mathematics have become merely list of procedures to follow and results only in practicing usual routine in algebraic manipulations without proper procedural fluency to enable learners to apply them in a variety of situational problems. Weaker students get frustrated easily over the manipulations required in Pure Mathematics (Tucker & Leitzel 1995). Students expect a lengthy algebraic solution for problems which is common in Pure Mathematics (Gordon, 2004). In teaching Pure Mathematics , teachers focus more on the procedures rather than understanding of the underlying concepts (Zachariades et al., 2007). According to Axtell (2006), teaching Pure Mathematics using the traditional approach fails to help students understand the basic concepts of Pure Mathematics . Gordon (2004) and Axtell (2006) concluded that the Pure Mathematics curriculum should be improved by focusing on the conceptual understanding of Pure Maths in particular.

Differential calculus, an important aspect of Mathematics that is of interest to this research work, is one of the ten learning areas of Advanced level Mathematics curriculum in Zimbabwe. Taught for about three to four weeks in the second term of every academic year, it carries a weighting value of about 10-15%. Expectedly, Lower 5 learners should be able to, among others, use and understand the principles of differential calculus to determine the rate of change of simple,

non-linear functions and to solve simple optimization problems (DoE, 2012). Unfortunately, as vital and inter-connected as differential calculus is to other learning areas of Mathematics and various other disciplines, it is discouraging to realize that many learners in most parts of the world do not perform well in it. Studies have revealed that a lot of learners struggle with the conceptual understanding of differential calculus (Habre & Abboud, 2006; Maharaj, 2013; Siyepu, 2013). The learners continually find it difficult to apply the basic principles of derivatives to solve even routine problems (Selden, Selden, Hauk & Mason, 2000). They also find it difficulty inacquiring the graphical knowledge of the derivative (Berry & Nyman, 2003; Orhun, 2012).Expressing their worries about the increasing failure-rates in differential calculus, Michchelmore and White (1996) sadly express that majority of the learners study the concept by rote learning. Bezuidenhout (2001) thus advises that well-constructed mental representations of the network of relationships among differential calculus concepts are essential for a thorough conceptual understanding of its underpinning principles.

Like their several other counterparts learning differential calculus in other parts of the world, the case of high school Lower 5 learners in Zimbabwe is also not a remarkable overall performance. Corroborating this claim are the 2024 ZIMSEC National and 2023 June Cambridge international Examiners" Reports on the performances of the learners in questions set on differential calculus in pure mathematics. Majority of the learners who registered the examinations generally demonstrate a poor understanding of differential calculus particularly the aspect of stationary points and thereby fail in the questions set on it.

As noted by the examiners while marking the learners' answer scripts, some of the candidates' common errors and misconceptions are: they confuse x-intercepts with turning points and still struggle to realize that, at a stationary point, the derivative is zero. Learners lack the ability to apply the basic rules of differentiation and have difficulty in interpreting correctly higher order questions involving setting up two simultaneous

equations. Furthermore, candidates could not display mastery of the derivative graph of a function and a good grasp of the derivative (gradient) when applying it to the shape and turning points of a(cubic) functions. They do not know how to get the x-coordinate of a point from a given x-intercept of a derivative graph. Worse still, candidates give bad interpretations of questions requiring application of Pure Mathematics to real-life problems.

Many teachers agree that there is a challenge amongst A Level learners over the years in the concept of stationary points in differential calculus. The researcher could observe that many learners had apparently determined to give less attention to the topic during their study hours and even in the tests and examinations simply because they considered the topic a great challenge to them. Bearing in mind the usefulness of differential calculus and the fact that whether these L5 learners like it or not, those of them whose career paths require any level of mathematical sophistication (such as engineering, technology, or any science-based courses) are required to take differential calculus as a course in their first year in the university, the researcher had decided to conduct the research on this aspect of Mathematics.

Thus, in an attempt to find appropriate learning strategies that could equip the affected learners with a reasonable measure of conceptual understanding of the concept, the researcher had resolved to investigate the impact that the constructivist instructional approach might possibly have. As part of the preliminary efforts made to this end, the researcher consulted with some Advanced level mathematics teachers and some university lecturers handling differential calculus in some Zimbabwe universities to find out what the learners‟ performances were in the first year course. The researcher gathered that the first-year students did demonstrate poor problem-solving skills and weak performances in the differential calculus modules every year. In view of this development, the study chose to make use of the instructional theory of constructivism (the main paradigm within which Mathematics and science educators have been working for few decades now) to investigate strategies by which the L5 learners might learn stationary points in differential calculus.

For many years, the traditional approach to the teaching and learning of Mathematics making use of the chalkboard and textbooks had been in use. In an average traditional Mathematics classroom, learners are involved in few class activities. A good part of the instructional time is devoted to individual seatwork and whole class recitations led by the teacher. The instructions and curricula rest seriously on transmission and absorption. Learners passively absorb mathematical structures created by others, as written in texts or as taught by other authoritative sources. Hence, teaching is done by way of transmitting sets of established facts, skills and concepts to learners. Obviously, traditional learning approach is an instructional method in which the teacher, seen as the authority source, transmits knowledge to learners considered tabula-rasa who receive it as complete and correct. It is easy then to point out that effective learning might not be sufficiently achieved in such a situation.

A worthwhile alternative therefore might be to shift the focus of the classroom from being teacher-dominated to learner-oriented using the constructivist instructional approach believed to be effective in enhancing learners' understanding and achievement (Akkus, Kadayıfçı, Atasoy & Geban, 2003;Gallery, 2001; Jonassen, 2011; Nancy & Palmer, 2005). In a constructivist learning environment, learners are expected to play an active role in the learning process while the teacher assumes the duty of a facilitator assisting learners to get to their own understanding of a concept by directing them towards "developing new insights and connecting them to their prior knowledge" (Doolittle, 1997).

For a proper understanding of the constructivist instructional approach used in the learning of the concept, the approach is compared with the traditional instructional

approaching Table 1-1 below according to Akkus, Kadayıfçı, Atasoy and Geban(2003), Gallery (2001),Jonassen (2011), Kilavuz(2005),Kim (2005), (2014),Nancy and Palmer (2005), Nayak (2012)and Nkhoboti (2002).

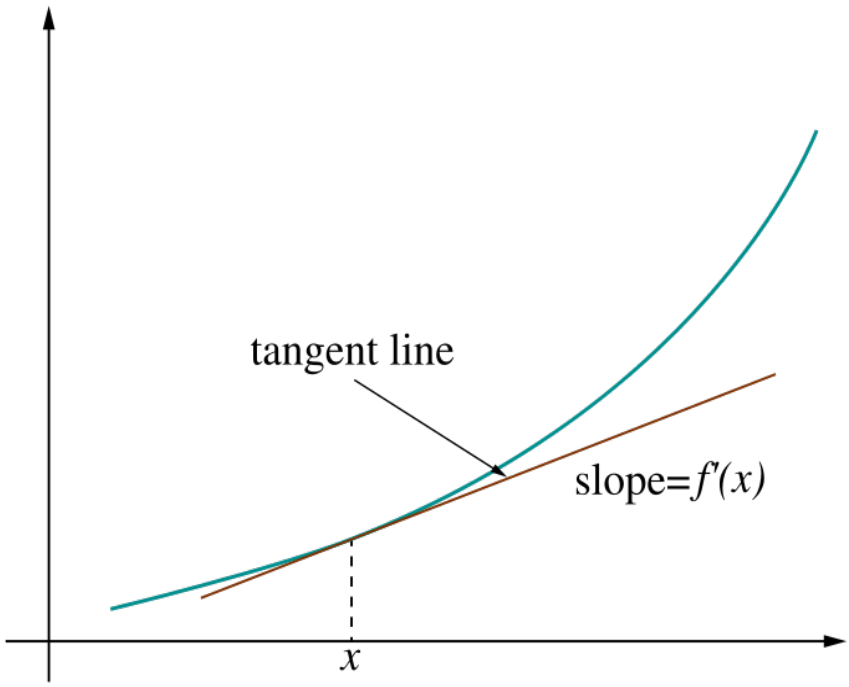
1.2.1 The aspect of differential calculus in mathematics.

Differential calculus is the study of the definitions, properties and applications of the derivatives of functions. The process of finding the derivative is called differentiation. Given a function and a point in the domain, the derivative at that point is a way of encoding the small-scale behavior of the function near that point. By finding the derivative of a function at every point in its domain, it is possible to produce a new function, called the derivative function or just the derivative of the original function. The common symbol for derivative is an apostrophe-like mark called prime(Clar,2013).

1.2.2 The Derivative

Suppose that x and y are real numbers and that y is a function of x, that is, for every value of x, there is a corresponding value of y. This relationship can be written as $y = f(x)$. If $f(x)$ is the equation for a straight line (called a linear equation), then there are two real numbers m and b such that $y = mx + b$.

Figure 1.1



In this "slope-intercept form", the term *m* is called the slope and can be determined from the formula:

$$M = \frac{Change\ in\ y}{Change\ in\ x} = \frac{\Delta y}{\Delta x}$$

where the symbol Δ (the uppercase form of the Greek letter Delta) is an abbreviation for "change in". It follows that $\Delta y = m\,\Delta x$.

Geometrically, the **derivative of *f* at the point *x* = *a*** is the slope of the tangent line to the function *f* at the point *a* (see figure). The derivative of the function of f is often denoted *f '*(x) pronounced f prime in Lagrange's notation or $\frac{dy}{dx}\Big|_{x=a}$ in Leibniz's notation.

1.2.3 Usefulness of Differential Calculus

Pure Mathematics is used in actuarial science, computer science, statistics, engineering, economics, business, medicine, demography and every branch of the physical sciences. It is also used in fields wherever a problem has to be mathematically modeled to obtain an optimal solution. Pure Mathematics is widely used in aspects of Physics such as mechanics and electromagnetism; to find the best fit linear approximation for a set of points in a domain in linear algebra; to determine the probability of a continuous random variable from an assumed density function; to find high points and low points (maxima and minima), slope, concavity and inflection points in analytic geometry etc.
As a learning area of Mathematics dealing with the study of the rates at which quantities change, the purpose of studying differential calculus is simply to introduce learners' minds to the scientific method of analysis. Through science, practical problems can be identified, explanations generated and logical solutions selected. The aim is for the learners to understand how to apply their minds in a systematic manner towards understanding the world around them (Hussain, 2012). Hussain further observes that the purpose of studying differential calculus is two-fold. First, it is a way of introducing the basic concepts of Mathematics used in studying almost any type of changing phenomena within a controlled setting. Second, studying differential calculus will develop invaluable scientific sense and practical engineering problem-solving skills in learners. It makes learners understand how to think logically to reduce even the most complex systems to a few interacting components.

Differential calculus has applications in computations involving velocity and acceleration, the slope of a curve, and optimization. It also helps in offering a clear understanding of the nature of space, time and motion. In the study of motion and area, there had for long been the puzzles involving division by zero or sums of infinitely many numbers. It is differential calculus that provides useful tools that resolve the puzzles. It is also of interest to note that the applications and generalizations of derivatives appear in several fields of Mathematics, such as: complex analysis, functional analysis, geometry, and algebra. In fact, derivative (or differentiation) has wide applications not only in Mathematics but also

in nearly all quantitative disciplines such as: Physics, Chemistry, Engineering, Operation Research, Economics, Statistics, Computer Science, Medicine, Demography and in other fields where a problem can be mathematically modeled and an optimal solution is demanded.

In science, one major goal is to predict what will happen in future. To do this, we must determine how changes in certain quantities will affect the future behaviour of other associated quantities. This is what differential calculus does best as the study of change, more precisely, changing quantities. Moreover, it involves the manipulations of functions. In fact, Introductory Calculus lays the foundation for understanding all sorts of scientific models (James, 2001). As pointed out by Clar (2013), apart from its centrality to Mathematics, differential calculus can be used in conjunction with other mathematical disciplines. For example, it can be used with linear algebra to find the best fit linear approximation for a set of points in a domain. Also, it can be applied in probability theory to determine the probability of a continuous random variable from an assumed density function. In analytic geometry, the study of graphs of functions, differential calculus is needed to find high points and low points (maxima and minima).

As specified in the Advanced level syllabus, the current policy statement for teaching and learning in Zimbabwean schools, those aspects of differential calculus learners are expected to learn include: limits, rate of change or gradient of a function at a point; derivatives of functions from first principles; applications of the product and quotient rules etc.; equations of tangents to graphs; graphs of cubic and other suitable polynomial functions; stationary points, x-intercepts, and practical problems involving optimization and rate of change. However, for in-depth study and to make the inquiry manageable, the research concerned itself with application of differentiation in obtaining stationary points.

1.2.4. Stationary points in differential calculus.

In mathematics, particularly in calculus, a stationary point or a critical point of a differentiable function of one variable is a point on the graph of the function where the function's derivative is zero. In simple terms it is a point where the function stops to either increase or decrease, thus there is no change hence the name stationary**. (**Chiang, Alpha 1986 in Wikipedia (2017). A stationary point of a function corresponds to a critical point of its graph for the projection parallel to the x-axis. Conversely, the critical points of the graph for the projection parallel to the y-axis are points were the derivatives is not defined. Saddler et-al (2011) explains a turning point as a place were the derivative changes its sign. A turning point can be either a relative maximum or a relative minimum, also known as the local maximum and minimum. If the function is differentiable, then the turning point is a stationary point.

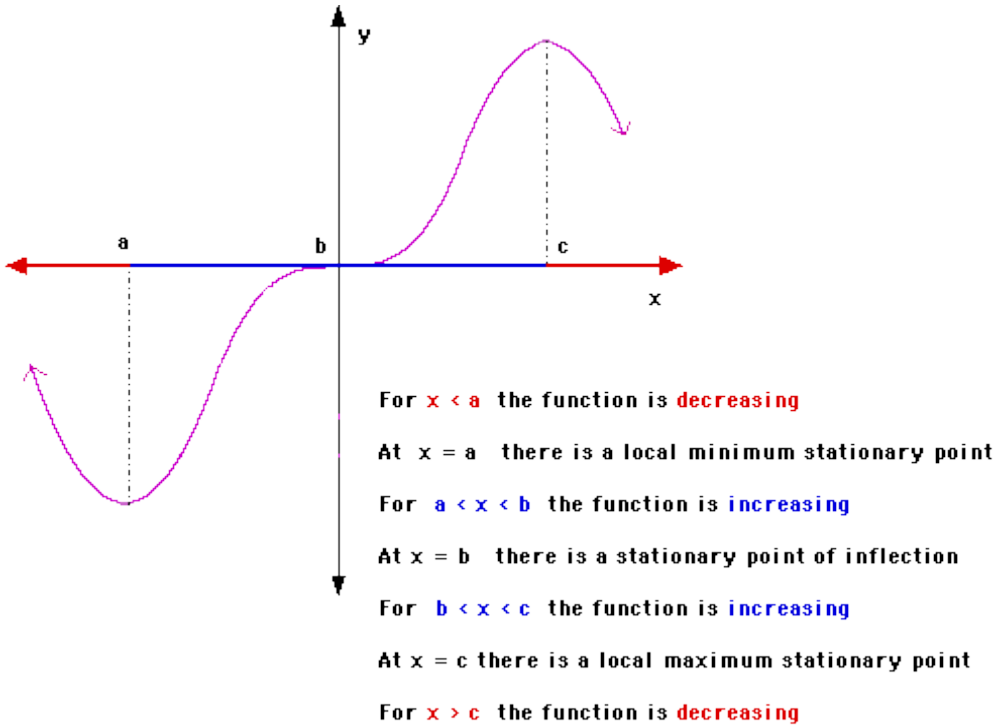
If the gradient of a curve at a point is zero, then this point is called a stationary point. This can be a maximum stationary point or a minimum stationary point. They are also called turning points. For a stationary point f '(x) = 0

Stationary points are often called local because there are often greater or smaller values at other places in the function.

Another type of stationary point is called a point of inflection. With this type of point the gradient is zero but the gradient on either side of the point remains either positive or negative.

The diagram below shows examples of each of these types of points and parts of functions.

Figure 1.2

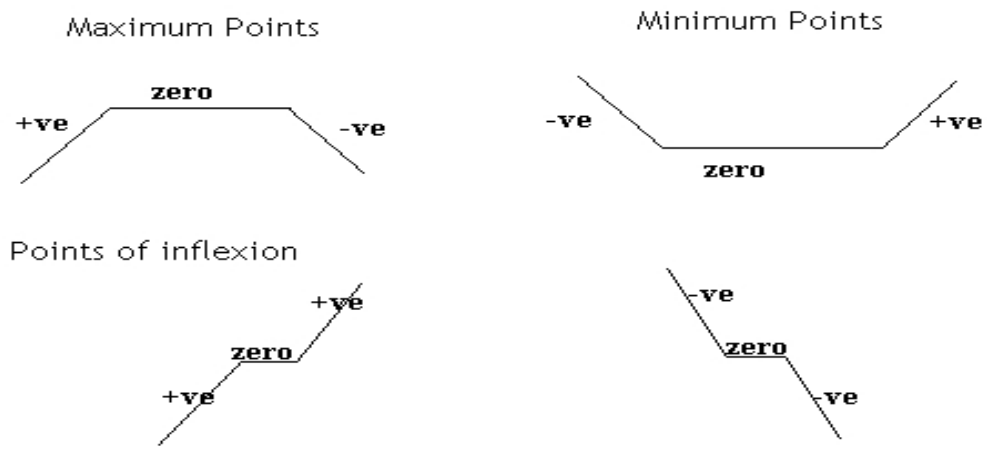


Adopted from source: Best Maths.com (2014)

1.2.5 Maximum, Minimum or Point of Inflection

At all the stationary points, the gradient is the same (= zero) but it is often necessary to know whether you have found a maximum point, a minimum point or a point of inflection. Therefore the gradient at either side of the stationary point needs to be looked at (alternatively, we can use the second derivative).

Figure 1.3

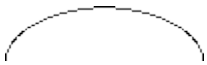
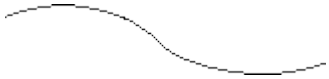


At maximum points, the gradient is positive just before the maximum, it is zero at the maximum and it is negative just after the maximum. At minimum points, the gradient is negative, zero then positive. Finally at points of inflexion, the gradient can be positive, zero, positive or negative, zero, negative. This is illustrated here:

1.2.6. Concavity

Every curve contains something called **concavity**, which may be either

Table 1.1

Concave down	Concave up	Both types of concavity
		

A point of inflection is one where the curve changes concavity. A point of inflection does not have to be a stationary point, although as we have seen before it can be.

1.2.7 Finding Stationary Points and Points of Inflection

Stationary points and points of inflection are found using the first and the second derivative tests as shown below:

1.2.8. First derivative test

Differentiating once and putting f '(x) = 0 will find all of the stationary points. Examining the gradient on either side of the stationary point will determine its nature, i.e. Maximum, minimum or point of inflection.

Example

Table 1.2

	Method	**Example** **Find the coordinates and nature of the stationary point(s) of the function $f(x) = x^3 - 6x^2$**
Step 1	Differentiate the function to find f '(x)	$f'(x) = 3x^2 - 12x$
Step 2	Put f '(x) = 0 as the gradient at the stationary points is zero.	$3x^2 - 12x = 0$
Step 3	Solve this equation to find the x-value of the stationary point.	$3x(x - 4) = 0$ $x = 0$ or $x = 4$

<table>
<tr>
<td>Step 4</td>
<td>Substitute this value into the original function to find the y-value of the stationary point.</td>
<td>When x = 0, y = 0 – 0 = 0
When x = 4, y = 64 – 96 = -32
Coordinates of the turning points are
(0, 0) and (4, -32)</td>
</tr>
<tr>
<td>Step 5</td>
<td>Examine the gradient on either side of the stationary point to find its nature.
<table>
<tr><td>neg.</td><td>0</td><td>pos.</td><td>Local minimum point</td></tr>
<tr><td>pos.</td><td>0</td><td>neg.</td><td>Local maximum point</td></tr>
<tr><td>pos.</td><td>0</td><td>pos.</td><td>Increasing point of inflection</td></tr>
<tr><td>neg.</td><td>0</td><td>neg.</td><td>Decreasing point of inflection</td></tr>
</table>
</td>
<td>Choose values either side of the turning points
<table>
<tr><td>When x = -1</td><td>When x = 0</td><td>When x = 2</td><td>When x = 4</td><td>When x = 5</td></tr>
<tr><td>f '(-1) is positive</td><td>f '(0) is zero</td><td>f ' (2) is negative</td><td>f '(4) is zero</td><td>f '(5) is positive</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
</table>
This shows a maximum point followed by a minimum point.
</td>
</tr>
</table>

Adopted from source: Best Maths (2014)

1.2.9. Second derivative test

Differentiating once determines the gradient of a curve and putting f '(x) = 0 will find all of the stationary points.

Differentiating a second time to find f ''(x) and evaluating it for a particular point will determine the nature of the point.

Table 1.2

f '(x)	f '' (x)	Nature of point
0	positive	A local minimum point
0	negative	A local maximum point
positive	0	increasing point of inflection
negative	0	decreasing point of inflection
0	0	Rare, but could be any of the above! Test the gradient on either side.

The second derivative determines concavity:

y '' < 0	Concave down
y '' > 0	Concave up

1.3 Importance of Stationary Points

The concept of stationary points is an essential part of differential calculus considering its usefulness. Among its other uses, it is applied in solving practical optimization problems. Whenever Mathematics is being used to model our physical world, we often express physical quantities in terms of variables. Functions are therefore used to describe how the variables change. Scientists, engineers and economists, for instance, show significant interests in the ups and downs of a function, its maximum and minimum values (i.e. its turning points). In several applications, these professionals are interested in such points obviously for maximizing power or profit and also for minimizing losses or costs. More generally, they use the idea of stationary points for the selection of the best element (considering

certain criteria) from some set of available alternatives. As future professionals, it is thus necessary that these L5 learners have to learn and understand stationary points and their uses.

1.4. Statement of the Problem

Students are experiencing difficulties in learning derivatives and understanding the concept stationary points in differential calculus. This could be due to the fact that they are introduced to this concept as merely a rule that should be applied with no justification or without mental construction for the procedure. In other words the traditional teaching methodologies of direct instruction may be contributing to lack of understanding of the concept of stationary points in differential calculus. So lack of mental structures obstruct students' understanding of the concept under study.
The focus of this study is to investigate the extent to which constructivist pedagogical approach could improve the learning of stationary points in differential calculus at learners in Chegutu District of Mashonaland West Province in Zimbabwe. Thus the study aims to measure the learners' understanding of the concept of stationary points through what they will achieve in their performance scores and the way they display their practical skills . In an attempt to look into the research problem, the following research questions have been formulated:

1.5. Research Question

Does the constructivist instructional approach facilitates learners understanding of differential calculus?

Sub-questions

1. Does constructivist instructional approach improve Lower 5 learners ' performance in the concept of stationary points in differential calculus?

2. Does the approach triggers leaners' procedural fluency in solving stationary points related problems in advanced level Pure Mathematics?
3. How does the constructivist instructional approach facilitates understanding of the concept of stationary points in differential calculus?

The hypotheses below, stated at .05 probability level of significance, were used to guide the study:
H_0: There will be no significant statistical difference between the study participants' pre-test and the post-test mean scores after the intervention.
H_1: There will be a significant statistical difference between the study participants' pre-test and the post-test mean scores after the intervention.

1.6. Research Objectives

The overall objective of the study is to assess the influence of the use of constructivist learning approach on Lower 5 learners' understanding of stationary points in differential calculus. More specifically, the research objectives will be to find out if constructivist instructional approach can:

- effectively enhance the development of procedural fluency in learning of the concepts of stationary points in differential calculus;
- significantly improve the performance of the concerned learners in the post-test set on the concept
- trigger leaners interest in learning and understanding of stationary points in differential calculus.

1.7. Significance of the study.

This study shall be significant to mathematics educators as they need to understand that the learning of stationary points does not have to be by memorization of rules, formulae and algorithms of differentiation or those of stationary points, or by passive reception of the knowledge transmitted by the teacher. It will help to illustrate that Lower 5 learners can actively construct their own knowledge of the concept by making links between their previously acquired ideas and the new concept through experience. It also wants to establish that these learners can learn the concept by interacting, experimenting, hypothesizing, exploring and reasoning individually and on group basis. The study will show that a deeper understanding of the concepts of stationary points in differential calculus can enhance its practical applications; The study also contributes to a constructivist approach to learning mathematics which helps in improving the understanding of some mathematical concepts and improving on the mathematics results at all levels of mathematics education. It also provides a base for similar studies in other secondary teacher training colleges. Fellow researchers will also benefit by using it as reference material or by way of raising areas of further research emanating from it. It will contribute to existing body of knowledge in similar researches and also to serve as a basis for further studies on the topic. More so it add on to the researcher's professional and intellectual maturity as an academic.

1.8. Delimitations of the study

The research will be carried out at Rio Tinto Mhondoro High and Presbyterian Mhondoro High in Chegutu District of Mashonaland West Province. The participants will be all advanced level mathematics students in these schools main study. The schools were selected for convenience because of their proximity to the researcher to alleviate the challenge of financial resources. Almost all the streams have difficulties in understanding the concept of stationary concept.. So there was need to make an exploration of the concept since it appears to be a perennial problem. The concept under study was chosen because it is covered at this level as prescribed by the syllabus. The concept is also relevant to main study as well as to applied mathematics. Since this will be done for convenience

and for financial reasons, the results of the study will peculiar to the schools under study. However the results may be inferred to other schools with a similar case in Zimbabwe.

1.9. Limitations of the study

The following aspects may limit the researcher's efforts in executing the research:
Cooperation from students as they would be too busy with other learning areas since they will be preparing for their Projects and other subjects. That will limit their availability and collection of data. The effect of this may be curbed by obtaining the necessary permission from the school authorities to carry out research in the respective schools. The researcher may also be affected by employment obligation as she is a fulltime teacher at a secondary school , so that would affect time and effort to carry out the research. This effect may be curbed by
applying for leave for the period of data collection. The researcher would ask respective Advanced level teachers to do the teaching while he sits and observes the students learning. Thus more time to observe how students are interacting and going about mental constructions of the stationary points. The researcher and the
teachers will agree on what and how to teach and with what method to use.

1.11. Definition of key terms.

For clarity and easy understanding of the study, it is proper at this point to explain the key terms used in the work. This is done as follows:
Impact: influence or effect
Constructivism: is a teaching philosophy based on the concept that learning (cognition) is the result of 'mental construction' - students construct their own understanding by reflecting on their personal experiences, and by relating the new knowledge with what they already know. Each student creates his or her own 'schemas' or mental-models to make sense of the world, and accommodates the new knowledge (learns) by adjusting them. One of its main principles is that learning is search for meaning,

therefore, to be effective, a teacher must help the student in discovering his or her own meaning.lbusinessdictionary.com(2017)

Constructivist Instruction: lessons or learning based on the principles or philosophy of constructivism.

*Traditional Instruction***:** is a form of learning that is based on transmission or absorption with the belief that learners passively absorb mathematical structures invented by authoritative sources or as recorded in texts. Class periods are lecture based and involve note taking, usually through the use of a chalk board or white board. In this instructional style, it is expected that students will answer questions generated by their teachers. (Sungur & Tekkaya, 2006)

Procedural fluency: is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014)

Calculus: refers to a branch of mathematics developed by Newton and Leibneiz that concerns the rate of change of the dependent variable and the slope of a curve, (Augad, 1987). In this study it is the study of the behaviour of functions in mathematics and involves some operations in differential and integral calculus;

Differential Calculus: a branch of mathematics concerned chiefly with the study of the rate of change of functions with respect to their variables especially through the use of derivatives and differentials.(merriam-webster.com retrieved 2017). The aspect deals with the study of the definitions, properties and applications of the derivatives of functions.

Stationary Points: are points on the graph where the gradient is zero. At any stationary point, the tangent to the graph is horizontal or parallel to the x-axis. The three types of stationary points are: maximum point, minimum point and point of inflection/inflexion.

Prior Knowledge: is learners' entry behaviour or previous experience capable of facilitating the learning of a new idea.

Understanding: refers to the performance or academic achievements of the learners under study.

Academic Achievement: is the learners' successful accomplishment of a learning goal which, in this study, is their encouraging performance in the application of differentiation in finding stationary points.

1.11. Organisation of study

The organization of the study is as follows: Chapter 1 is on the introduction and background of the study. Chapter 2 is on literature review as well as the theoretical framework of the study with Chapter 3 focusing on the research methodology. Chapter 4 will be focusing on data presentation, analysis, interpretation and discussion. Finally Chapter 5 will be on summary, conclusion and recommendations.

1.12. Summary

In this first chapter the researcher has given an overview on the background of understanding of the concept of stationary points by using the constructivist approach of teaching Lower 5learners in Zimbabwe and the problems that students have as they learn this concept of differential calculus, as seen by different authorities. Focus was on the background of the study, the statement of the problem, the research question and guiding sub research questions. The delimitations and limitations of the study, operational definitions and ethical considerations were also looked into.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This chapter will start by a review of theory that informs this study. It will also review relevant literature to gain insight into certain issues critical to the research. These essential issues include: traditional instructional approach , its shortcomings , an overview of the learning theory of constructivism, with

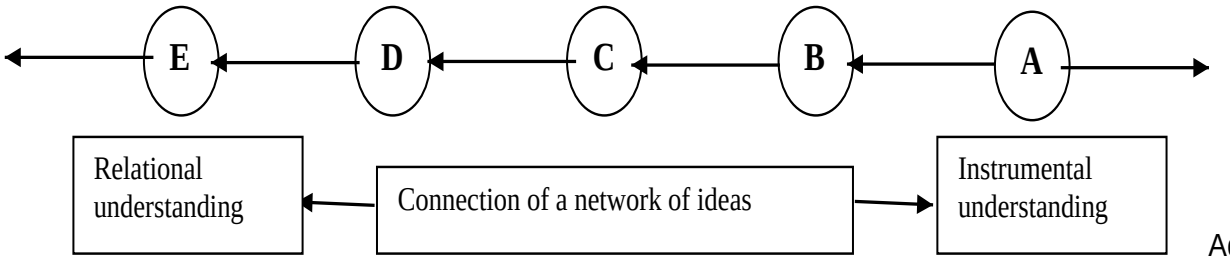
any analysis on two basic types namely radical constructivism and social constructivism, the pedagogy of constructivist learning theory and a survey of studies in support of the constructivist classroom teaching of stationary points. The various useful analysis of the opinions of several researchers and scholars shall be used in this regard. It will investigate some possible answers to the research questions that have been put forward.

2.2 Theoretical framework

This research is based on the constructivist theory, an approach which was philosophically founded by John Dewey (1933-1998), as many authors cite. Brunner (1990) and Piaget (1972) are considered the chief theorist among the cognitive constructivists while Vygotsky is the major theorist among the social constructivists. This is a "moderate" version, compatible with the way most mathematicians see mathematics, and a social constructivist version, inspired by the work of Vygotsky, which takes into account sociocultural perspectives,(Selden, 1999). The theory proposes that, "As long as there were people asking each other questions, we have had constructivist classrooms. Constructivism, the study of learning, is about how we all make sense of our world,..." (Brooks 1999 in Mascol & Fischer, 2005). All constructivist agree that there is no absolute knowledge, but just how we interpret it . The acquisition of knowledge however require individuals to consider the information and, based on their past experiences, personal view and cultural background, construct an interpretation of information that is being presented to them. 'Constructivism is a postmodern theory of knowledge with the potential to transform educational theory' (Fleury 1998, in Matthews 2000). It is not surprising then that, 'For several years now, across the world, pre-service and in-service teachers have been considering constructivism as a referent for their philosophies of education' (Bentley 1998, in Matthews 2000). Matthews, is also of the view that constructivism is not just a theory about education, it is a theory about one of culture's greatest and most enduring achievements, namely science. Expanding these thoughts in the teaching and learning of Pure Mathematics Walle (2014) suggests that understanding exists along a continuum (Figure 2.1) from an instrumental understanding knowing something by rote or without meaning (Skemp, 1978)

,to a relational understanding, knowing what to do and why. Instrumental understanding, at the right end of the continuum, shows that ideas (e.g., concepts and procedures) are learned, but in isolation (or nearly so) to other ideas. Here you find ideas that have been memorized. Due to their isolation, poorly understood ideas are easily forgotten and are unlikely to be useful for constructing new ideas. At the right end of the continuum is relational understanding. Relational understanding means that each new concept or procedure is not only learned, but is also connected to many existing ideas so there is a rich set of connections.

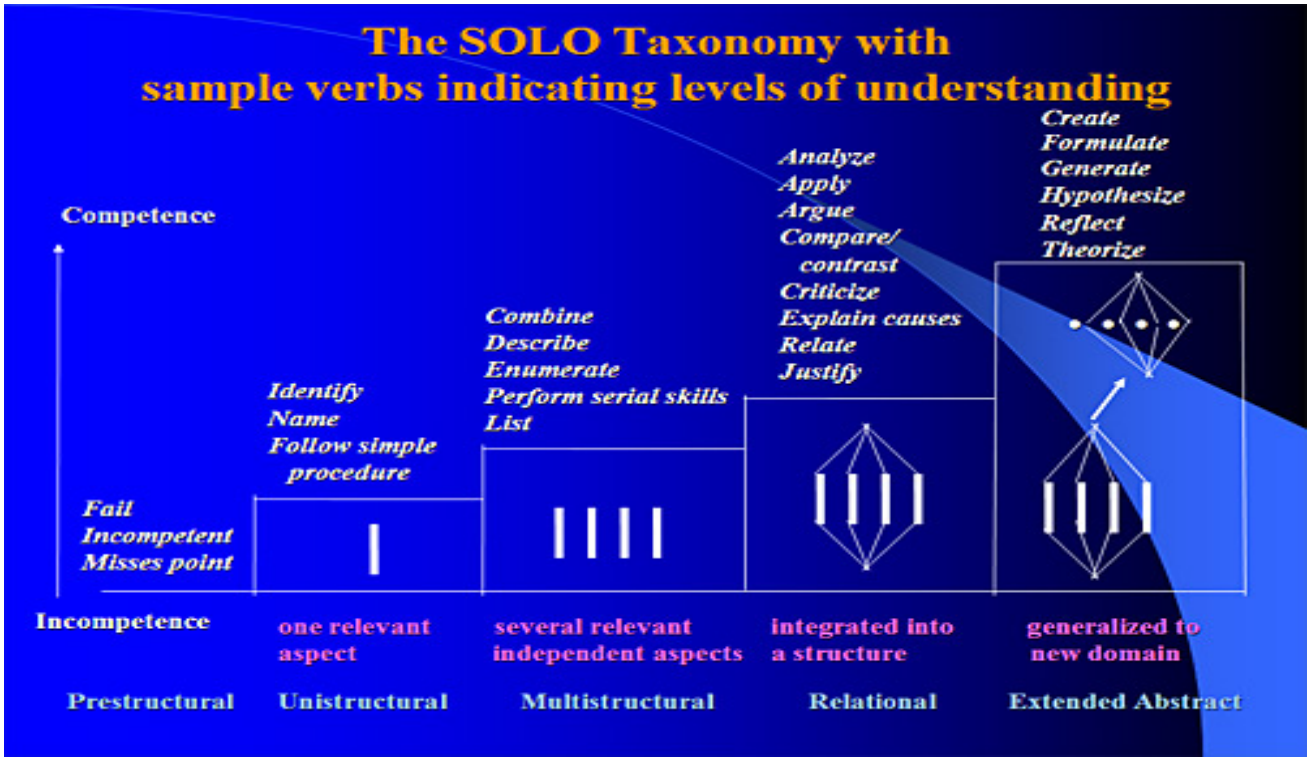
Figure 2.1.



Adopted from South African Institute for Distance Education (SAIDE), (2008)

In support of this constructive view of relational understanding, Desfitri (2016), argues that some of the goals of teaching and learning Pure Mathematics in high school include understanding the concepts , nature of derivatives of trigonometric and algebraic functions and to apply them to determine stationary points (points of maximum, minimum and turning point); and to analyze forms of mathematical models in the forms of equation of functions, as well as applying the concepts in nature of the derivative and the tangent line of the curve in estimating the value of the functions and values of the roots of algebraic equations. Therefore the Structure of Observed Learning Outcomes Taxonomy (SOLO) (figure 2.2) by Brigg and Collins (1982), is a necessary tool in the learning of stationary points since it describes the stages of learning that students go through to reach a real depth of understanding on a topic. It outlines the journey from surface to deep learning. Desfitri (2016)

Figure 2.2



Adopted from John Bigg (2015)

Hattie(2013) and Desfitri (2016) agree that SOLO supports students in developing metacognition, self-regulation, self-efficacy, engagement and resilience when learning Pure Mathematics . Using SOLO, students and teachers can give and discuss feedback more effectively.

2.2.1. Procedural fluency

Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures. Procedural fluency is the ability to apply procedures accurately, efficiently, and flexibly; to transfer procedures to different problems and

contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another. To develop procedural fluency, students need experience in integrating concepts and procedures and building on familiar procedures as they create their own informal strategies and procedures. Although conceptual knowledge is an essential foundation, procedural knowledge is important in its own right. All students need to have a deep and flexible knowledge of a variety of procedures, along with an ability to make critical judgments about which procedures or strategies are appropriate for use in particular situations . Procedural fluency builds from an initial exploration and discussion of number concepts to using informal reasoning strategies and the properties of operations to develop general methods for solving problems (NCTM, 2014). Effective teaching practices provide experiences that help students to connect procedures with the underlying concepts and provide students with opportunities to rehearse or practice strategies and to justify their procedures. Practice should be brief, engaging, purposeful, and distributed (Rohrer, 2009). Too much practice too soon can be ineffective or lead to math anxiety (Isaacs & Carroll, 1999). Analyzing students' procedures often reveals insights and misunderstandings that help teachers in planning next steps in instruction. In the same way, worked examples can serve as a valuable instructional tool, permitting teachers to understand how students analyze why procedures work or don't work and consider what procedure might be most appropriate in a given situation (Booth, Lange, Koedinger, & Newton, 2013).

2.2.2 Traditional Instructional Approach and Its Shortcomings

Kalu (2012) describes the traditional classroom as a one-person show with largely uninvolved learners, seen as empty knowledge-seekers. He notes that traditional classes are usually dominated by direct and unilateral instructions from the teacher, who seeks to transfer thoughts and meanings to the passive learners thereby leaving a limited chance for learner-initiated questions, independent thoughts or interactions among learners. Within this conception, learners are expected to blindly accept the information from the instructor without questioning. Corroborating this, Stofflett (1998) remarks that

the traditional approach followers assume there is already a proven body of knowledge that learners have to abide. The views expressed by Kalu and Stofflett (2012) about the traditional instructional approach might be worthy of belief. The learning approach is a passive transmission of information from the teacher to the learners. It takes learners as empty vessels meant to be filled by the teacher who is recognized as the sole dispenser of knowledge. So, learners have to absorb these ideas and knowledge transmitted to them only to reject such during examinations. In this conventional instructional practice, the learning process is fully controlled by the teacher thereby making it impossible for learners to exercise independence. Besides, the structure of the traditional classroom depends heavily on textbooks, workbooks, worksheets, established facts and the curriculum content. Other studies by Kilavuz, 2005; Kim, 2005; Moyo, 2014; Nayak, 2012; and Nkhoboti, 2002) have indicated that traditional instructional approach has failed to achieve maximal attainment of learning outcomes. With the noticeable deficiency of the traditional approach, most learners have regrettably been unable to connect what they learn in the Mathematics classroom to real life. To the learners, Vander Berg and Louw (2006) observe that, practical knowledge and school knowledge are not really inter-related. In a research conducted by Mochesela (2007), she informs that traditional approach has produced learners whose performances in Mathematics are not satisfactory and who are not sufficiently equipped with critical problem-solving skills that are necessary in this dynamic world. In another investigation by Stofflett (1998), he discovers that the traditional, teacher-dominated, knowledge-dissemination approach evidently promotes rule-bound, rote and mechanistic learning. Being ruled by memorization and learning of isolated concepts and procedures, it usually results to poor knowledge transfer, low academic achievements and under-performances.

In the light of the above, the traditional teacher, as information-giver to passive learners, and the textbook-guided classroom have been unable to bring about the expected outcome of producing thinking learners (Young & Collins, 2003). This is because the approach emphasizes the learning of answers more than the exploration of questions, memorization at the expense of logical thinking, bits and pieces of information as against understanding in context. In addition, the learning approach does

not involve learners in active knowledge construction. It fails to encourage them to work together, to contribute and share ideas with one another; hence, the clamor for the learner-centered constructivist instructional approach. With this realization, there is a need to modify or replace the learning approach with a more applicable and more gainful ones.

2.2.3. Radical Constructivism

In response to the objective-driven, didactic teaching methods traditionally used in mathematics classrooms (Hiebert et al. 2003; Stigler and Hiebert 1999, 2004), some researchers turned to a more cognitive model focusing on conceptual change (Confřey 1990). These researchers felt that purely fact-based mathematics lessons promoting drill-and practice pedagogy did not reflect the personal experiences of the learners (Cobb 1994) and saw them to be too opaque or without character (Steffe 1990) in that it did not reflect the value of the individual's constructions. Researchers who labeled these traditional practices insufficient for mathematics education have recently been drawn to a perspective of teaching called radical constructivism (Steffe and Kieren 1 994 in Maeghan (2012). This framework describes learning as the act of building up cognitive structures based upon one's own views of reality. Radical constructivists do not deny that one, objective reality exists, but claim that there is no way a person can ever know what it is (Goldin 1990; Hardy and Taylor 1997).

Objective reality lies beyond our experience and our cognitive ability to experience it. Instead, our senses and previous conceptual adaptations help us to obtain knowledge, Maeghan (2012). They state that ", knowledge consists of mental constructs which have satisfied the constraints of objective reality" (Hardy and Taylor 1997, p. 137). Because their conception of knowledge is different, their definition of learning must also be different. Learning is defined as the neutralization of a disturbance brought by sensory perceptions (Hardy and Taylor 1997). This neutralization is accomplished when a person reorganizes their model of experience and the activities associated with that experience. In other words, constructivists see knowledge as how a person individually makes sense of their experiences within their interpretation of reality (von Glasersfeld 1990 and Maeghan (2012)). They also suggests that when

a person experiences a situation in which prior understanding conflicts with their current sensory experience, the changing of their understanding is learning. Thus, for a radical constructivist teaching mathematics, knowledge consists of the model of experience and its associated mathematical activities. Radical constructivists believe that the human mind can only know what it has constructed (von Glasersfeld 1990), and that knowledge is not passively received. Rather, new mathematical knowledge is made by the action of the knower. One wonders, then, whether or not a learner could choose to construct any model of reality they wish. Hardy and Taylor (1997) debunk this notion by stating that models of reality constructed by an individual are only considered knowledge when they explain perceived reality. When the mathematical model a student has constructed fails to work, it is replaced. This replacement of failed models by newly constructed models that better explain perceived reality is defined as learning.

2.2.4. Social Constructivism

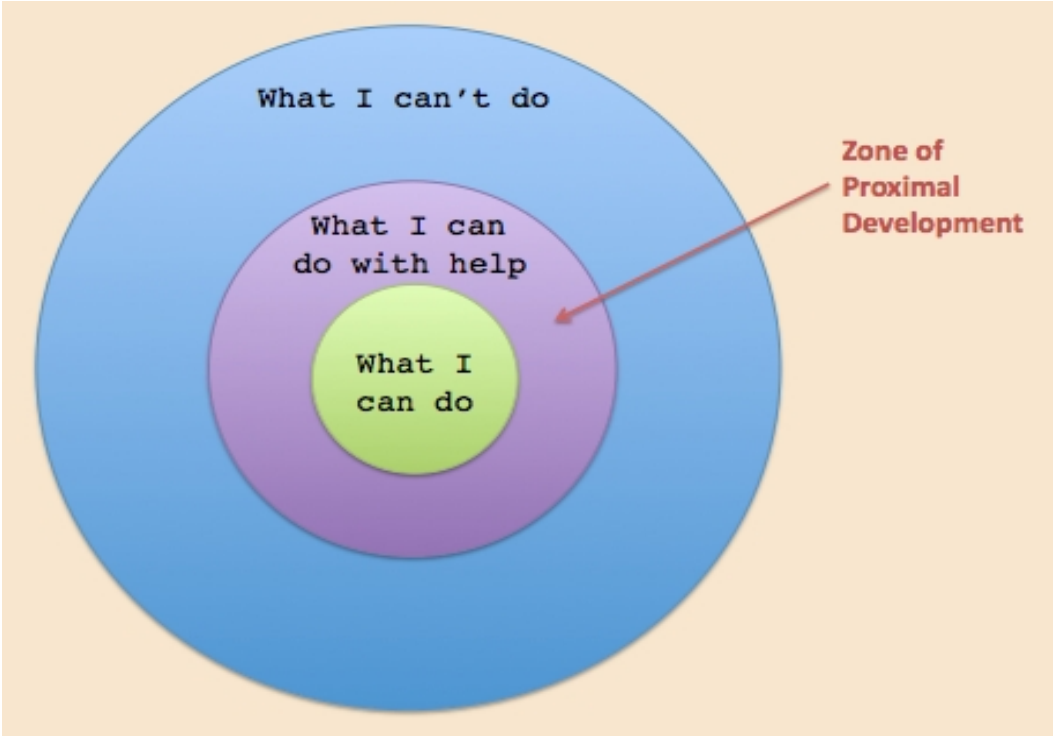
Radical constructivism is seen to have many faults by a number of researchers (Goldin 1990; Cobb and Yackel 1996; Garrison 1997). For example, Garrison (1997) disagrees with von Glasersfeld's notion that people construct their own reality because this individual reality presents only a person's subjective view. Radical constructivists claim that only nature plays a role in the development of conceptions (von Glasersfeld 1996); however, social constructivists believe that society and social situations facilitate conceptual development (Confřey 1990; Garrison 1997 in Maeghan (2012). Garrison posits that an individual's subjective reality is built through a shared reality that everyone experiences. Although radical constructivism does not deny that a social component influences the building of a shared reality (Hardy and Taylor 1997), they do not explain its influence on individual learning. Social constructivists, in contrast to radical constructivists, do explain the influence that a shared reality has on learning.

Maeghan (2012) argues that discussion of ideas is critical to learning, for two reasons. First and foremost, without language, meanings cannot be made because there would be no mechanism for mutual understanding (Confrey 1990; Garrison 1997). Making meaning is a property of social behavior,

then, because we are dependent upon a shared language for the construction of meaning (Confrey 1990; Garrison 1997). Vygotsky (1979 in Denhere etal (2013) also stressed the essence of language in cognitive development. To him, cognitive development results from an internalization of language. Secondly, context and social dialog play a role in the convergence of meaning (Steffe 1990; Loong 1998) because we develop the skills of interpretation and the ability to construct meaning through socialization (Resnick 1989). Social constructivism considers the contextual nature of a learning situation, stating that people perceive their experiences as it makes sense to them within a given social context. As Garner and Hansis (1994) put it, "...people are not governed by recognition of some invariant objective reality; rather, they construct the events that unfold around them, sometimes using stereotypes and other schémas without awareness". Theoretically, the practice of social constructivism in math classrooms sounds enticing because teachers want to promote student discussion in order to enable better learning.

Vygotsky as cited by Denhere et-al (2013) postulated that the knowledge a learner constructs has to first exist in the social context and setting of that learner (Plourde & Alawiye, 2003;Woolfolk, 2010). In view of this, learning occurs when the learner interacts with other people, adults or more knowledgeable peers in his or her social and cultural setting (Kansellar, 2002). This automatically recognizes the essence of language as a vital tool for acquiring knowledge within the social constructivist context (Nicaise & Barnes, 1996). The Vygotsky social constructivist theory identifies two levels of development: the actual level of development and the Zone of Proximal Development (ZPD). At the actual level of development, the learner is capable of constructing knowledge without the help of another person (Woolfolk, 2010).

Figure 2.3



Adapted from (InnovativeLearning.com)

At the ZPD level, to be able to solve a problem, the learner requires the assistance of another person , an adult or a more knowledgeable learner (Kansellar, 2002; Nicaise & Barnes, 1996). The main implication of the Vygoskian social constructivist theory for teaching and learning is that learners must construct their own knowledge and meaning through social interaction with peer learners and adults or their teachers. Researchers have long found that the presence of effective discussions about content promote increased learning (Chinn and Anderson 1998; Commeyras 1993; Murphy et al. 2009), and these findings are similar in mathematics contexts (Loong 1998). While Cobb and Yackel (1996) found that students increased their mathematical understanding by using a constructivist framework for discussions, they do not offer any insights into what aspects of the discussion increased students' learning ,nor are we assured that misconceptions can be identified and corrected , Maeghan (2012). Social constructivism moves in the right direction in that it allows the teacher the ability to steer students toward accepted and shared understandings. However, as will be pointed out in later sections,

the adoption of this paradigm does not adequately represent or describe the actions of the teacher in a mathematics classroom, nor does it alleviate the problem of students relying on prior erroneous beliefs to create mathematical

By augmentation of these two major thoughts of constructivism, we now have the constructivist instructional approach which is at the center of this study. Constructivist teaching encourages reflection by both teachers and students and promotes interactive mathematical communication. In essence, the hallmark of constructivist teaching is that activities are thoughtfully chosen by the teacher to allow the students to construct their ideas.

Summary of comparison of constructivist and traditional approaches. Table 2.1

	Directed/Traditional Instructional Models	**Constructivist Models**
Teaching/Learning Methods	Stress individualized work. Have specific skill-based instructional goals and objectives; same for all students. Teachers transmit a set body of skills and/or knowledge to students. Students learn prerequisite skills required for each new skill. Sequences of carefully structured. presentations and activities to help students understand (process), remember (encode and store), and	Stress group-based, cooperative work. Have global goals such as problem solving and critical thinking; sometimes differ for each student. Have students generate their own knowledge through experiences anchored in real-life situations. Students learn lower order skills in the context of higher order problems that require them. Learning through problem-oriented activities (e.g., "what if" situations); visual formats and mental models; rich, complex, learning

	transfer (retrieve) information and skills. Traditional teacher-directed methods and materials are used: lectures, skill worksheets.	environments; and learning through exploration Nontraditional materials are used to promote student-driven exploration and problem solving
Assessment Methods	Traditional assessments (e.g., multiple choice, short answer) with specific expected responses; student products (e.g., essays) graded with checklists of rubrics.	Nontraditional assessments (e.g., group product such as web pages, multimedia projects) with varying contents or portfolios; student products graded with self-report instruments, rubrics.
Instructional Needs and Problems Targeted	Accountability: All students must meet required education standards to be considered educated. Individualization: Help meet individual needs of students working at many levels. Quality assurance: Quality of instruction must be consistently high across teachers and schools in various locations. Convergent thinking: All students must have the same skills.	Higher level skills: All students must be able to think critically and creatively and solve problems. Cooperative group skills: Help students learn to work with others to solve problems. Increase relevancy: Students must have active, visual, authentic learning experiences that relate to their own lives. Divergent thinking: Students must think on their own and solve novel problems as they occur.

Criticisms	Breaking topics into discrete skills and teaching them in isolation from each other is directed more at basic skills than at higher level ones; students cannot apply skills later (i.e., knowledge is inert). Learning is repetitive and predictable: students often find it uninteresting and irrelevant; lack of motivation leads to lower achievement and higher dropout rates. Not all topics lend themselves to directed approaches. Students cannot solve novel problems or work cooperatively with others to solve problems.	When students are allowed to demonstrate knowledge in varying ways, teachers cannot certify students' individual skill levels, as required by today's accountability standards. Letting students generate their own knowledge is time consuming and inefficient; students may lack prerequisite skills to handle constructivist problem-solving environments effectively. Not all topics lend themselves to constructivist approaches. Despite learning being anchored in authentic problems, students may not transfer skills to real-life situations.

Adopted from source: Allyn & Bacon(2010).

2.2.5. Practices of Constructivism in the Mathematics Classroom

Mathematics teachers who intend to teach in the constructivist paradigm inevitably must consider the social aspects of the paradigm, as they foster a learning environment that is based on the interaction of their students. In addition, they work under the assumption that there is a reality of mathematical concepts and rules that exist outside the students' minds that they would like their students to learn. A successful mathematics classroom in the constructivist paradigm will incorporate many mathematical ideas that are accepted by the class because of shared events they have interpreted together as

implying certain mathematical rules or law. Maeghan (2012) added that ". . .a learning classroom context can be a fruitful breeding ground for conceptual understanding when it gives students the opportunity to verbally express their conceptions and explanations, to compare, question, criticize, evaluate them, as well as to write in support of thinking, reasoning, and communicating''.

The views expressed by Kim (2005), Kalu (2012), Opoh and Iwok (2014) about constructivist pedagogical practices in a Mathematics classroom can as well be found helpful to the teacher. As they claim, in a constructivist lesson, the teacher has to set up suitable hands-on task and find means of engaging the interest of the learners in the topic. He has to introduce certain basic ideas that give life and form to the subject matter. He then revisits and builds on these repeatedly. He can accomplish this by giving a demonstration, telling a relevant short story, presenting some data or showing a short film. The teacher must know that learners come to class with certain ideas about the natural world. He has to take these ideas into account and provide activities capable of enabling learners to develop their current understanding to a more plausible idea. He needs to ask open-ended questions to probe learners‟ pre-conceptions of the topic. He can even present some ideas challenging, contradicting or conflicting with their existing understanding and experiences.

As further noted by the Kim (2005), Kalu (2012), Opoh and Iwok (2014), the teacher should divide the learners into small groups where they will have the opportunity to formulate their own hypotheses and experiments that can make them link their previous understanding with current constructions. In the process, the learners are required to think critically, constructively and intelligently. Thus, the learning environment has to be such that is challenging and supportive of learners‟ thinking. During the small group interaction time, it is required of the teacher to move round the classroom to guide, help or ask probing questions capable of assisting the learners to come to an understanding of the principle being studied. As identified by Brookfield (2005), some of the benefits of learners interacting, collaborating and working together are: learners explore a diversity of perspectives; their intellectual ability increases; learners‟ voices and experiences gain respect and value; habits of collaborative learning are promoted and learners develop skills of synthesis and integration.

Still in line with the opinions of the researchers above, after being given enough time for discussions and interactions, the small groups present their ideas and findings in turn to the whole class so as to reach a consensus on the right ideas to imbibe. Nancy and Gallery (2001) agree that allowing learners to play a much more active role in learning motivates them to learn, to identify and resolve their personal misunderstandings, and to apply what they are learning to situations relevant to their own lives. It enables learners to understand better, and develop strengths and interests in their learning, which in turn provide for life-long learning and career opportunities.

For a constructivist teacher to successfully guide and monitor the learners towards constructing new knowledge, Brownstein (2001) submits that the teacher as the facilitator is expected to display a totally different set of skills from that of a teacher. Corroborating his view, Rhodes and Bellamy (1999) comment: a teacher tells, a facilitator asks; a teacher lectures from the front, a facilitator supports from the back; a teacher gives answers according to a set curriculum, a facilitator provides guidelines and creates the environment for learners to arrive at their own conclusions; a teacher mostly gives a monologue, a facilitator is in continuous dialogue with the learners. Edinyang (2013) summarizes it that the primary task of a constructivist Mathematics is to improve the learning gains of the learner in the subject through the application of appropriate instructional techniques, as the success or failure of the learners in any given Mathematics examinations is often attributed to the ability or inability of the teacher to use good instructional strategy to achieve predetermined instructional objectives

The essentially interactive nature of constructive learning is also extended to the process of assessment. Instead of considering learner assessment as a process carried out solely by the teacher, it is conducted as a two-way process involving both the learners and the teacher. Authentic assessment is best achieved through teaching, interactions between both the teacher and the learners, and learners and learners; and as well by observing learners in meaningful tasks. Holt and Willard-Holt (2000) stress the concept of dynamic assessment in a constructivist learning environment. They explain it as a way of determining the true potential of learners that differs significantly from conventional tests. Learner

assessment should therefore avoid standardized tests and grades such as achievement tests designed with multiple choices to test subject specific knowledge.

Assessment should be made part of the learning process as learners are supposed to play an important role in examining their own progress. The teacher has to enter into dialogue with the learners being evaluated so as to determine their present performance level in a learning task and deliberate with them on possible means of improving upon such performance on subsequent occasions. Types of assessment in line with the constructivist views include: reflective journals/portfolios, case studies, group-based projects and presentations**.**

2.3 Related studies.

Although it is so discouraging to note that very few (less than 30%) of the teachers use learner-centered teaching methods during their teaching of mathematics (Rudhumbu, 2014), most researchers generally acknowledge that constructivist learning strategies contribute significantly towards bringing about effective learning (Arredondo, 2011; Boudourides, 2003; Plourde & Alawiye, 2003; Soanes, 2007). (Kunter 2004) whose study was aiming to describe effects of constructivist teaching approaches on students' learning and interest development in a large, representative sample. In addition, students' individual perception of challenge was considered in order to examine the theoretical assumption that constructivist teaching styles may foster students' interest indirectly via this sense of challenge. The results confirm the theoretical assumptions of self-determination theory (Deci & Ryan, 2000; Ryan & Deci, 2000; Ryan & Powelson, 1991), which states that students' experience of autonomy and competence is one underlying cause for interest development, and they also point to the possibility that this sense of challenge may be enhanced in constructivist learning environments. The present findings thus indicate that a content-oriented perspective to the learning environment may be promising: an instructional approach providing students with the opportunity to restructure their own preconceptions and elaborate existing concepts seems to support the development of intrinsic motivational tendencies via the experience of challenge, even in regular classrooms. The results thus

supplement the existing body of research on instructional quality, which has thus far focused mainly on structural elements of classroom organization. They cannot, however confirm the results from studies on the beneficial effects of constructivist teaching approaches in general, as no direct effects of constructivist teaching elements could be found on interest development, and the small effects on achievement observed in the sample did not persist when comparing students in the different school tracks.

Sunzuma et-al (2012) carried out a research to investigate the factors that hinder the implementation of the student-centered methods in the teaching and learning of mathematics at secondary school level. The study was a descriptive survey, involving questionnaires and interviews with twenty five in service mathematics teachers. From the findings the constraints included examination assessment requirement, time factor, resources especially textbooks, language barrier, teachers' subject matter , content, and other minor ones such as the size of classes and the heavy planning involved. Teachers regarded the student-centered methods as useful and rewarding teaching and learning strategies. Teachers also showed that they are fully aware that their pupils get highly motivated by being involved in student-centered activities. It was therefore concluded that the implementation of student-centered methods in the teaching and learning of secondary school mathematics was far from being realized in the district under study.

More so Alipio (2014) conducted a research which was designed to ascertain teachers‟ perceptions on learner-centered teaching in the discipline of mathematics in basic education. His study was aiming to determine the extent to which teachers background training contributes to perception of learner-centered approach in the teaching of mathematics. He also focused on the extent to which teachers professional experience contributes to perception of learner centered approach in the teaching of mathematics as well as the extent to which teachers background training contributes to learner-centered practices in the teaching of mathematics. The results showed that teachers teaching experiences as well as type of training did not have significant effect on their perceptions of learner-centered teaching. The results of this study have also shown that teachers' professional experiences as

well as the type of training they have received during their training in teaching methods have no significant effects on the type of approach (teacher or learner-centered approach). Teachers of basic education prefer to use both approaches when they teach mathematics. Results from observation schedule show that teachers do not use learner-centered approach. Instead, they use teacher-centered teaching.

The study by Rudhumbu (2014) aimed to investigate the application of motivational teaching methods in the teaching of mathematics in Zimbabwe. Motivating students during their learning of mathematics has been viewed in literature as critical to successful learning of mathematics by students. Students find the learning of mathematics too abstract, mechanical and difficult. This problem has been compounded by teachers' obsession with teacher-centered methods like drill and practice which inhibit students to be creative and to demonstrate problem solving skills. A great deal of research has been carried out on how to teach mathematics as well as on how to incorporate psychological principles of motivation into the teaching of mathematics. The main finding of this study was that teachers in Zimbabwean schools mostly use teacher-centered teaching methods rather than learner-centered teaching methods in their mathematics and this is negatively impacting their ability to motivate students to effectively learn mathematics.

One of the most recent study by Omoniyi (2016) was carried out to investigate whether the application of constructivist instructional approach in the learning of stationary points in differential calculus by Grade 12 learners in South Africa would improve conceptual learning. Three high schools constituted the research fields , one served as the control group while the other two represented the experimental group. Being a mixed-method research, quantitative data was gathered through pre-test and post-test while qualitative data was collected from classroom observations. The results obtained indicate that the experimental group demonstrated a better understanding of the concept of stationary points than the control group.

2.6 Summary

The findings of the literature study reveal that the learning theory of constructivism is a popular paradigm that has proven to be capable of offering useful learning strategies that can improve learners' academic proficiency in Mathematics classes. As active participants in the learning process, learners construct knowledge by themselves based on their prior knowledge, past experiences and perception of the new concept. In Mathematics, this means learners are required to create personal mathematical ideas, interpret and solve real-life problems by their own perceived methods and intuitive mathematical thinking. To do this successfully, enough time should be given to them for small-group discussions, interactions with fellow learners, the teacher and even the learning task, communications, suggestions, questioning, inquiry, exploration and experimentations.

The submission by Nayak (2012) is thus pertinent at this juncture: classroom teaching practice is likely to be more effective when it is informed by an understanding of how students learn and also that learning proves to be more effective if learners are opportune to clarify or explain their own ideas. It is therefore important that the major implications of the learning theory of constructivism should be reflected in classroom practice. However, the question on impact of constructivist instructional approach on the problem-solving approach and procedural proficiency of the learners is still a cause of concern to be scrutinized in this study.

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 Introduction

This chapter will typically focus on how data is going to be collected, what instruments will be used and the intended means for analysing the data in answering the research questions.

3.2 THE STUDY DESIGN

A research design is referred to as a detailed outline of how an investigation or study will take places(Business dictionary, 2017) , pointed out that research design is the road map for the study as it identifies the research's certainty and clarifies the procedures. Makore-Rukuni (2001) sited in Tichapondwa (2013) defines research design as a plan or structure for investigation. The researcher is intending to use the mixed method approach, that is both the qualitative and the quantitative approaches. Quantitative method will be applied because the researcher want to measure the achievement scores of the study participants after the application of the intervention. He intends to justify this by verifying that constructivist instructional approach will be used in the experimental group and the traditional approach will be followed in the control group. Furthermore, he wants to be able to account for the qualitative factors such as learner motivation, and teachers attitude as an indication of the results from the achievement scores.

(Yeasman and Rahman, 2012) state that mixed method research is a methodology for conducting research that involves collecting, analysing integrating quantitative example experiments, surveys and qualitative example focus groups. Denzin (1970) reiterated that by mixing both the qualitative and quantitative paradigms the researcher gains breadth and depth of understanding, while off setting the weakness inherent to use each approach by itself. This yield to triangulation which is encouraged to a researcher when carrying out a study. Although qualitative and quantitative methods differ on how they access knowledge and research questions they address, they are complementary and help to provide a more complete analysis of the research problem (Maree 2007 in Tichapondwa 2013) and offer the best chance of responding to the specified questions. In using the mixed method approach, one ensures that the triangulation of the qualitative and quantitative approaches and data sources in a single study, convergence and corroboration of results from the different methods on the same phenomenon (Tichapondwa 2013)

The researcher is intending to use a pre-test post-test matching control quasi-experimental design. The descriptive research design will also be adopted in this study. The study participants will be randomly assigned into control and experimental groups. A pre-test is going to be administered to both control and experimental groups to determine their initial cognitive level as well as to assess their level of procedural proficiency in the concept of stationary points in differential calculus. Then, post-test will be conducted to gather data on the achievement scores of the study participants.

The descriptive research design will also be involved, by doing classroom observations to collect data on the natural setting of the research fields. Classroom visits are made to ascertain that classes in the experimental schools were conducted in line with the ideology of constructivism. Here, the behaviour of the study participants will also observed and described without necessarily influencing it. The data obtained from the classroom observations will be used to provide support and justification for the learners' scores in the pre-test and the post-test.

3.3 Population of study, Sample and Sampling Technique

3.3.1 Population of study

Nakona, (2006) views that deciding on the population of interest relevant to a study and getting the right respondents that fall within the set categories is crucial to obtaining quality data for it. Hence, a population in the research sense is the complete set of unit of analysis that is under investigation(Davis, 2005). According to Chiromo (2012), population refers to all the individuals, units, objects or events that would be considered in a research project. The population constitutes all the characters under investigation. In this study the target population consist of all lower 5 students in Mhondoro circuit of Chegutu District.

3.3.2 Sampling procedure

Sampling will be done purposively. Hoberg (1992) says that in purposive sampling researchers handpick the case to be included in the sample on the basis of their judgments. In this way they build up a sample that is satisfactory to their specific needs. Barreiro and Albandoz (2001) explain it as the one in which the person who is selecting the sample is who tries to make the sample representative, depending on his opinion or purpose, thus being the representation subjective.www.experiment.resource.com/home also posits that in purposive sampling, subjects are chosen to be part of a sample with a specific purpose in mind. Thus in this study purposive sampling was seen suitable for lower 5 students at Rio Tinto Mhondoro High and Sandringham High since they have a wider catchment area and spheres of influence in Zimbabwe, the results are most likes to be well representative. The researcher is also taking into consideration that these are school which are convenient to him in order to minimize travelling costs due to the geographical location of his school, Presbyterian High

3.3.3 Sample

A sample is a part of the population under study. It is selected from a targeted population. Best and Kahn (1993) describe a sample as a portion of the population selected for observation and analysis. Sampling is a process of selecting a number of individuals for study in a way that they

represent the population. The intention of sampling is that the sample accurately represents the whole population and be representative enough so that results may not be doubted. It is usually difficult to work with the whole population, hence the choice of a sample. The main objective of drawing a sample is to make inferences about the larger population from the smaller group which should reflect the characteristics of the total population.

Out of the targeted 184 lower 5 mathematics learners in Chegutu district, a sample of 52 students from the two will be selected as a sample. 35 students from school A will be the experimental group while 17 from school B will be a control group. A total of mathematics teachers from the respective teachers will also be selected as part of the sample to collect qualitative information from them in the form of interviews.

3.4 Validity and Reliability

The validity and reliability of measurements made in research will also be considered.

3.4.1 Validity

In quantitative research, this is achieved through measurement of the validity and reliability **.**Heale (2015) defines validity as the extent to which a concept is accurately measured in a quantitative study. For example, a survey designed to explore procedural fluency in calculus but which actually measures attitude would not be considered valid is concerned with the soundness or effectiveness of a measuring instrument. A valid statement should give a true measurement, description or explanation of what it claims to measure or describe. Thus, for this study, validity is referring to the effectiveness of data generation instruments like interviews, document analysis and the questionnaire in exploring understanding of stationary points by learners.

In this study all the questions in the tasks and questionnaires all ask about information on differentiation and stationary points in particular data will be valid so they provide a true picture of

what is being measured. Data generated in the questionnaires and the tasks actually give relevant information on the students' understanding of stationary points in differential calculus.

3.4.2 Reliability

According to Heale (2015) the second measure of quality in a quantitative study is *reliability*, or the accuracy of an instrument. In other words, the extent to which a research instrument consistently has the same results if it is used in the same situation on repeated occasions. Reliability deals with the accuracy of measuring instrument. One would want to find if the same result would be obtained if the same instrument were used under similar conditions. Haralambos, Holborn and Heald (2008) postulate that data are seen to be reliable if other sources using the same investigative methods will produce the same result. In this study, reliability will be enhanced by creating an atmosphere that is free from distractions.

For the interview sessions and answering of the questionnaire the participants will be of the purpose of the research as well as the potential benefits of the study. A conducive environment will be created for respondents to feel free during the interview sessions and completion of the questionnaire.

Validity and reliability in this research will be achieved through triangulation. Three instruments will be used, the questionnaire, achievement test and classroom observation checklist. The results obtained using these instruments will be compared and a similarity analysis will be done. The questionnaire for students will consist of open ended questions that the researcher would ask. All the respondents would be asked the same initial questions. Questions will be structured in such a way that the respondent will be required to give his/her opinion or feeling on a particular notion.

3.5 Research Instruments

The measuring instruments to be prepared for the study are – pre-test/post-test, classroom observation checklist and questionnaires

3.5.1 Pre-test/Post-test Instrument

The pre-test and the post-test, will be drawn from ZIMSEC and Cambridge past question papers on the concept of stationary points in differential calculus. The tests will be administered to both control and experimental groups. Each test consist of 6 questions . The questions will be of a reasonable level of difficulty and their solutions demanded a variety of strategies. The questions will be demanding procedural and conceptual understanding. The procedural questions demanded simple procedures, algorithms or rules. On the other hand, the conceptual questions require students' understanding of obtaining stationary points by differentiation. The pre-test and the post-test will consist of exactly the same set of questions, though the questions will and numbered differently in the post-test.

The two groups will be allowed to write the pre-test on the first day of the 4-week empirical investigation; that is before they are taught the topic at all. It will arranged that way to determine the learners‟ pre-knowledge of the topic. After that, the experimental group will be taught with the constructivist pedagogical approach, and the control group through traditional instructional method. Both groups will write the post-test. The post test will assess the possible influence of the of the teaching approaches in developing an understanding of procedures and concepts and their application in real life problems.

3.5.2 Classroom Observation Checklist Instrument

Classroom observations will be carried out to collect relevant data on how the study participants were taught stationary points in differential calculus, especially with the use of the constructivist pedagogical approach. A suitable observation checklist will be developed for this purpose. Classroom observation checklist contains a list of factors to be considered while observing a class. It gives a structure and framework for the observation. The classroom observation checklist will be carefully constructed by the researcher in line with the existing ones already done by experts and with the

thorough guide of his Supervisor. Among the characteristics to be specified on the checklist for observations are:

- how the teacher reviewed the previous lesson,
- introduced the new one and linked both for effective learning
- how he engaged learners in group discussions;
- how he facilitated the discussions and the subsequent group/individual class presentations;
- how he timed and paced the whole lesson;
- the teacher's ability to use questioning to get learners actively involved in class activities
- the extent to which he allows learners' ideas and initiatives.

The checklist constructed for the classroom visits will be validated by specialists in the field of education. This will allow reconstruction and removal of some items of the instrument.

3.5.3. Questionnaire

The researcher will administer the questionnaires personally to ensure a 100% return as well as explaining and clarification of some grey areas which may need more light.

The questionnaire was used as a research instrument and was administered during the students' regular lecture time in order to explore their understanding of differential calculus. It will also help to gather their views on the topic content, method of delivery and their attitude. It is also important to understand how they do this so as to inform effective teaching and learning of the concept. The questionnaire is a power full tool as it enabled the researcher

to gather data from the selected 52 lower 5 mathematics students within a short space of time space of time. The questionnaire for teacher will also solicit for their perception on the traditional and constructivist teaching methodologies as well as on reasons why they use their usual teaching methodologies.

3.6 Data Collection Methods

In the study, the mixed-method approach involving both quantitative and qualitative data collection techniques will be used. At the quantitative level, the pre-test and the post-test and questionnaires will be administered to the study participants. Then, the qualitative part took the form of classroom observations. Pre-test will be administered on the first day of the investigation and the post test after the intervention strategies. Since all the questions will be taken from past examinations, the learners' answers will be assessed and the researcher will make sure he will considered the step-by-step process of getting each answer. This will enable him able to see what the learners' achievements and procedural proficiency before and after the intervention.

The researcher will also do classroom visits to the two schools under study during the normal school hours to gather useful data on how the learners were taught stationary points in differential calculus through traditional and constructivist instructional approaches by their regular Mathematics teachers. This will also make it possible for him to collect data on learner co-operations, contributions and levels of participation during the lessons.

3.7 Data Analysis Procedures

Since the data in this study will be qualitative and quantitative in nature, both qualitative and quantitative data analysis methods will be considered. The data analysis approach in each case will be as follows:

3.7.1 Pre-test/Post-test Data Analysis

The pre-test and post-test scores will be standardized (i.e. converted to percentage). SPSS (A statistical package) will be used to analyse the data with descriptive statistics the descriptive and inferential statistics.

3.7.2 Classroom Observation Data Analysis

Analysis of the data collected from classroom observations followed the qualitative data analysis strategy. After each classroom observation, the data gathered on field notes will be screened using

the classroom observation checklist. The data will be coded, collated and transferred into a spreadsheet. The data were then sorted and the emerging themes noted.

3.8 Ethical considerations

All researchers are expected to take into cognisance ethical issues when conducting their studies. Ethics, as defined by Musazi & Kanhukamwe (2003) are a code of behaviour or expected behaviour of an individual or a group of people in an organization. Ethics and morals are simply concerned with the right and wrong within the context and delimitation of one's study. To ensure objectivity and integrity of the study some ethical considerations had to be taken into account. Informed consent is one of the issues to be observed by the researcher in this study. Thus participants who are to take part in in responding to the questionnaires and interviews as well as the written tasks will fully be informed and the permission to interact with them will be sought from the school authorities. Participants will also be informed that participation is voluntary and they are free to withdraw at any time. Participants will also be assured of anonymity and confidentiality in the study. This issue of confidentiality is to be respected as well. In addition the principle of beneficence will be communicated to the participants. They will be informed of the potential benefits of the study of this such as to improve the academic arena that is, teaching and learning in high schools. The researcher also mentioned that the information collected would only be used for academic purposes. Thus participants were assured that the exercise was a truly academic work and data collected would be used for research purposes only.

3.9 Summary

This chapter dealt with the research design, sampling procedures, research instruments, data collection procedures, validity and reliability and data analysis and presentation techniques. The design involved both quantitative and qualitative methods. The population of study was also defined and sampling procedures were outlined. The next chapter will deal with data presentation ,analysis and interpretation of results.

CHAPTER 4

DATA ANALYSIS AND PRESENTATION OF RESULTS

4.1 Introduction

In the previous chapter, the research methodology adopted in conducting the investigation was discussed. Here in this chapter, the techniques and the results of the data analysis are presented.

4.2 Research Questions

I . Does constructivist instructional approach improve Lower 6 learners' performance in the concept of stationary points in differential calculus?

Findings:

Learners in group B showed a clearer understanding of what stationary points are and why they occur .Instead of memorizing procedures , they were able to explain that stationary points occur where the first derivative is zero and relate this to the gradient of the curve .Group B learners were more successful in distinguishing between maximum points , minimum points and the points of inflection using the second derivative or sign change methods .During lessons group B learners actively participated in group discussions , asked meaningful questions , and shared different solution strategies .This level of engagement was higher than expected in teacher- centered lessons .Active participation allowed learners to learn from peers and correct misconception collaboratively .Common errors such as confusing stationary points with intercepts or failing to interpret derivative results correctly were significantly reduced in Group B. Learners expressed more interest and confidence in Differential Calculus after being taught using the constructivist approach . M any learners reported that learning through discussions and exploration made Calculus less abstract and more meaningful .Positive attitudes contributed to sustained improvement in performance.

2. Does the approach trigger leaners' procedural fluency in solving stationary points related problems in Pure Mathematics?

   Findings:

Learners in Group B consistently followed correct steps when solving stationary point problems .They accurately differentiated functions , equated the first derivative to zero ,and solved for critical values with fewer errors .The Constructivist approach helped learners practice procedures repeatedly in meaningful contexts , improving speed and confidence .Learners in Group B were able to choose between different procedures such as the second derivative test or the sign change method depending on the problem .They explained why a particular method was suitable .This demonstrates procedural flexibility , a key aspect of procedural fluency .Learners in Group B increasingly solved stationary point problems independently without relying on teacher -provided steps .This independence indicates strong procedural fluency development .Common procedural errors such as forgetting to equate the derivative to zero , incorrect differentiation , or misclassification of stationary points were reduced in Group B .Constructivist activities allowed learners to correct mistakes through discussion and guided practice .

3. How does the constructivist instructional approach facilitates understanding of the concept of stationary points in differential calculus?

Findings :

The Constructivist instructional approach starts from what learners already know , such as gradient , curves , and basic differentiation .Learners are guided to link these ideas to stationary points by exploring how the gradient of a curve changes .Constructivist approach methods encouraged learners to explore graphs and functions by sketching curves , analyzing turning points , and observing where the curve becomes horizontal .Through guided discovery , learners see the connection between the algebraic process of differentiation and the graphical behavior of a function .This makes stationary points meaningful and less abstract .Learners in Group B could justify their answers using logical reasoning , such as explaining why the second derivative is positive or negative at a stationary point .This strengthens analytical thinking and deepens conceptual understanding .Constructivist teaching makes use of algebraic , graphical , and verbal representations .Learners relate :

* the equation of a function ,

*its derivative

*and the shape of its graph .

These multi- representation approach deepens learner's understanding of stationary points .Because learners actively construct knowledge through experience , discussions ,and reflection , their understanding of stationary points is more durable and transferable to new problems .Learners are more likely to recall and apply the concept correctly in examinations .

4.3 Data Analysis Strategies

As indicated in the previous chapter, the study adopted the mixed methods involving both quantitative and qualitative data collection methods. This accounted for the adoption of the quantitative and qualitative data analysis techniques for the data collected. The data collected from the pre-test and post-test scores were analyzed by quantitative data analysis methods, while the data gathered from the classroom observations and interviews were analyzed using qualitative data analysis method. The results of the quantitative data analysis shall be presented first for both the schools, followed by the qualitative data analysis results.

4.3.1 Quantitative Data Analysis Strategies

The quantitative data collected were captured on Excel and exported into MINITAB and SPSS for analysis. The analysis were done using the descriptive and the inferential statistical tools.

4.3.2 Descriptive Data Analysis Strategy

Descriptive statistical analysis were used to describe the initial results of the analysis for each of the schools. This took the form of comparison of the two sets of mean and standard deviation of the scores obtained by the study participants in both pre-test and post-test in all the groups. Graphs were also used to show the distribution of the scores.

Table 4.1 < Descriptive Statistics for Pre Test Results

Descriptive Statistics

	N	Minimum	Maximum	Mean	Standard Deviation
Group A Pre Test	20		38	14.65	12.304
Group B Pre Test	20		27	9.75	7.312
Valid N (list Wise)	20				

Although GRP A had a higher maximum as compared to GRP B, the performance in both groups were below standard. This will be clarified further by the mean marks in Table 4.3.

Table 4.2 Descriptive Statistics for Post Test Results

Descriptive Statistics

	N	Minimum	Maximum	Mean	Standard Deviation
Group A Post Test	20	10	68	41.05	17.84
Group B Post Test	20	28	83	55.75	18.168
Valid N (list Wise)	20				

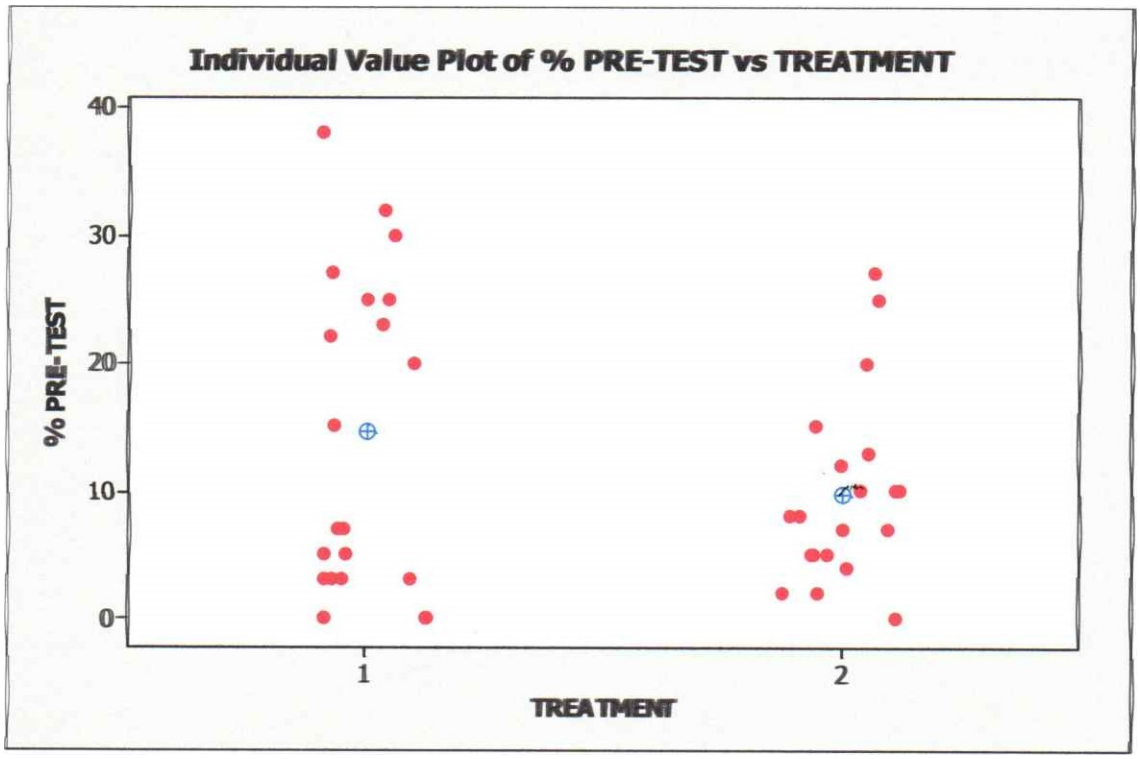
Post test maximum score was 68% in Group A and 83% for Group B. the results in these tables indicate that there was improvement in both groups. How ever there was a remarkable improvement in the experimental Group B, from 27% to 83% on the maximum scores for the pre-test vs the post-test. The noted positive variance is 56% whilst that one in the control group is 30%.

The following analysis is aimed to address whether the constructivist instructional approach improve L6 learners performance in the concept of stationary points in differential calculus?

4.3.3 Pre-test results.

The pretest result are presented by the scatter, which shows the distribution of marks

Figure-4.1 Pre-Test < Individual Value Plot



From Figure 4.1, the pretest marks of the treatment Group A), the control group are widely spread as compared to those from treatment 2 (Group B) which is the experimental group- The higher marks are found in the control group, while those in the experimental group are evenly spread with a lot of participants clustered around 1 however the scatter plot indicates that the general performance in both the experimental group are generally the same .

The two-sample test was used to test for significant differences in the mean scores of the two treatments and results are summarized in Table 4.3.

4.3: Two Results

Two-sample T for % PRE-TEST

Treatment		Mean	Standard Deviation	S E Mean
I Group A	20	14.7	12.3	2.8
2 Group B	20	9.75	7.31	1.6

Difference = μ (l) - μ (2)

Estimate for difference: 4.90000

95% CI for difference: (-1.63620, 1 1.43620)

t-Test of difference 0 (vs not =).• t-Value= 1.53 I^{D}-value = 0.136 DF = 30

The p values is 0, 136,(p=.136 > u=.05) in the confidence interval of 95%. This indicates that there is no statistical significant difference between the performances of the two groups. The performance in the experimental group was the same as the performance in the control group before the intervention of the teaching methodologies. This implies that learners from both groups were at the same cognitive level before the experiment, hence this validate the results to be obtained after the intervention of the teaching strategies.

4.3.4 Post-test results

After the implementation of the two different teaching methodologies, that is the traditional teacher centered in Group A, and the constructivist learner centered in Group B, the following results were obtained.

4.4. Two Sample T Test on Post Test Results.

Two-sample T for POST-TEST

Treatment		Mean	Standard Deviation	S E Mean
Group A	20	41.1	17.7	4
2 (Group B)	20	55.8	18.2	4.1

Difference = μ (1) - μ (2)

Estimate for difference: -14.7000

95% CI for difference: (-26.2364, -3.1636)

t-Test of difference = 0 (vs not =): t-Value = -2.58

P-Value = 0.014 DF = 37

The mean and standard deviation for group I were 41.1 and 17.3 whilst in group 2 were 55.8 and 18.2 respectively. The P values was 0.014 which is less than 0.05, (p=.014 < u=.05). This shows that there is a statistically significant difference in performance between the two groups. There was better performance in Group B (mean = 55.8) than in Group A (mean = 41.1).

The findings are in agreement with the submission of Nhamo (40) that classroom teaching practice is most likely to be more effective when it is informed by an understanding of how students learn and that learning becomes more effective if learners are allowed to clarify or explain their own ideas. For learners to perform their best, the learning theory of constructivism should be reflected in mathematics classroom practices in a more learner-focused way.

4.3.4.1 Change of Performance (Within Groups).

The Paired Sample Test was used to analyze change in performance between the Two Groups. This was done to see whether the teaching strategies under study brings a significant change in performance.

4.5 shows the changes in performance after the intervention.

4.5 Paired for Change

Paired Samples Test

	Mean	Standard Deviation	Std Error Mean	95% Confidence Interval of the Difference			df	Sig (2tailed)
				Lower	Upper			
Pair 1 Group A Pre Test - Group A Post Test	—26.4	17.957	4.015	-34.804	-17.996	-6.575	19	0.000
Pair 2 Group B Pre Test - Group B Post Test	-46	17.773	3.974	-54.318	-37.682	11.574	19	0.000

Table 4.5 indicates that there was a significant difference in performance change from both groups (p=.0 < = .05). This implies that the both teaching strategies brought about significant changes in performance although there is a difference on the level of change between groups. It is however necessary to analyze the level of change using the means as shown in Table 4.6.

Table 4.6 Performance change, Two Sample Test

Two-sample T for CHANGE

Treatment		Mean	Standard Deviation	S E Mean
1	(Group A) 20	26.4	18.0	4.0

2	Group B	20	46.O	17.8	4.0

Difference = μ (l) - μ (2)

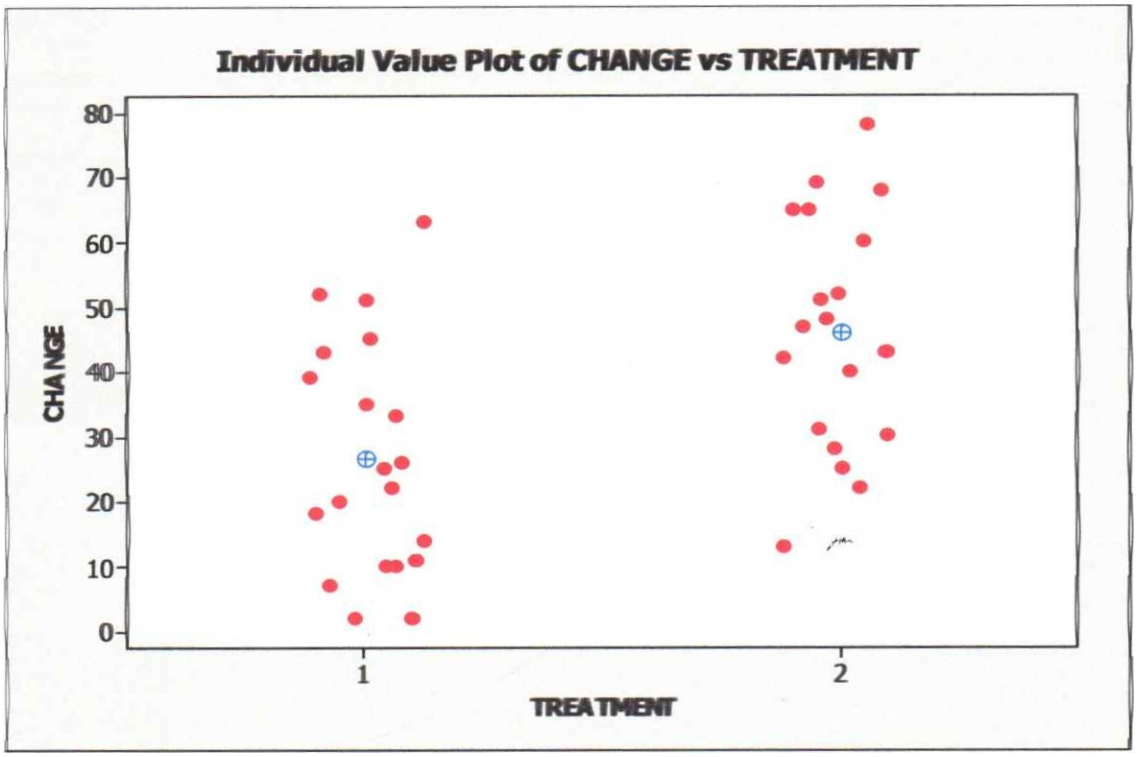
Estimate for difference: -19.6000

95% Cl for difference: (-31.0472, -8.1528)

t-Test of difference = O vs not =): t-Value —- -3.47 P-Value = O.001 DF = 37

The P value being 0.001 which is far less than 0.05 shows that there is a significant difference in performance change at 95% level of significance, a=.05). The change in Group B (mean=46.0) is significantly than tie in Group A (mean=26.4)- This means the constructivist approach yielded better results as compared to the traditional approach.

Figure 42 Distribution on Performance Change



The quantitative results in Figure 4.2 shows that the constructivist teaching approaches has far much better achievement results as to traditional teaching strategies. The change after the intervention in treatment I(Group A) was far much lower than in treatment 2 (Group B). Hence we can conclude that although both teaching methodologies change, constructivist method of instruction is more effective than the traditional approach .

The results are quite in line with the literature study done for the investigation as they conform to the findings of different researchers like Akkus et al, 2003; Guthria Humenick, 2004; Ncharam, 2012; and Taber, (2011) constructivism approach has proven to be capable of developing deep understanding of Mathematics in learners at all levels. Cekolin (2001),also concur with the results as he

found out that constructivist pedagogy improves achievement in Mathematics and the level of confidence for middle school learners and also Kim (2005) who obtained from his study that constructivist learning is more effective than traditional teaching in terms of academic achievement.

4.3.5 Procedural Fluency Analysis.

To address whether the constructivist approach trigger leaners' procedural fluency in solving stationary points related problems in Pure Mathematics , a separate analysis on Post Test scores of two questions which needed application was done on MINITAB.

In the post test, question 2 and 3 (shown below) needed learner to be able to think critically and apply differential and algebraic concepts.

Q2. An open rectangular box has length 2 m and width cm.

a. Given that its volume is 128cm^3 , find in terms of a, the depth d cm, and the total surface area, A cm^2 of the box. (ZIMSEC J2012).

b. Find the dimensions of the box correct to 3decimal places such that the surface area is a minimum. Justify that the surface area is a minimum.

Q3.a. Given that the open box have an Area of 972cm^2 . Given also that the length is twice the width,

i. find the maximum volume of the box.

ii. Show that the volume is maximum.

b. Find the stationary points of the curve y = sin2x +1 and determine their nature for 0 S x S 7t

The questions were in the last two categories of the SOLO taxonomy, namely relational and extended abstract. They were selected to assess the procedural fluency. These questions were analyzed separately and results in Figure 4.7 were obtained.

Table 4.7 Two Sample Test For procedural Fluency.

		Mean	Standard Deviation	S E Mean
Group A	20	33.5	16.9	3.8

Group B	20	58.1	25.5	5.7
Difference = μ (GROUP A) - μ (GROUP B) Estimate for difference: -24.65 95% CI for difference: (-38.56, -10.74) T-Test of difference 0 (vs not =): T-Value = -3.61 P-Value = 0.001 DF=33				

Since the p value is less than 0.05, (p=.001< u=.05), we can therefore conclude that there is a significant difference in the level of procedural fluency between the two groups. As a result, we can strongly conclude that constructivist teaching approach significantly improves procedural fluency in the teaching of stationary points in mathematics.

The results indicated that those in the experimental classroom were in a better position to apply taught concepts on real relational and extended abstract questions of the SOLO taxonomy. Rohrer (2009) also support the notion that effective teaching practices provide experiences that help students to connect procedures with the underlying concepts and provide students with opportunities to rehearse or practice strategies and to justify their procedures. He also suggests that practice should be brief, engaging, purposeful, and distributed.

4.4 Qualitative Data Analysis

For the analysis of qualitative data, information from lesson observations, key informant interviews and questionnaires will be looked into in detail.

4.4.1 Results of the Qualitative Data Analysis for GRP B (Lesson Observation).

In an attempt to assess whether the constructivist instructional approach facilitate understanding of the concept of stationary points in Pure Mathematics, classroom observations were carried out primarily to ascertain that the class activities in the experimental group were in conformity with the dynamics of a constructivist classroom. Group B had two lessons which were observed, with the first one on 15 October from 09:00 to 10 00hours and the second one on 29 October 2025 from 1 1 00 to 12 00hours.

Each observed lesson on the concept of stationary points was of one hour duration with an average of 20 learners in attendance. At the beginning of every lesson, the teacher also asked relevant questions in

introducing each day's lesson in order to arouse learners' interest and curiosity in this topic. He also wanted to find out the learners' prior knowledge about the topic and as well to lead them into making possible enquiries. The teacher informed the learners beforehand the topic for the lesson for them to prepare for it.

In each lesson, the teacher divided up the learners into small discussion groups of an average of 4 learners per group to facilitate effective interactions among them. In their various discussion groups, the learners were allowed considerable time to interact making use of their previous knowledge and experience to solve assigned tasks. They had the opportunity to communicate with one another, make guesses, suggestions and contributions such as, " . .I think we begin by differentiating the function then we equate to 0 and solve for the x values of the turning point".

The teacher was moving around the classroom, monitoring the progress of each group and facilitating the discussions. Without interfering in the learners' group discussion, he was observing and listening to the interactions made by learners and giving them assistance were necessary. After the discussions, all the groups were called upon one by one to present the solutions jointly agreed upon before the class. While each group was presenting its common answers to the class, other learners in the class were allowed to make their individual contributions and the teacher was moderating the discussion. The teacher was quoted saying from the workings and solution got from the two groups which have presented, it shows that they researched deeply in the topic, let's clap hands for them".

The teacher used the answers given by the learners as a basis for explaining the topic making use of useful illustrations and examples to make the concept clear and real. Later, the whole class agreed on the final solutions which the learners were to write down in their notebooks.

The researcher noted that the class activities displayed during the class observations conformed to the constructivist learning principles. The teacher gave the learners ample opportunity to ask and answer questions, make guesses , suggestions, contribute and share ideas. He encouraged learner autonomy , initiatives and also related each lesson to learners' interests, needs and aspirations.

It was also noted that the learners enjoyed the learning environment which they were exposed to. Learners displayed a higher level of motivation and interest in the subject. Thus the constructivist teaching approach triggers inter teaching in the teaching and learning of stationary points. This complies with the findings of Nancy and Gallery (2001) that when learners play a much more active role in learning, they get motivated to

learn, to identify and resolve their personal misunderstandings, and to apply what they learn to situations relevant to their own lives.

4.4.2 Results of the Qualitative Data Analysis for Group A (Lesson Observation)

The lessons in Group A were done using varied traditional approaches such as lecture method and individual work. Participants could hardly speak and did not have more opportunity to interact on their own. A few questions were directed to the teacher who responded authoritatively without giving chance to other learners to explore their ideas.

The classroom atmosphere was tense and was not learner friendly. The researcher observed that some learners were even dosing during the time of lesson. Learners were not enjoying lesson as compared to their counterpart in the experimental group. They were inactive and were not being actively involved. When some were being called to work out some problems on the chalkboard, they experienced challenges in trying to follow the procedures.

The teacher was not telling learners on the forthcoming topic or content to prepare them for the next lesson. From the introduction of the lesson, it was observed that learner had no idea about the topic for the day. This contributed to low morale and lack of motivation on learners. From the lesson observation results, it can be concluded that the traditional teaching approaches contributes to a negative attitude of the subject. The methodologies does not significantly develop learners' procedural fluency since participants had a challenge in solving other problem which needs application.

4.4.3 Key Informant Interviews

Five learners were selected randomly from each group for interviews to make a total of ten. Main areas under analysis were teaching methodologies, procedural fluency and student perceptions on the topic of stationary points.

It was observed that the majority of learners pointed out that the most dominant methodologies from both schools were the traditional methods of teaching such as the lecture method. Some learners indicated that "...at times teachers mix the teaching approaches", that is the use of learner centered and teacher centered in a single lesson or lecture although learner centered approaches were rarely used.

Some learners in the group indicted that "...normally the teacher introduces a topic, work out some problems, then ask some individuals who might have understood to go and work out some examples and followed by work or assignments to do".

Despite the use of teacher centered approaches which favored by most teachers, for convenience purposes, the majority of the leaners stated that "...we enjoy working in groups, working at our own exploring various strategies of solving problems". A number of learners support the constructivist teaching approaches, and need a teacher as a facilitator not the sole source of knowledge.

The majority, of learners from group A confirmed that they have a challenge in applying the taught concept on real life problem, whilst a few from Group B indicated the same. One student were quoted saying ". . . when the teacher present his examples on the chalkboard it may appear as if its an easy thing, but when trying to do the problems later alone they will be very complicated". This has also been noted on the second and third problems in the Post- test from learners in group A, the control group. Only a few learners from group A were able to get the questions correct since it need application of concept. The majority of learners in group B managed to apply. Thus we can conclude that the constructivist approach is effective on improving procedural fluency.

It was also observed that learner perception have a bearing on the understanding of stationary points. Most learners from group A did not really appreciate that the concept of stationary points can be applicable in real life situations. This results in lack of interest among learners of exploring on various concepts of problems or concept. Most of learners from the constructivist class showed that they have an idea on the application of the topic in other real life areas such as maximizing the area of a rectangular garden ,in physics and business.

Student perception on the usefulness of the topic concept in real life application triggers interest in mathematics and improves performance. A number of the learners from traditionalist class did not consider the topic of stationary to have real life application, whilst those in the constructivist class highlighted that it is useful in area like business, engineering and many other areas. A student from the Group A, was quoted saying, "...I do this topic for the sake of passing the exam but otherwise I don't even know its application in any field in real life". The learner indicated some displeasure on this topic.

Another from the constructivist class said, " my desire to become one of the engineers in life probed me to build an interest in this topic since it helps to model and solve some engineering problems". As a result, it can

be concluded that if students are well informed on the application of the mathematical concept, they tend to have more passion in the area and hence improves performance.

4.4.4 Discussion of Teacher Interviews

From the questionnaires which we responded by the A level teachers, the observations were noted. Learners from both schools had the same aptitude level as indicated by their Ordinary level grades where almost similar.

It was also noted that all the teachers in the study agreed that the most effective method of teaching approach is the constructivist method. Although the dominating methods are traditional approaches, educators believe that the constructivist is the best.

Teachers with administrative duties tend to have less time to assist learners on using the modern approach. It was also noted that teachers have a positive perception on the topic of differential calculus. They concur with each other that the topic is very essential and can be applied in many other topics in the Advanced level syllabus.

In this research, the analysis of students' procedures revealed insights and misunderstand that help teachers in planning next steps of instruction. In the same way, Booth, Lange, Koedinger, & Newton, (2013) added that worked examples can serve as a valuable instructional tool, permitting teachers to understand how students analyses why procedures work or don't work and consider what procedure might be most appropriate in a given situation.

The teacher in the control school hardly introduced his lessons with leading questions. The leaners could rarely give correct answer It was observed that this finding conforms to the conclusion of Murray, Olivier and Human (1998) that leading learners to discuss their own views of a problem and tentative approaches raises their self-confidence and provides them opportunities to reflect and devise new and more viable conceptual skills.

4.5 Summary

This chapter discussed the results and findings of the study. As the study was aimed to assess the impact of constructivist instructional approach on lower six performances in the teaching and learning of stationary

points in Pure Mathematics. The research questions were answered. The presentation of the findings was followed by an analysis and discussions of each of the questions given to the students. The way the data was presented is reader friendly. The next chapter constitutes to conclusions, discussions and recommendations

CHAPTER 5

SUMMARY, CONCLUSION, AND RECOMMENDATIONS

5.1 Introduction

This study explored on the impact of the constructive teaching and learning approach on students' performance in Chegutu District. This chapter discusses the findings of the study. The first part of the chapter summarizes the entire study. The summary highlights the main purpose of the study, the statement of the problem, the significance of the problem and the results of the study. This is followed by conclusions drawn from the findings and results of the data analysis. The third part is a discussion by the researcher, explaining the findings and the conclusions reached by the study. The final section gives recommendations for action or practice and for further studies.

5.2 Summary

This study was carried out to assess the impact of constructivist instructional approach in the teaching and learning of Pure Mathematics on lower 6 learners in Chegutu District, Mashonaland west. The researcher used lower 6 leaners from two different Schools in Chegutu district as participants. One school was used as the control group (Group A) whilst the other one (Group B) was the experimental group. The main objective was to establish if the constructivist approach have an impact on, performance, procedural fluency and arouse interest in the teaching and learning of stationary points in differential calculus. In order to have an in depth study of the areas of interest, the research was facilitated.

In an attempt to address the first research question, two schools were selected. Those from school A were assigned into Group A and those from school B were assigned to Group B. Group A was the control group whilst Group B was the experimental group. The study constituted Lower 6 learner, thus 20 from each group.

A pretest on stationary points was given before any interventions to establish the level of aptitude and performance from both groups. The test was made up of 7 questions which were set using the SOLO Taxonomy, by Brigg and Collins (1982), (refer to figure 2.2). This was done to assess the stages of learning that learners go through in order to reach a real depth of understanding on a topic. The learners had not been exposed to the topic. The pre-test results had a mean on 14,7 and 9.75 in Group A and Group B

respectively. The P value was 0.136, (P>0,05) meaning there was no any significant difference between the results from both groups.

In the control group, traditional teaching approaches were employed in the teaching of stationary points in differential calculus. In this approach the teacher spent a lot of time in front of the class explaining concepts and giving examples whilst the learners were busy coping notes and listening. The teacher was the center of knowledge. In Group B, the constructivist approach was used. There was inquiry based leaning were by the roles of the teacher were restricted to guiding the learner. Classroom activities which were most dominating in this class were mainly group work and a democratic interaction between the learners and their facilitating teacher.

After the intervention of the teaching strategies, a post-test was administered to assess the level of understanding. was observed from the post-test results that the mean mark for Group A was 41,1 whilst that one in Group B was 55,7. The p value was 0.001, (P<0.05) implying that there was a significant difference in the performance of the two groups at 95% confidence interval.

Response to question two was addressed by setting higher order questions which needed to assess the development of procedural fluency by the constructivist approach. Question two and three were analyzed separately to establish this fact. Key informant interview guides were also used to make an assessment on the development of procedural fluency.

The quantitative results had the mean of 33 and 58.1 in Group A and Group B respectively, with a p value of 0.001 (P=0,001<0.05) implying that there was a significant difference between the development of procedural fluency between the two groups at 95% confidence interval. Augmenting this is the qualitative data from the KIIG. The majority of learners from Group A testified that they faced a lot of challenges in applying taught concept in higher order questions whilst the minority from Group B were facing .some challenges on procedural fluency.

The third question was addressed by the classroom observations were carried out primarily to ascertain that the class activities in the experimental group were in conformity with the dynamics of a constructivist

classroom. It was observed that learners in the traditionalist classroom had low morale in the lesson since they were not actively involved in the teaching and learning process. Some of the learners were even dosing in class while others appear to be absent minded. The lower the motivation amongst the learners, the lesser their minds' ability to grasp and understand the taught concepts.

In the constructivist classroom, almost every learner was actively involved and was enjoying the lesson. The classroom environment was democratic and learner friendly. Due to this high level of motivation, the learners' level to understanding was far much better as compared to those ones in the control group.

5.3 Major Findings of the Study

Before the recommendations of the study are fully given, highlighted below are the major findings of the investigation from the lower 6 classes under study:

- Constructivist instructional approach improved the study participants' post intervention performance in the concept of stationary points in Pure Mathematics .

- Constructivist instructional approach develops the learners' procedural fluency in the teaching and learning calculus.

- The instructional approach of constructivism has triggered a considerable level of classroom cooperations, interactions, participations and individual as well group knowledge constructions and understanding.

5.4 Conclusion.

From the findings of this research, it can be inferred that constructivist instructional approach might have .facilitated the learning of the concept of stationary points in Pure Mathematics and also brought about the improved performances of the experimental group in the post-test. The findings concur with the submission of Nayak (2012) that classroom teaching practice is likely to be more effective when it is informed by an understanding of how students learn and that learning becomes more effective if learners are allowed to clarify or explain their own ideas. It is therefore important that the major implications of the learning theory of constructivism should be reflected in classroom practices in a more learner-focused way.

Constructivism being an instructional approach that places learners at the center of their learning, learners therefore need to ensure they show as much interest and commitment to any learning tasks that may be assigned them particularly during their class group discussions and even other class activities. They have to work collaboratively and accept the responsibility for their own learning. Learners should therefore, as much as they can, control those learner variables that can prevent them from being actively involved in the learning process. The information from previous chapter shows that the approach under study brought about positive results on procedural fluency.

5.5 Recommendations.

Based on the findings of this study to assess the impact of constructivist instructional approach on teaching and learning of stationary points in differential calculus by lower six students, it has been observed that the teaching approach is very effective in the cognitive development and understanding if correctly implemented by the teacher.

It should however be noted that even though the results of the current study corroborates the findings of previous studies in that they also advocate the use constructivist-based teaching methods over traditional teaching methods on stationary points facilitating learners' performance in Mathematics, it must be cautioned that Mathematics educators in Zimbabwe should not solely cling to the assumption that the constructivist-based approach is a solution for all the Mathematics learning difficulties. At times some components of the traditional approach needs to augment the teaching, though it needs to be minimized. Teaching should necessitate commitment on the part of the learners, teachers, educational managers and administrators.

Teaching entails giving learners the opportunity to identify the gaps and limitations of their own construction of mathematical knowledge to evaluate their own ideas in applying the new knowledge to find solutions to problems in everyday life.

It is against this backdrop that the researcher would do to make the following inferences and recommendations:

- The teaching of mathematics should aim at encouraging a group learning approach, constructive mathematical discourse in their classroom instruction. The knowledge that the learner constructs by himself is more meaningful than that the one that is transmitted to him by the teacher or someone else;
- Educators should critically analyze the learner procedure apart from the concept mastery and solutions. Procedural fluency builds from an initial exploration and discussion of number concepts to using informal reasoning strategies and the properties of operations to develop general methods for solving problems. Guided instruction is needed to assist the learner through analysis of step by step problem solving.
- Mathematics educators should desist from assuming that they are the sole source of knowledge. Educators should play a facilitating role and learn together with learners as they will be guiding them.
- For effective learning to take place, a reasonable amount of work is needed to help the learner construct and develop new concepts. Too much work load will be ineffective since it leads to mathematics anxiety and learners will end up losing interest in the subject. On the other hand too little work will not expose to a variety of problems which are needed by the learner in order to develop procedural fluency and a deeper understanding of the concepts.
- A friendly and democratic classroom environment should be created by the educators. This helps to arouse interest of the subject, develops critical thinking in the learner and promotes innovativeness in the learners' mental cognition.
- Mathematics educators should create awareness among other teachers that the traditional instruction is becoming less and less relevant to achieving the goals of Mathematics education in this modern dispensation. Mathematics teachers should be encouraged to implement the constructivist methods in their classroom instruction for innovation and creativity so as to meet the new curriculum goals. 5.5.1 Suggestions for further studies.

Forthcoming studies can focus on other Advanced level learning areas such as statistics, sequences and series, vectors, geometry and also some aspects on Pure Mathematics such as integration. Greater samples should also be considered since this one was limited to low numbers of mathematics leaners in nearby schools.

Research tools can also be further developed so as to collect all relevant data for the research area. For example some covariates such as sex, age, parental background, cultural background, attitude, learners aptitude and many others may also influence the results apart from the pedagogical approach.

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APPENDICES

Appendix A

PRE-TEST

(QUESTIONS ON STATIONARY POINTS IN DIFFERENTIAL CALCULUS)

INSTRUCTION: Answer all the questions showing clearly your working.

TIME ALLOWED: 1 hour 30 minutes.

I . A curve $y = x^3 — 5x^2 + 8x - 7$

a. Find the gradient of the curve at (2;-3) [3]

b. What does the answer above tell you about the point (2;-3)[1]

(Camb Nov 2007)

2. Find the coordinate of the turning points of the graph of y= 12x -12 .

determine the nature of the turning points, hence sketch the graph of y against x. [61

b. Use your graph to write down the set of values of k for which the equation $x^3 — 12x - 12 - k = 0$ has more than one real root.](ZIMSEC J2012).

3. Given: $f(x) = x^3 + x + 10$

a. write down the coordinates of the y-intercept of f(x). (I)

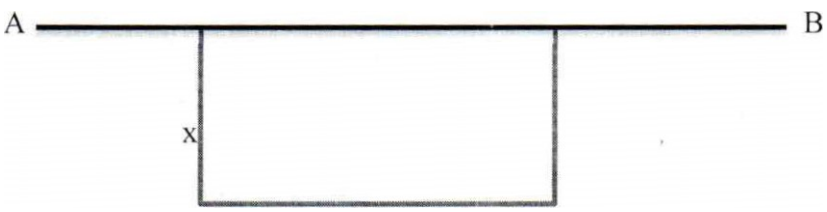
b. show that (2; 0) is the only x-intercept of (4)

c. calculate the coordinates of the turning points of f(x) (6)

d. sketch the graph of f(x) in your answer book

Show all the intercepts with the axes and all turning points (3) (Camb J2013)

4. A farmer wants to make a rectangular pen for keeping sheep, he uses a wall AB as shown in the diagram below.



(28-2x)

The length of the pen is (28-2x) meters and the width is x meters.

a. If the area of the pen is Ym^2, express the area Y in terms of x

b. For the area, Y in (a)

i. $\frac{dy}{dx}$ ii. Find the value of x for which Y is the maximum.

iii. Explain how you know that this value gives the maximum point. [5]

c. find the largest possible area of the pen. [2]

5. An open rectangular box has length $\frac{32}{x^2}$cm and width x cm.

a. Given that its volume is 128cm^3, find in terms of x, the depth d cm, and the total surface area, Acm^2 cm of the box. (ZIMSEC J2012). [4]

b. Find the dimensions of the box correct to 3 decimal places such that the surface area is a minimum. Justify that the surface area is a minimum. [6]

6a. Given that the open box have an Area of 486cm^2. Given also that the length is twice the width,

i. find the maximum volume of the box.

ii. Show that the volume is maximum. [6]

b. Find the stationary points of the curve y= sin2x +1 and determine their nature for $0 \leq x \leq \pi$ [6] (N2013).

7. A curve have the equation y =(x + 2)(x^2 - 3x + 5). Find the coordinates of the minimum point and justify. [5]

TOTAL MARKS [60]

Appendix B

POST-TEST

QUESTIONS ON STATIONARY POINTS IN DIFFERENTIAL CALCULUS

INSTRUCTION: Answer all the questions showing clearly your working.

TIME ALLOWED: I hour 30 minutes

I. A curve have the equation $y=(x + 2)(x^2 - 3x + 5)$. Find the coordinates of the minimum point and justify. [5]

2. An open rectangular box has length .cm and width a cm.

a. Given that its volume is $128cm^3$, find in terms of a, the depth d cm, and the total surface area, A cm^2 of the box. (ZIMSEC J2012). [4]

b. Find the dimensions of the box correct to 3decimal places such that the surface area is a minimum. Justify that the surface area is a minimum. [6]

3.a. Given that the open box have an Area of $972cm^2$. Given also that the length is twice the width,

i. find the maximum volume of the box.

ii. Show that the volume is maximum. [6]

b. Find the stationary points of the curve y = sin2x +1 and determine their nature for $0 \leq x \leq \pi$ [6] (N2013).

4. A curve has equation $y = \frac{8}{x} + 2x$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx}$ [3]

(ii) Find the coordinates of the stationary points and state, with a reason, the nature of each stationary point. [5] [UCLES 2016]

5. The function $y=f(x) = -2x^3 + a^2 + bx + c$ is sketched below. The turning points of the graph of f(x) are T(2;-9) and S(5;18) .

a. Show that a = 21 , b =-60 and c = 43. (7)

b. Determine an equation of the tangent to the graph of f at x = 1. (5)

c. Determine the x-value at which the graph of f has a point of inflection. (2)

6. Given: $f(x) = x; + x + 10$

a. write down the coordinates of the y-intercept of f(x). (1)

b. show that (2; ()) is the only x-intercept of f(x) (4)

c. calculate the coordinates of the turning points of f(x) (6)

d. sketch the graph of f(x) in your answer book

Show all the intercepts with the axes and all turning points (3) (Camb J2013)

TOTAL [601

Appendix C

CLASSROOM OBSERVATION CHECKLIST (DESIGNED IN LINE WITH THE LEARNING PRINCIPLES OF CONSTRUCTIVISM)

Observer: _____

School: _____

Educator: _____

Level: _____ Number of Learners Present: _____

Topic Taught: _____

Date of Observation: _____ Time: _____

The following are the areas for observation in each classroom visit and the rating scale. (NA means Not Applicable)

Observable Characteristic	YES	NO	N/A
A. Classroom Resources			
.1 . The classroom is provided with suitable resources as original sources of information for problem-solving			
2. The classroom is very spacious and learners are comfortable seated			
3. Materials presented are very appropriate to the level of learners			
4. Materials presented are related to the objectives of the learning area			
B. Presentation and Methodology			
I . Teacher begins class at the appropriate time			
2. Initial class activities include:			

(a) Making learners sit in small groups (b) Review of previous work by using questioning approach (c) Statement of lesson's objectives (d) Introduction of the day's learning materials to be presented			
3. Learners en a e in rou discussions as the construct new knowled e			
4. Teacher oes round each learners[er] discussion ou			
5. Each group presents its understanding of the aspect of stationary points bein learnt			
6. Teacher aces lesson a ro riatel			
7. Real-life roblems are discussed and solved in each rou			
8. Study participants present the solutions as discussed and understood in each rou			
9. Teacher encoura es learner autonom and initiatives			
10. Teacher relates lesson to educational goals, learners' personal needs and societal concerns			
I l. Teacher summarizes salient oints of the da 's work			
12. Teacher rovides suitable assi nment re are learners for the next to lesson			

Field Notes

_____

_____

_____

_____

_____

D

KEY INFORMANT INTERVIEW SCHEDULE QUESTIONS FOR STUDENTS

Discussion To ic	Ke conce t	Guide uestions
Teaching methods	a. Most used methodologies b. Useful methodologies to the learner	a. Which teaching methodologies are being mostly used by teachers in teaching differentiation? b. Are the teaching methods helpful in simplifying the topic of differentiation? c. What would you do in order to make sure the teacher understands that you have not understood some concepts of differentiation? d. Which teaching approach do you think should be implemented to help learners understand better
Procedural fluency	a. application of concepts b. challenges faced	a. Where you able to apply the concepts of derivatives on various questions on stationary points? b. Did you face and challenges on the procedures when tackling problems on stationary points?
Written work	a. Tests, exercises, homework and group work.	a. Were the tests and exercises helpful in getting you to understand the principles of differentiation? b. Comment on the exercises, homework, group work and class discussioos which were held during the learnin of differentiation? Were the hel ful?
Student Perception	a. Application in real life	a. Do you think of learning differential calculus and stationary points are a necessity in real life? b. If yes, what are its real life applications?

Appendix E

QUESTIONNAIRE FOR TEACHERS.

The aim of this questionnaire is to collect useful information on the investigating the impact of constructivist learning approaches in the teaching and learning of stationary points in differential calculus at advanced level mathematics. All the responses will be treated as confidential. Therefore there is no need to write your name.

Instructions. May you please fill in the spaces provided or tick the appropriate box.

A. Teacher's qualification and experience.

l. Do you have a teacher training certificate? [yes]

2. What other professional qualifications do you have? …….

3. What is your area of specialization?……..……..

4. How many years have you been in the teaching field? ..……

B. Learners and their aptitude level

l . How many students are taking A-Level mathematics at your school? .....

2. May you please indicate the selection criterial below on their ordinary level passes.

Grade				Total
Number of students				

C. Classroom activities and methodologies

1. Which teaching methods do you use when teaching?.....
2. Which method would you prefer most? ..……

	Never lessons	Some lessons	Most lessons	Every
In your mathematics lessons, how often do you usuall ask students to do the followin ?				
a. ex lain the reasonin behind an idea .				
b. work on problems for which there is no immediatel obvious method of solution ...				
c. work individually without assistance from the teacher				

d. work individually with assistance from the teacher				
e. work together as a class with the teacher teachin the whole class				
f. work together as a class with students res ondin to one another				
g. work in pairs or small groups without assistance from the teacher ..				
h. work in pairs or small groups with assistance from the teacher				

D. Learner assessment and level of understanding.

l . Please fill in the required information?

Tasks in a month	Number	Average pass rate
Exercises		
assi nments		
Home-works		

Tests		

2. What difficulties do they encounter when it comes to the topic of stationary points?

3. Which methods do you use in order to gauge the students' understanding when it comes to this topic?

4. Do students have a challenge in applying the taught concept when approaching other questions in the topic? [Yes] [No]

5. If yes, what are the challenges

6. How do you ensure that students who have not understood the topic of differentiation during classes end up understanding?.....

d, Teacher's perception on the subject/topic

	Strongly disa ee	disagre e	agr ee	Strongly a ree
To what extent do you agree or disagree with each of the following statements?				

a. Mathematics is primarily a formal way of representing the real world. .				
b. Mathematics is primarily a practical and structured guide for addressin real situations.				
c. If students are having difficulty, an effective approach is to ive them more ractice b themselves durin the class				
d. A liking for and understanding of students are essential for teachin mathematics.				
e. differential calculus is very useful to other A-Level mathematics to ics				

In a typical month of lessons for your mathematics class, what percentage of time is spent on each of the following activities? Write in a percentage for each activity The total should add to 100%

a) administrative tasks (not related to lesson's content/purpose .....

b) homework review .....

c) lecture-style presentation by teacher..... .

d) teacher-guided student practice ....

e) re-teaching and clarification of content/procedures.....

f) student independent practice.....

g) tests and quizzes ....
h) other .....
Total
100%

THANK YOU for the thought, time, and effort you have put into completing this questionnaire.

Appendix F

LEARNER'S INFORMED CONSENT

Information for Research Participant

As a postgraduate student at Bindura university of Science Education (student number: B240134A), I am conducting an academic research on the topic: Impact of Constructivist Instructional Approach on L6 Learners' Understanding of Stationary Points in Differential Calculus. As the research topic implies, the purpose of the study is to determine the extent to which the application of the theory of constructivism enhances L6 learners" understanding of stationary points in differential calculus. It is in the light of this that your consent to participate in the research work has been sought. Please, note that any information supplied shall remain

strictly confidential and anonymous, and shall be used for the purpose of this investigation only. If you are willing to participate in the research, please sign this informed consent form. Thank you for your interest and cooperation.

Researcher's Name: Chitsuro M.

Signature: Date:…….

Participant's Declaration

I ……(optional) hereby confirm that I have been well informed by the researcher about the nature, conduct, benefits and fisks of the study. I have also read and understood the above information. I am aware that the outcome of the study shall be anonymously processed into a research report. I understand that my participation is voluntary and that I can, at any level of the study, without prejudice, withdraw my consent and participation in the study. I had sufficient opportunity to ask questions and therefore, of my own volition, declare my intention to participate in the study.

Research Participant's Signature:_____ Date:…….

Appendix G

EXAMINORS' REPORT

MATHEMATICS

SUBJECT 9164

PAPER 1

NOVEMBER 2010

GENERAL COMMENTS

The paper was quite standard, covering most parts of the syllabus. Candidates must manage time in answering the questions since many could not finish the paper. Some last questions were not attempted by many candidates and these being questions with most marks Olfihe question paper means that the total marks were low for such candidates.

Candidates are reminded of the importance of rounding their answers to the required degree of accuracy if marks are not to be lost unnecessarily. For example, some marks were lost in answering 9(c) and 14 (iii) for this reason.

SPECIFIC COMMENTS ON QUESTIONS

QUESTION 16

In pert (a) (i) most candidates were not able to make the correct connections of the chain rule. Hence deriyation of the differential equation was achieved by very few. In part (a) (ii), many candidates were able to separate the variables and obtain the initial part of the general solution correctly. Some were not able to make h the subject by failing to exponentiate correctly. Some were not able to deduce the maximum value of h. In (b), most candidates were not able to go beyond the general solution since they were not sure about how to apply the initial conditions

SPSS OUTPUT

PRETEST RESULTS

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation

GRP A PRE-TEST	20		38	14.65	12.304
GRP B PRE-TEST	20		27	9.75	7.312
Valid N (listwise)	20				

POST TEST

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std, Deviation
GRP A POST-TEST	20	10	68	41.05	17.840
GRP B POST-TEST	20	28	83	55.75	18.168
Valid N (listwise)	20				

Paired Samples Statistics

	Mean	N	Std. Deviation	Std. Error Mean

Pair 1	GRP A PRE-TEST	14.65	20	12.304	2.751
	GRP A POST-TEST	41.05	20	17.840	3.989
Pair 2	GRP B PRE-TEST	9.75	20	7.312	1.635
	GRP B POST-TEST	55.75	20	18.168	4.063

Paired Samples Test

	Paired Differences					t	df	Sig. (2tailed)
				95% Confidence Interval of the Difference				
	Mean	Std. Deviation	Std. Error Mean	Lower	Upper			

Pair 1 GRP A PRE-TEST GRP A POST-TEST	-26.400					-6.575		.000
Pair 2 GRP B PRE-TEST GRP B POST-TEST		17.957	4.015	-34.804	-17.996		19	
		17.773	3.974	-54.318	-37.682	-11.574	19	