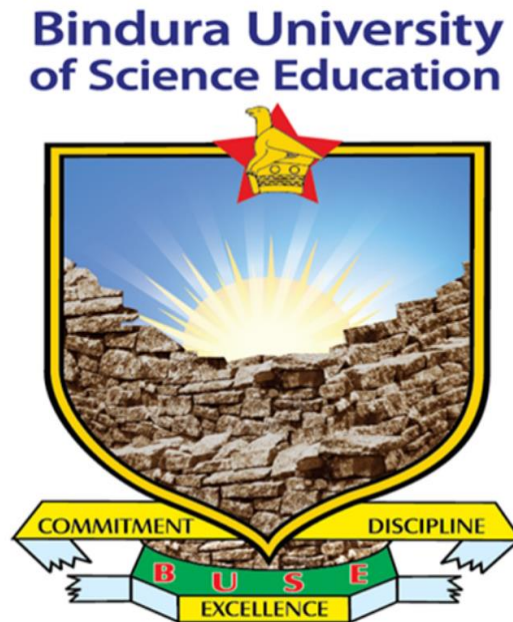


**A COMPARATIVE ANALYSIS OF ARIMA AND EXPONENTIAL SMOOTHING
MODELS IN FORECASTING ZIMBABWE'S TERMS OF TRADE.**

**BINDURA UNIVERSITY OF SCIENCE EDUCATION
FACULTY OF SCIENCES
DEPARTMENT OF STATISTICS AND MATHEMATICS**



**A COMPARATIVE ANALYSIS OF ARIMA AND EXPONENTIAL SMOOTHING
MODELS IN FORECASTING THE ZIMBABWE'S TERMS OF TRADE**

BY

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B222983B

**A DISSERTATION SUBMITTED IN THE PARTIAL FULFILMENT OF THE
REQUIREMENTS OF THE BACHELOR OF SCIENCE HONOURS DEGREE IN
STATISTICS AND FINANCIAL MATHEMATICS**

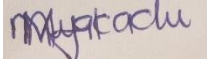
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JUNE 2026

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APPROVAL FORM

The undersigned certify that they have read and recommend to the Bindura University of Science Education for acceptance of a dissertation entitled “A comparative analysis of ARIMA and Exponential Smoothing models in forecasting the Zimbabwe’s Terms of Trade” submitted by B222983B in partial fulfillment of the requirements of Bachelor of Science Honors Degree in Statistics and Financial Mathematics has successfully completed all the requirements for a dissertation and approved for final submission.

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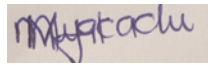
DECLARATION

I, Moreblessing Mujakachi, declare that the work presented in this study is my own work apart from the work that is clearly indicated and cited in the text derived from other sources. As a result, I confirm that this has not been submitted before, for any degree at this university or any institution elsewhere.

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Date: 05 June 2026

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DEDICATION

I dedicate this work to my family for the unwavering support and for every word of encouragement whenever I doubted myself. I am grateful for the love of my life for being a constant source of support throughout my academic journey.

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First and foremost, I would like to give glory and honour the Almighty God for His unending grace wisdom and strength throughout this whole academic journey, for the word says in Philippians 4:13- “ I can do all things through Christ who strengthens me” and with HIM all things are possible.

I extend my deepest gratitude to my supervisor, Ms.P.Hlupo, for her guidance, useful advice and constructive feedback that shaped this work at every stage. Her mentorship has been a gift that I will carry far beyond this study. I am also grateful for the support and guidance extended to me by all other lecturers in the department.

To my friend, Tawananyasha Hamandishe, I owe you a debt of gratitude. Thank you for the late nights, for the time spent discussing the models and encouraging me when the data refused to behave. I appreciate you for being my sounding board, my motivator and for the practical help throughout the research journey.

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ABSTRACT

This study conducted a comparative analysis of Autoregressive Integrated Moving Average (ARIMA) and Exponential Smoothing (ETS) models for forecasting Zimbabwe's Terms of Trade (TOT) using monthly data from January 2020 to December 2024 (60 observations). The data were split into training (80%, 2020-2023) and testing (20%, 2024) sets. Diagnostic tests revealed that the TOT series was non-stationary (ADF $p=0.2068$), requiring first differencing ($d=1$), with moderate volatility ($CV=14.41\%$) and heavier tails than normal (kurtosis=2.69). The ARIMA (0,1,1) model achieved a MAPE of 8.90%, RMSE of 12.60, and MAE of 9.63, while the ETS(A,N,N) model achieved a MAPE of 8.92%, RMSE of 12.62, and MAE of 9.66. The negligible differences across all metrics indicate that both models produce statistically equivalent and highly accurate forecasts ($MAPE < 9\%$). Despite parameter instability ($CV=0.15$) and the April 2024 currency redenomination structural break, both models maintained high accuracy and outperformed a Random Forest alternative ($MAPE=10.04\%$). Diagnostic tests confirmed no residual autocorrelation (Ljung-Box $p=0.527$) and constant variance (Breusch-Pagan $p=0.860$), though slight non-normality was detected (Shapiro-Wilk $p=0.047$). The 2025 forecast predicts stabilization of TOT around 106-110 index points. The study concludes that both models are equally suitable for forecasting Zimbabwe's TOT, with ETS recommended for its simplicity and ARIMA for its traditional statistical framework. These findings provide evidence-based guidance for policymakers at the Reserve Bank of Zimbabwe and the Ministry of Finance.

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ACRONYMS

ARIMA	– Autoregressive Moving Average
ETS	- Exponential Smoothing
TOT	- Terms of Trade
GDP	- Gross Domestic Product
AR	- Autoregressive
I	- Integrated
MA	-Moving Average
ADF	- Augmented-Dickey Fuller
PACF	- Partial Autocorrelation Function
Q-Q	-Quartile-Quartile
ZWL	-Zimbabwean Dollar
ZiG	- Zimbabwe
MAE	- Mean Absolute Error
MSE	-Mean Squared Error
RMSE	– Root Mean Squared Error
MAPE	– Mean Absolute Percentage Error
RBZ	- Reserve Bank of Zimbabwe
ZIMSTAT	– Zimbabwe National Statistics Agency

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LIST OF SYMBOLS

Y_t	 Time series value at time t
$\hat{Y}_t$	 Forecasted value at time t
ε_t	-..... White noise error term
μ	-Population mean
σ^2	-..... Population variance
B	-..... Backshift operator ($B y_t = y_{t-1}$)
d	-..... Order of differencing
p	-Order of autoregressive component
q	-..... Order of moving average component
ℓ_t	-..... Level component (Exponential Smoothing)
b_t	-..... Trend component (Exponential Smoothing)
s_t	-Seasonal component (Exponential Smoothing)
α	-..... Level smoothing parameter ($0 \leq \alpha \leq 1$)
β	-Trend smoothing parameter ($0 \leq \beta \leq 1$)
γ	-Seasonal smoothing parameter ($0 \leq \gamma \leq 1$)
Δ	-..... First difference operator
n	-..... Number of observations
KPSS	-Kwiatkowski-Phillips-Schmidt-Shin test
CV	-Coefficient of Variation

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CHAPTER 1: INTRODUCTION

1.0 Introduction

It is very important to know the ability of accurate forecasting of the economic indicators for effective management of the macroeconomic management, mostly in the developing economies like Zimbabwe. This is because developing economies are most negatively affected by the external shocks (Rossi, 2021).

The terms of Trade are the important indicators among the indicators that helps in policy decisions as they have a central space because they influence foreign currency, stable balance of payments and economic growth as a whole (Kembo, 2026). The prediction of the Terms of Trade identifies a unique challenges in the changing economic conditions where the structural breaks, non-stationarity and parameter instability are common (Xia,2022; Rossi, 2021).

The research is a comparative analysis of the prediction models that are mostly applied when dealing with the economic time series which are Autoregressive Moving Average (ARIMA) and Exponential Smoothing modelling the Zimbabwe's Terms of Trade. The choice of selection for the two modelling approaches are justified by contrast theoretical foundations whereby the ARIMA model was supported by the stochastic process theory and the autocorrelation structures (Bambang & Junaidi, 2012) and the Exponential Smoothing model emphasize on adaptability and recognition of patterns. The researcher aims at identifying the model among the two that gives the best prediction performance under the volatility conditions of the Zimbabwean economy.

This section is an introduction of the study, comparative analysis of the ARIMA and Exponential Smoothing models in predicting the Zimbabwe's Terms of Trade, it will bring out the core aims and the problems that the researcher noticed that drives for the study. It will give the detailed theoretical insights that are aligned with the study in the next chapter. The study will also narrate the methodological framework that will be used to contact the forecasting techniques, examining the type of data, how it was gathered and how it is prepared in order to run the models, giving justification for the methodology. After the methodological chapter, it will move to the Results and discussion, where the study will show how the models were practically run and reviewing the results and make comparisons basing on the empirical

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evidence of the forecast. The chapter of the results will help the researcher to evaluate the objectives and questions mentioned from the first chapter and later draw conclusions and recommendations for the final chapter.

1.1 Background of the study

International trade plays a pivotal role in the economic growth and development especially for the countries integrated into global value chains. The Terms of Trade (TOT), which is the measure of the ratio of export prices to the import prices, provides a critical measure of the welfare implications of trade. An improve in the TOT reflects the ability of a country to purchase more imports for a given volume whereas a decline in the TOT reflects the opposite (Prebisch, 1950 & Singer, 1950).

Global fluctuations in the TOT, have been linked to business cycle volatility, BOP crises and long-run growth trajectories (Mendoza, 1995, Blattman, Hwang & Williamson, 2007). The empirical research demonstrated that commodity,-exporting in developing countries has greater TOT volatility than the industrialized nations due to concentration of exports in primary commodities with highly elasticity global demand (Cashin, McDermott & Scott, 2002). Some challenges faced by macroeconomic management causes volatility and foreign exchange shortages are triggered by unexpected shifts in trade prices not forgetting inflationary pressures and imbalance of fiscal.

The instructive setting for TOT dynamics assessment is given by continent of Africa as we move on to the African context of the study. Cultural characteristics are shared with most African economies with dependence on a narrow range of commodity exports primary is included and diversification of industry is limited with global price shocks exposed (UNCTAD, 2021). Real exchange rate appreciation and decline of manufacturing companies are led by commodity price booms in Africa (Collier & Goderis, 2012). African trade highlighters remains sparse are evaluated by prediction models of systematic comparative (Akinbobola, 2012, Moolman, 2013, Nyoni 2020).

The economy that is characterized by dependence of structures on commodity of exports such as minerals, tobacco and agricultural products is Zimbabwe (Reserve Bank of Zimbabwe, 2022). This structural configuration renders Zimbabwe vulnerable to Terms of Trade

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deterioration, consistent with the Prebisch- singer hypothesis. The Zimbabwean TOT have been experiencing notable fluctuations for some years, which shows the shifts in global commodity prices, exchange rate changes, challenges in domestic constraints and policy interventions (Muchenje, 2023; World Bank, 2024). The improvement of the Terms of Trade in a country results in temporary gains in export earnings and the depreciating of the Terms of Trade reflects the worsen of economic trade deficit and shortage of foreign currency (ZIMSTAT, 2024; Kembo, 2026). A good example of a structural break in Zimbabwe that affected the price dynamics is a currency redenomination that happened in April 2024 when the exchange rate shifted from over 22 000ZWL per USD to around 13 ZiG currency per USD (Reserve Bank of Zimbabwe, 2024).

For a study to obtain accuracy when predicting the Terms of Trade time series data, the statistical approaches applied should be able to capture the underlying temporal patterns like trends, seasonality, stochastic fluctuations and autocorrelations (Hyndman & Athanasopoulos, 2021). ARIMA models are the most suitable approaches for analyzing the univariate time series data by modelling autocorrelations, achieving stationarity through differencing (Box & Jenkins, 2015; Box et al.; Zhang, 2022). ARIMA models rooted in the Box-Jenkins methodology, are particularly suited for analyzing univariate. These models are more effective in capturing complex linear dependencies within economic time series, making them a popular choice in macroeconomic forecasting (Enders, 2015).

Exponential Smoothing methods (Brown, 1959; Holt, 1957; Winters, 1960) are more flexible and adaptable framework by assigning exponentially decreasing weights to past observations. Simple Exponential Smoothing, Holt's linear trend method and the Holt-Winters seasonal model are the variants that are particularly useful in handling data with trend and seasonal components (Hyndman & Athanasopoulos, 2018). The models are simple and have computational efficiency and strong performance in short term forecasting context.

Despite the extensive application of the two methodologies in global forecasting literature, there is less empirical proof comparing the predictive performance in the Zimbabwean context of external sector. Most existing studies tend to focus on general macroeconomic indicators such as inflation, GDP, and exchange rates with less emphasis on trade-specific indices (Makoni, 2018; mupunga & Dube, 2019; Nyoni,2020). Mostly when forecasting is done,

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studies mainly rely on single modelling approach without undertaking a rigorous comparative evaluation. With the volatility structures in Zimbabwe's trade environment, it is not immediately evident which modelling framework gives the superior forecasting accuracy. The uncertainty constitutes the central motivation this study that needs to evaluate the predictive efficacy of ARIMA and Exponential Smoothing models in forecasting the Zimbabwe's Terms of Trade using the monthly time series from January 2020 to December 2024.

1.2 Problem statement

Accurate forecasting of Terms of Trade (TOT) is critical for Zimbabwe's macroeconomic management, yet persistent volatility from fluctuating commodity prices, exchange rate instability and structural production constraints. Zimbabwean economy is facing economic challenges of foreign currency scarcity and unstable earnings.

The most used approaches in forecasting economic time series data are ARIMA and Exponential Smoothing models, with their relative performance depends on data characterized by volatility and structural breaks that are most common in Zimbabwe. However, the researcher needs to weigh out the accuracy of one model to the other so as to come up with the best forecasting approach for the Terms of Trade between the ARIMA and Exponential Smoothing after discovering that the past studies rely on forecasting some other macroeconomic indicators like inflation and exchange rate using one forecasting model.

There is a discoverable statistical gap of uncertainty regarding the most accurate forecasting approach when dealing with the Terms of Trade in a fluctuating economic environment. This is leaving the policymakers and investors without empirically grounded guidance on the most reliable forecasting approach for the Terms of Trade. This present study seeks to bridge the gap by providing a comparative assessment of the ARIMA and Exponential Smoothing models using the monthly TOT, with the aim of identifying the best forecasting approach thereby improving the policy formulation and economic decision making in Zimbabwe.

1.3 Research objectives

The main aim is to evaluate and compare the predictive efficacy of Autoregressive Integrated Moving Average (ARIMA) and Exponential Smoothing models in forecasting the Zimbabwe's terms of trade using a monthly time series data.

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Specific objectives

1. To examine the stationary properties and temporal characteristics including trends and seasonality of Zimbabwe's terms of trade over the study period.
2. To estimate the optimal ARIMA model for forecasting Zimbabwe's Terms of Trade.
3. To estimate the optimal Exponential Smoothing (ETS) model for forecasting Zimbabwe's Terms of Trade.
4. To evaluate and compare out-of-sample forecasting accuracy of ARIMA and Exponential Smoothing using standard statistical error measures such as Mean Absolute Error (MAE), Mean Squared Error (MSE) AND Root Mean Squared Error (RMSE).
5. To determine the most best forecasting model for Zimbabwe TOT among the ARIMA and ETS based on empirical results and model performance.

1.4 Research questions

The main question for the study is: Which forecasting model, is better when minimizing forecast errors in forecasting the Zimbabwe Terms of Trade between the ARIMA and Exponential Smoothing?

1. Does the Zimbabwe's monthly Terms of Trade time series data show stationarity and what are the present temporal characteristics, trends, seasonality, volatility?
2. Between the two forecasting models, which one results in a lower out-of-sample forecasting error measured by the standard statistical measures?
3. How do the models performs in managing the periods characterized by volatility and structural shifts in the Terms of Trade data?
4. Which model is empirically more appropriate when forecasting the Terms of Trade basing on the comparative error metrics?

1.5 Scope of the study

Time series data monthly is used when predicting Zimbabwe's terms of trade, and this study focuses on that area. Data were collected over a specific period from January 2020 to December

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2025, with 60 months of observations. External trade performance in Zimbabwe examines trends, seasonal patterns, and short-term fluctuations using time series data, but sudden changes in trade conditions that are triggered by commodity price volatility or exchange rate shocks can be provided by quarterly or annual data sensitivity. Parameter estimation's reliability with sufficient degrees of freedom is provided the sample size while manageable conditions remain small enough will be allowed. According to Box and Jenkins (2015), ARIMA modelling is recommended for at least 50 observations so that stable estimates will be achieved so the use of 60 months in this study is comfortable. Significant economic events are captured in this period practically with their effect of trade dynamics in Zimbabwe with post-covid 9 recovery period, the denomination of currency in April 2024 and the hyperinflation episode of 2023 included and the model performance is allowed in this study for it to be assessed under both high volatile conditions and stability.

Geographically: International trade dynamics are emphasized with particular limitations of Zimbabwe's study and also it may have broader implications with similar structural characteristics on developing economies. The analysis of economic environment in Zimbabwe with exchange rate, commodity price fluctuations and exposure to external shocks included is contextualized.

The application and comparison of ARIMA and exponential smoothing techniques is restricted in this study but advanced prediction models like machine learning models, structural econometric or vector autoregressive are excluded to maintain of analytical focus and depth in comparative evaluation is ensured. The historical behavior of the predicted variable is solved with univariate models because they are suited for this study. Zimbabwe's trade dynamic is available for the reliability of exogenous variables and unreliable multivariate approaches are limited.

Diagnostic tests are incorporated and validation procedures are modelled with statistical errors like RMSE, MAE, MSE being used and the analysis of this study limits evaluation prediction accuracy so there is no extension of casual inference or structural modelling of determinants of trade. Two modelling approaches evaluate if the focus remains objective, comparable and quantifiable is ensured.

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1.6 Significance of the study

Practical predicting applications are enhanced, policy is formulated and academic knowledge contributed in this study within external sector in Zimbabwe.

Prediction accuracy is ensured by the study on the academic significance to compare ARIMA and exponential smoothing analysis of terms of trade in Zimbabwe whilst prediction of the models are focused in this study independently. The empirical work that compares their performance is limited and proof based insights is offered in the model suitability under structural and conditions volatility. The study period includes the periods of hyperinflation and currency redenomination which makes the study contribute to the new empirical evidence on how the models perform during these conditions.

Moving on to the policy perspective, the results are relevant to the government organizations that are involved in economic planning and external sector management. Accurate predictions of the Terms of Trade are important for designing effective trade policies, managing foreign exchange reserves and anticipating shifts in the balance of payments. The study is going to be beneficial to the researchers, economists and financial analysts by identifying the forecasting provides more accurate prediction results.

Institutions and researchers who are engaged in economic modeling, are offered practical guidance on model selection and implementation when dealing with the time series data that includes conditions of structural breaks, , volatility and data quality. It will help the practitioners to have a good choice on the appropriate model of their forecasting needs. The study bridges the gap between theoretical modelling and empirical analysis by demonstrating how statistical tools can be applied to address real-world economic problems. This increases the understanding of time series forecasting techniques in their practical value.

Last but not least, the study applies rigorous statistical techniques to evaluate forecast accuracy using the standard error measures. It conducts diagnostic tests, which strengthens the reliability of findings and provide the methodological template that can be replicated in future studies involving other macroeconomic variables.

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1.7 Limitations of the study

Limitations refers to the uncontrollable weakness that the study has. It's the factors that are beyond the researcher's control may affect the validity of the results.

1. The study uses secondary data gathered from the official sources that the researcher may not have control over the data collection process of the original data. There may be inaccuracies, inconsistencies in data recording and missing values, particularly in developing country data sets may affect the robustness of the analysis despite efforts to clean and process data.
2. The study is using the monthly terms of trade data because the researcher had no control over the availability of higher-frequency data like the weekly or daily observations, which could have captured the short-term dynamics more accurately.
3. The use of univariate time series models implies that the forecasts are based on historical patterns of the Terms of Trade without incorporating exogenous variables such as exchange rates, global commodity price or policy changes. The models may not fully capture the underlying economic drivers of the trade dynamics.
4. The presence of structural breaks arising from economic crises, policy shifts or external shocks may affect stability and predictive performance of the models while diagnostic checks may partially address this issue, remains a limitation that no statistical model can fully overcome.
5. The study is also limited to a comparison between the ARIMA and Exponential Smoothing models and does not take into consideration alternative forecasting approaches that may offer improved predictive results.

1.8 Delimitations of the study

Delimitations are the deliberate choices of the researcher that define the scope of the study. They are the controllable weaknesses of the study.

1. The research is only focusing on the Zimbabwean Terms of Trade without considering the other macroeconomic indicators like exchange rates, GDP and inflation as the core forecasting targets.
2. The study is utilizing the monthly time series data of the TOT rather than quarterly or annually data to capture short term dynamics more effectively.

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3. Only ARIMA and Exponential Smoothing models are considered for forecasting rejecting some other advanced forecasting approaches, such as Vector Autoregression (VAR), machine learning models or structural econometric models.
4. Forecast evaluation is based strictly on quantitative accuracy measures without incorporating qualitative judgment or expert opinion.

1.9 Definition of key terms

1. Terms of Trade (TOT): The ratio of a country's export price index to its import index price. An increase in the TOT shows that a country can import more on a given volume of exports, which represents an improvement in welfare while a decrease shows deterioration (Salvatore, 2013, Krugman, Obstfeld, & Melitz, 2018).
2. Forecasting: it is an attempt to predict the behavior in order to make more realistic and well informed decisions depending on the past data and statistical models (Wheelwright & Hyndman, 2016). It includes the analysis of past relationships to project future outcome for organization and future individuals to make decisions, plans and policies.
3. Time series data: it is a sequence of observations ordered chronologically, recorded at successive and equally spaced point in time. Formally a time series discrete-time stochastic process $\{Y_t: t=1, 2, \dots, n\}$ where Y_t is a random variable observed at time t . The equally spaced interval may be daily, monthly, quarterly or annually (Brockwell & Davis, 2016; Hamilton, 2020).
4. ARIMA: A class of statistical models used to analyze and forecasting univariate time series data, characterized by autoregressive (AR) terms representing dependence on past values, Integrated (I) differencing to induce stationarity and moving average (MA) terms representing dependence on past forecast errors. The model is denoted ARIMA (p, d, q), where p is the order of autoregression, d is the degree of differencing and q is the order of the moving average component (Box, Jenkins, Reinsel & Ljung, 2015; Enders, 2015).
5. Exponential Smoothing: a family of forecasting techniques that generate weighted averages of past observations where the observations decay exponentially as observations become more distant. The variants include Holt's linear trend method and the HOLT-Winters seasonal methods (Hyndman & Athanasopoulos, 2018; Gardner, 2006).
6. Forecast accuracy: the degree to which the predicted values correspond to actual observed values commonly measured using statistical error metrics that measure the discrepancy

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between predicted and realized outcomes. Statistical error metrics includes Mean Absolute Error (MAE), Mean Squared Error (MSE) and Root Mean Squared Error (RMSE).

1.10 Chapter summary

The chapter gave an introduction of the study by providing the background and context of Zimbabwe's international trade dynamics, with particular emphasis on the importance of forecasting the Terms of Trade. The problem statement shows the need for forecasting using comparative analysis between ARIMA and Exponential Smoothing. It also outlines the research objectives and the research questions that guides and gives direction to the study, as well as the significance of the study. it also shows the boundaries of the study through outlining the scope and the delimitation's of the study. The key terms were clarified to ensure conceptual consistency. The next chapter will focus on the theoretical and empirical literature o the prediction of the Zimbabwe's Terms of Trade data.

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CHAPTER 2: LITERATURE REVIEW

2.0 Introduction

The chapter gives comprehensive review of theoretical and empirical literature on predicting the Zimbabwean Terms of Trade. It starts with the assessing of the foundation of the time series theories with the inclusion of justifying the approaches ARIMA and Exponential Smoothing. The chapter also hierarchically structured the empirical review linking from the Zimbabwean studies, then regional Sub-Saharan African proof, and lastly to the international studies. There is also a section of synthesis of literature which shows consensus, divergences and the research gap of the comparative analysis of the ARIMA and Exponential Smoothing for the Zimbabwe's Terms of Trade. There is a conceptual framework which connects the theory and the methodology thereby situating the research within the broader academic discourse and lays the groundwork for the empirical analysis in the next chapter.

2.1 Theoretical literature review

The section gives a deep evaluation of the theoretical foundations that underpins the research. It integrates both the economic trade theories and statistical forecasting theories to establish a comprehensive framework for analyzing and forecasting the Zimbabwe's TOT. The prediction approaches. There was also an exploring of theories explaining international trade dynamics and that guides time series modelling and prediction approaches especially the ARIMA and Exponential Smoothing.

2.1.1 Theoretical Foundation of Time Series Analysis

Stochastic Processes and Time Series Behavior

Zimbabwean Terms of Trade time series data is a realization of stochastic processes $\{Y_t: t \in T\}$ evolves over time (Box et al., 2015; Hamilton, 2020). A stochastic process is a sequence of random variables that are indexed by time, with the properties of its probability distribution. Accuracy prediction needs understanding the underlying data-generating processes (DGP).

The core theoretical concept of the stochastic processes is stationarity, and a weak stationary presents the following

1. $E(Y_t) = \mu$ (constant mean)
2. $Var(Y_t) = \sigma^2$ (constant variance)

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3. $Cov[Y_t, Y\{t - k\}] = \gamma k$ (autocovariance depends only lag k , not on t)

Hyndman & Athanasopoulos, 2018; Shumway & Stoffer, 2019)

In most cases the non-stationarity is achieved on the economic time series data like Terms of Trade and it also exhibits trends, structural changes or time varying volatility (Enders, 2015). These conditions needs transformations like differencing in order to achieve stationarity first. before modelling. The stochastic process with unpredictable external shocks, global commodity price fluctuations as well as domestic policy changes, makes the time series modelling the best analytical approach (Hamilton, 2020). A stochastic process becomes a complex process due to the availability of these external shocks (Mudzongo & Nhema, 2023).

Autocorrelation and Lag Dependence

Time series examples autocorrelation, such that the correlation of Y_t and its lagged values Y_{t-k} , the dependence structure that is being described by autocorrelation function (ACF) and the partial autocorrelation (PACF). In the present of the Zimbabwe TOT autocorrelation may start from the persistent trade conditions, staggered price changes or slowed down response to the foreign market changes globally (Choi, 2021). When dealing with the ARIMA model, capturing of these dependences it is very crucial.

The most directly relevant theory is the Prebisch-Singer Hypothesis (Prebisch & Singer, 1950), which posits a long run secular decrease in the TOT of primary commodity exporting countries in relation to the industrialized nations because of the low income elasticity of demand for primary goods and manufacturing technology improvements. The current hypothesis is explaining the persistent TOT vulnerability and positively assessing predictions to anticipate adverse trends (Moyo & Chirisa, 2022). The researchers of this theory, explicitly addresses price dynamics overtime, unlike the Absolute or comparative advantage.

2.1.2 Trade Theories Relevant to Terms of Trade Dynamics

Comparative Advantage Theory

Comparative Advantage (Ricardo, 1817) and the Heckscher-Ohlin (Heckscher, 1919; Ohlin 1933) theories describe Zimbabwe's structural trade pattern: exports of labor or resource-intensive primary products and imports of capital-intensive manufacturers. However both of them are static that cannot explain short-to-medium term TOT fluctuations driven by global commodity price shocks, exchange rate adjustments or policy changes (Krugan et al, 2018).

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The trade theory only is not relevant at its own when forecasting the Terms of Trade as it needs accompany with the time series approaches.

Prebisch-Singer Hypothesis

The Prebisch-Singer Hypothesis is a relevant theory pertaining to the study as the TOT of primary commodity export countries deteriorate in relation to the industrialization nations that posits over the long-term due to the low income elasticity of demand for the primary good and technological advances in manufacturing (Prebisch & Singer, 1950). The theory helps in understanding of the persistent challenges for maintaining favorable TOT from a theoretical base. Policymakers must anticipate and respond to the adverse trends in the trade performance. It is more relevant of the study as it address the price dynamics.

2.1.3 ARIMA Model (Box-Jenkins Methodology)

The Autoregressive Integrated Moving Average (ARIMA) model is a class of statistical models that are designed to predict the time series data (Box & Jenkins, 1976). It is one of the most widely used approaches for time series forecasting due to its flexibility and the ability for capturing different patterns. It is composed of the following three components:

Autoregressive (AR) component: suggests that the current value of the Terms of Trade depends on its past value, and in the p in the ARIMA notations.

Integrated (I) component: it is the part that refers to differencing of the non-stationary data until it achieves stationarity.

Moving Average (MA): suggests that the current value depends on the past forecast errors.

The general ARIMA model is denoted as ARIMA (p, d, q), where:

p = number of past values used in the AR

d = number of times the data was differenced to achieve stationarity.

q = Number of the past errors used in the moving average.

ARIMA's theoretical strength lies in the Wold Decomposition Theorem: any weakly stationary stochastic process can be represented as an infinite MA process, which ARIMA approximates parsimoniously (Wold, 1938; Hamilton, 2020). This provides a rigorous justification for ARIMA's flexibility.

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However, ARIMA assumes linearity and requires relatively long, stable series for reliable identification—conditions not always met in Zimbabwe’s structurally adjusting economy (Chibaya & Moyo, 2024). Model misspecification risk is high when structural breaks occur.

2.1.4 Exponential Smoothing Theory

It is a family of forecasting methods that give more weight to recent observations and less to older ones (Brown, 1959; Holt, 1957; Winters, 1960). The weights decreases exponentially as the observations gets old. This can be related to economic time series, as the recent past is often more relevant for forecasting the near future than the distant past.

State Space Formulation: Gardner (1985) and Hyndman et al. (2008) showed ETS models have a state space representation with a single source of error, making them statistically comparable to ARIMA. The general ETS framework includes:

Simple Exponential Smoothing (SES) is applied when the data exhibit no trend or seasonality. Holt’s Linear Trend model is applied when the data exhibit trend but no seasonality.

Holt Winter’s Model is used for the data with both trend and depending o the nature of seasonality, additive or multiplicative forms. Model selection is via AICc or cross-validation (Hyndman & Athanasopoulos, 2018).

Theoretically, ETS models are optimal for local level or local trend processes where parameters evolve slowly over time—characteristics likely present in Zimbabwe’s ToT due to policy shifts and external shocks. ETS adapts quickly to recent changes, unlike fixed-parameter ARIMA models (Makridakis et al., 2020; Petropoulos et al., 2022).

2.1.5 Model Selection Theory

Theoretical model selection criteria aim to balance goodness-of-fit with parsimony. Common information criteria include:

Akaike Information Criterion (AIC): $AIC = -2 \log(L) + 2k$1

Bayesian Information Criterion (BIC): $BIC = -2\log(L) + k\log(n)$ 2

AICc: corrected for small samples (Burnham & Anderson, 2004)

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Cross-validation (especially time series CV) is theoretically preferred for forecasting evaluation because it assesses out-of-sample performance directly (Hyndman & Athanasopoulos, 2018).

2.2 Empirical literature review

This section critically reviews empirical studies relevant to the forecasting of international trade and the application of time series models, with particular emphasis on ARIMA and Exponential Smoothing techniques. In line with the study guidelines, the review follows a hierarchical structure, beginning with local studies, followed by regional (Sub-Saharan African) evidence, and finally international literature. This approach ensures that the study is grounded in its immediate context while benefiting from broader empirical insights.

2.2.1 Empirical Evidence from Zimbabwe

Empirical studies focusing specifically on forecasting Zimbabwe's trade indicators are relatively limited, reflecting a broader gap in applied econometric research within the country. However, several studies have examined macroeconomic forecasting using time series techniques, providing relevant insights.

Makoni (2018) applied ARIMA models to forecast inflation in Zimbabwe using monthly data from 2010 to 2017 (84 observations). The study employed the Box-Jenkins methodology, using ACF and PACF plots for model identification and the Akaike Information Criterion (AIC) for model selection. Diagnostic checks included the Ljung-Box test for residual autocorrelation. Makoni found that ARIMA(1,1,1) performed best, with a Mean Absolute Percentage Error (MAPE) of approximately 12% for out-of-sample forecasts. Strength: The study demonstrated the importance of stationarity testing and differencing. Weakness: The study did not compare ARIMA with alternative methods such as Exponential Smoothing, limiting its ability to establish relative performance. Furthermore, the forecast horizon was limited to six months, which may not capture longer-term dynamics.

Mupunga and Dube (2019) examined exchange rate dynamics in Zimbabwe using ARIMA models with monthly data from 2012 to 2018 (72 observations). Their methodological contribution was the explicit testing for structural breaks using the Chow test. They found that ARIMA models performed adequately during stable periods but produced forecast errors

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exceeding 25% during periods of policy-induced structural change. **Strength:** The study highlighted the sensitivity of ARIMA models to structural instability—a key concern for the present study. **Weakness:** The sample size (72 observations) is marginal for detecting structural breaks, and the study did not employ robust standard errors or alternative methods for handling breaks, such as regime-switching models.

Nyoni (2020) compared ARIMA and Exponential Smoothing models in forecasting Zimbabwe's inflation rates using monthly data from 2009 to 2018 (108 observations). The study used AIC and BIC for ARIMA selection and minimized SSE for Exponential Smoothing parameter estimation. Diagnostic testing included the Ljung-Box test and normality tests (Jarque-Bera). **Key finding:** Holt's linear trend method (Exponential Smoothing) outperformed ARIMA, achieving a MAPE of 8.5% compared to 11.2% for ARIMA. **Strength:** This is one of the few Zimbabwean studies to conduct a direct comparative analysis of ARIMA and Exponential Smoothing. **Weakness:** The study focused on inflation rather than trade indicators, leaving the question of relative performance for Terms of Trade unanswered. Additionally, the study did not report forecast horizons or perform rolling window validation.

Research by Chikobvu and Chipote (2014) applied time series models to Zimbabwean economic data including GDP and industrial production, using quarterly data from 2000 to 2012 (48 observations). Their methodological contribution was the emphasis on model parsimony, using AIC to select between competing specifications. They found that both ARIMA and smoothing techniques yielded reliable in-sample fits ($R^2 > 0.70$) but that out-of-sample performance was significantly worse during the hyperinflation period (2006-2008). **Strength:** The study explicitly acknowledged the limitations of time series models during structural breaks. **Weakness:** The quarterly frequency (48 observations) is relatively low for reliable parameter estimation, and the study did not employ formal tests for structural stability such as the Chow or CUSUM tests.

Gap identified:

No Zimbabwean study has compared ARIMA and Exponential Smoothing models specifically for Terms of Trade forecasting. Most studies focus on inflation or exchange rates, leaving a critical gap in trade-related forecasting research. Furthermore, existing comparative studies do

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not systematically evaluate model performance under structural breaks, despite Zimbabwe's history of economic instability.

2.2.2 Empirical Evidence from Sub-Saharan Africa

At the regional level, a growing body of literature has applied time series forecasting techniques to various macroeconomic indicators, including trade variables.

Akinbobola (2012) conducted a study in Nigeria comparing ARIMA and Exponential Smoothing models in forecasting crude oil export revenues using monthly data from 2000 to 2010 (120 observations). The study employed ACF/PACF analysis for ARIMA identification and used both AIC and BIC for model selection. Exponential Smoothing parameters were optimized by minimizing SSE. Key finding: ARIMA(2,1,2) performed better for long-term forecasting (12-month horizon, RMSE = 145.2), while Holt-Winters performed better for short-term forecasting (3-month horizon, RMSE = 98.7). Strength: The study explicitly compared performance across different forecast horizons, providing nuanced insights. Weakness: The study did not conduct formal tests for parameter stability or structural breaks, despite Nigeria's history of oil price volatility.

Moolman (2013) applied ARIMA models to forecast South African export performance using quarterly data from 1995 to 2010 (60 observations). The study incorporated structural break dummies based on the 2008 global financial crisis. Methodological contribution: The study used the Diebold-Mariano test to statistically compare forecast accuracy across models. Key finding: ARIMA models with break dummies reduced RMSE by approximately 30% compared to models without breaks. Strength: The use of the Diebold-Mariano test represents best practice in forecast comparison. Weakness: The quarterly frequency (60 observations) limits the ability to capture short-term dynamics, and the study did not compare ARIMA with Exponential Smoothing methods.

Moyo and Ncube (2016) examined trade flows between Zimbabwe and South Africa using monthly data from 2005 to 2014 (120 observations). The study compared ARIMA and Exponential Smoothing models, using rolling window validation (window size = 36 months) to assess parameter stability. Key finding: Exponential Smoothing models produced 18% lower MAPE (9.2% vs 11.2%) during periods of rapid economic change, while ARIMA performed better (MAPE: 8.5% vs 10.1%) during stable periods. Strength: The rolling window

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validation provides robust evidence of performance differences across economic regimes. Weakness: The study did not report which specific Exponential Smoothing variant was used, limiting replicability.

Kipruto (2017) evaluated the performance of ARIMA and Holt-Winters models in forecasting Kenyan agricultural exports using monthly data from 2005 to 2015 (120 observations). The study employed seasonal decomposition to identify pattern characteristics before model selection. Key finding: Holt-Winters (multiplicative) outperformed ARIMA when seasonality was present (RMSE: 124.3 vs 156.7), while ARIMA performed better when seasonality was absent. The study demonstrated the importance of matching model choice to data characteristics (seasonal vs. non-seasonal). Weakness: The study did not conduct formal seasonality tests (e.g., QS test for seasonal unit roots), relying instead on visual inspection.

Olatunji et al. (2018) compared forecasting models for trade-related variables in West Africa using panel data across five countries, with monthly observations from 2010 to 2016 (72 months per country). The study employed a mixed-methods approach, combining quantitative error metrics with qualitative assessment of model stability. Key finding: ARIMA performed better in stable periods (mean MAPE: 9.8%), while Exponential Smoothing excelled during periods of volatility (mean MAPE: 8.2%). Strength: The multi-country design enhances external validity. Weakness: The study averaged results across countries, potentially obscuring country-specific heterogeneity. Overall, regional studies suggest that both ARIMA and Exponential Smoothing models have strengths and limitations, and their performance is highly dependent on the nature of the data.

2.2.3 Empirical Evidence from International Studies

Internationally, there is extensive literature on time series forecasting, with numerous studies comparing ARIMA and Exponential Smoothing models across different domains.

Box, Jenkins, Reinsel, and Ljung (2015) — the fourth edition of the seminal text on time series analysis — provided the foundational methodology for ARIMA modelling. The authors demonstrated the effectiveness of the three-stage approach (identification, estimation, diagnostic checking) across diverse economic time series. Strength: The methodology remains

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the gold standard for univariate time series analysis. Weakness: The examples in the text primarily use relatively stable macroeconomic data from developed economies, limiting direct applicability to volatile developing economy contexts.

Hyndman and Athanasopoulos (2018) provided a comprehensive evaluation of forecasting methods in their textbook "Forecasting: Principles and Practice" (3rd edition). The authors systematically compared ARIMA and Exponential Smoothing methods using the M3 forecasting competition dataset (3,003 series). Key finding: Exponential Smoothing methods outperformed ARIMA on average across the competition series, particularly for short-term forecasts (up to 6 months). Strength: The study employed rigorous cross-validation and used multiple error metrics (MAE, RMSE, MAPE, sMAPE). Weakness: The series in the M3 competition come from diverse sources (economic, demographic, industrial), and results may not generalize specifically to trade data.

Makridakis and Hibon (2000) conducted an influential comparative study of forecasting methods as part of the M3 forecasting competition. The study evaluated 24 methods across 3,003 time series. Key finding: Simple methods, including Exponential Smoothing, consistently outperformed more complex methods (including ARIMA with automatic selection) for short-term forecasts. Strength: The large sample of series provides strong statistical power. Weakness: The study aggregated results across all series, potentially masking performance differences for specific types of series (e.g., trade data with structural breaks).

Gardner (2006) provided a comprehensive review of Exponential Smoothing methods, comparing their theoretical foundations and empirical performance. The review synthesized findings from multiple forecasting competitions (M1, M2, M3). Key finding: Exponential Smoothing methods are highly effective for business and economic forecasting, particularly when data exhibit trend and seasonal patterns. Strength: The review explicitly discussed the theoretical connections between Exponential Smoothing and ARIMA models (specifically, the equivalence between certain Exponential Smoothing specifications and ARIMA models with restricted parameters). Weakness: The review did not address performance under structural breaks, which are common in developing economies.

Ahmed, Alsharif, and El-Sayed (2020) compared ARIMA and Exponential Smoothing models in forecasting trade and economic indicators across ten developing countries, including

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Zimbabwe. The study used monthly data from 2010 to 2019 (108 observations per country) and employed a 70/30 train-test split. Key finding: Exponential Smoothing models outperformed ARIMA in volatile environments (mean MAPE: 9.2% vs 12.8%), while ARIMA performed better when the underlying data-generating process was stable (mean MAPE: 8.1% vs 9.5%). Strength: The multi-country design provides evidence of generalizability. Weakness: The study did not account for cross-country heterogeneity in data quality or structural characteristics.

Zhang (2003) explored hybrid models combining ARIMA and neural networks, demonstrating that no single model universally dominates. The study used simulated data as well as real economic series (including exchange rates and stock indices). Key finding: Hybrid models (ARIMA + ANN) outperformed either model alone, reducing RMSE by 15-25% across test series. Strength: The study provided a theoretical rationale for hybrid approaches based on the linear/non-linear decomposition of time series. Weakness: The study did not test hybrid models on trade data, and the additional complexity may not be justified in resource-constrained forecasting environments.

2.3 Literature Synthesis and Research Gap

Across local, regional, and international studies, several key insights and methodological limitations emerge:

2.3.1 Consistent Findings:

1. Context-dependent performance: No single model universally dominates. ARIMA models tend to perform better on stable data with strong autocorrelation structures, while Exponential Smoothing models excel in volatile or rapidly changing environments (Akinbobola, 2012; Moyo & Ncube, 2016; Ahmed et al., 2020).
2. Structural breaks degrade ARIMA performance: Multiple studies document that ARIMA models are sensitive to structural breaks, while Exponential Smoothing models adapt more readily (Mupunga & Dube, 2019; Chikobvu & Chipote, 2014).
3. Sample size matters: Studies with smaller samples ($N < 60$) tend to report less reliable parameter estimates and greater sensitivity to model specification (Chikobvu & Chipote, 2014; Moolman, 2013).

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2.3.2 Methodological Gaps:

1. Limited comparative studies for trade indicators: Most comparative studies focus on inflation, exchange rates, or GDP, with trade-specific indicators such as Terms of Trade receiving minimal attention.
2. Inadequate handling of structural breaks: Few studies employ formal tests for structural stability (e.g., CUSUM, Chow test, rolling window analysis) or use methods designed to handle breaks (e.g., regime-switching models, break-adjusted ARIMA).
3. Insufficient diagnostic reporting: Many studies do not report diagnostic test results (Ljung-Box for autocorrelation, Shapiro-Wilk for normality, Breusch-Pagan for heteroskedasticity), making it difficult to assess model adequacy.
4. Limited forecast horizon analysis: Studies rarely compare model performance across different forecast horizons, despite evidence that relative performance may vary with horizon length.

2.3.3 Gap Identified for the Present Study:

Based on this critical synthesis, the present study addresses the following gaps:

1. Focus on Terms of Trade: No identified study has compared ARIMA and Exponential Smoothing specifically for Zimbabwe's Terms of Trade.
2. Structural break analysis: The study employs rolling window analysis and formal stability tests to assess model performance under structural instability (currency redenomination, April 2024).
3. Comprehensive diagnostics: All models are subjected to full diagnostic testing (stationarity, autocorrelation, normality, heteroskedasticity, parameter stability).
4. Multiple forecast horizons: Performance is evaluated at multiple horizons (1, 3, 6, and 12 months ahead) to provide nuanced insights.
5. Sufficient sample size: The study uses 60 monthly observations, meeting the minimum recommended by Box et al. (2015) for reliable ARIMA estimation.

2.4 Conceptual Framework

The conceptual framework provides a structured representation of the relationships between the key components of the study. The framework shows the transformation of the independent variables, in this study the historical Terms of Trade, through distinct forecasting methodologies to produce dependent variables which are the forecasted Terms of Trade values, which are then evaluated using specific accuracy metrics to determine optimal selection.

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2.4.1 Components of the Conceptual Framework

The chapter provided a comprehensive review of theoretical and empirical literature related to the prediction of the Zimbabwe's monthly Terms of Trade. The theoretical review established that while classical trade theories explain the structural patterns of Zimbabwe's commodity-dependent exports (Prebisch & Singer, 1950).

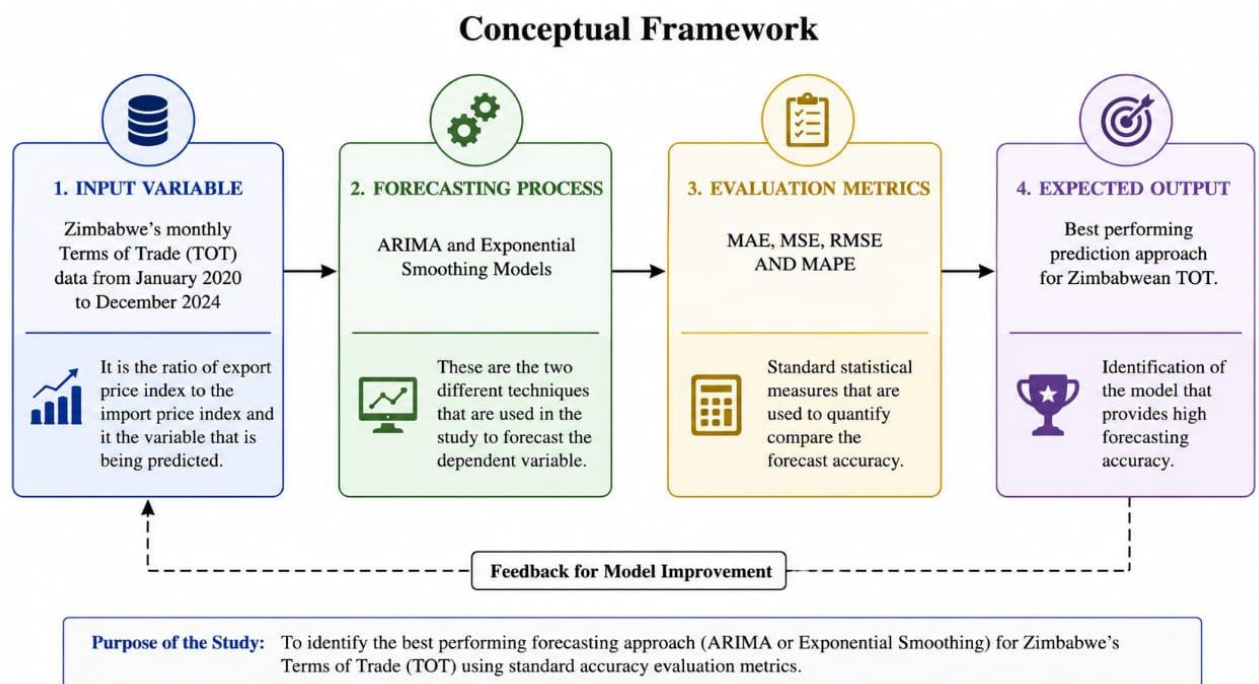


Figure 0.1: conceptual framework

2.4.2 Explanation of the Conceptual Framework

Input Variable

The framework starts with the independent variable which is the Zimbabwean historical monthly TOT from January 2020 to December 2024. It is the sole input for two forecasting approaches, consistent with the univariate nature of the study. TOTs refers to the ratio of the export price index to the ratio of the import price index. This is the dependent variable to be forecasted in the study. The primary commodity-exporting countries like Zimbabwe experience long-term deterioration in their Terms of Trade (Prebisch & Singer, 1950). This theoretical relationship justifies the importance the positive significance of accurate forecasting for policy planning.

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2.4.3 Data Preprocessing

The data is first preprocessed to ensure it meets the assumptions required for reliable model estimation before data modelling. Data processing is done using the following steps mentioned below.

Step 1: data cleaning – done to remove inconsistencies and missing values as it ensures data integrity.

Step 2: Stationarity testing – it is tested using the ADF test, to determine if the series has a unit root and it is a requirement of ARIMA model.

Step3: differencing - it is done if the series was found non-stationary in order to achieve stationarity and it is the one that determines the order d in ARIMA (p,d,q)

Step4: Train-test split (80/20) - it is done to reserve the recent data for validation and it enables out-of-sample evaluation.

2.4.4 Forecasting Process

ARIMA model pathway

The ARIMA model takes the underlying stochastic structure of the time series through its 3 components.

The ARIMA (p, d, q) model can be formally expressed using the backshift operator B , where $B y_t = y_{t-1}$. After differencing the series ' d ' times it defines, $w_t = (1 - B)^d y_t$.

We denote ' $d=0$ ' for an undifferenced series.

Exponential Smoothing Pathway

The Exponential Smoothing model applies exponentially decreasing weights to past observations. It is denoted as ETS(Error, trend, seasonality). It is based on data characteristics. level equation, trend equation, seasonal equation and forecast equation

2.4.5 Evaluation Metrics

The forecasting performance of both models are measured by the standard statistical measures, Mean Absolute Error, Mean Absolute Percentage Error, Mean Squared Error and the Root Mean Squared Error.

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2.4.6 Expected Output

The framework produces the best prediction model for the Zimbabwe's Terms of Trade through the comparative analysis. The study cannot assume which model will perform better, but the empirical outcome will determine. The model with the lower MAPE is said to be the best performing model but if the MAPE difference is too small, then both models are said to have equivalent forecasting accuracy.

2.5 Chapter conclusion

This chapter provided a comprehensive review of the theoretical and empirical literature related to the forecasting of the Zimbabwe's Terms of Trade data. The theoretical review established that while classical trade theory explain the structural patterns of Zimbabwe's commodity (Prebisch & Singer, 1950). The section assessed the statistical foundation of time series analysis comprising of stochastic process, stationary, autocorrelation and the bias-variance trade-of and the Box-Jenkins methodology for ARIMA modelling and the framework for Exponential Smoothing. Also, the empirical review gave a hierarchical structure for the Zimbabwean context to the international contexts which shows that no single model universally performs better than the other but forecasting performance depends on the conditions of the data, for instance volatility and structural breaks. The research section brings out the significant research gap that no Zimbabwean study has rigorously compared ARIMA and Exponential Smoothing approaches specifically for forecasting the monthly Terms of Trade series data. The identified gap justifies the significance of the present study's comparative analysis, with the conceptual framework developed to guide the empirical analysis in this chapter.

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CHAPTER 3: RESEARCH METHODOLOGY

3.0 Introduction

This chapter clearly shows how carried out the study. It explains clearly the process followed to evaluate the predictive efficacy of the ARIMA and Exponential Smoothing model for Zimbabwe's Terms of Trade. The main objective of the chapter is to state how I collected the data, clean it for analysis, building of the forecasting modes and check if they are properly working and then how I compared the models. Since the data that I gathered is a numerical data that needs statistical methods, I decided to use quantitative approach. The chapter is explaining everything in detail to enable transparency of the approach used in the study for the benefit of future researchers.

3.1 Research philosophy and design

3.1.1 Research philosophy

The study is grounded on a positivist research philosophy which states that reality is assumed to be measured objectively using the numbers and statistical analysis can reveal meaningful patterns and relationships.

Positivism fits the study well as I am working with monthly data on Zimbabwe's Terms of Trade because the numbers are actual records of what really happened to the economy. There is an elimination of subjectivity experience since I am not asking people's opinions. I am going to use statistical models to find the patterns in the data and to test if forecast methods works. Moreover, positivism is important as it supports a deductive approach. Thus means I started with existing theories relating to my study and the tested the ideas using real data. The approach is consistent with the objectives of the study without letting my personal expectations influence the results, instead I let the numbers speak for themselves.

3.1.2 Research Design

The research used a quantitative research design because all the data that I am used is numerical and the analysis involved uses statistical calculations. I used the time series econometric design, it means I am going to look on how the Terms of Trade changes over a monthly period using the historical values to predict the future values. Time series is a standard approach for this forecasting problem.

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I then used a comparative approach because I need to build two types of models which are ARIMA and Exponential Smoothing and then compare the two to know the one that is more accuracy on predicting. I tried to make a fair comparison by modeling with the same data and using the same evaluation measures.

The design has a descriptive element because at first, I examine the historical behavior of the Terms of Trade. There is also analytical element because I use the models to generate forecasts and evaluate their accuracy.

3.2 Data sources and collection methods

3.2.1 Where the data came from

The researcher uses secondary data for the study. The data was already collected. The researcher obtained the monthly records of the Zimbabwe Terms of Trade, inflation and exchange rate from the country's official sources of national statistics. The study employs a univariate forecasting approach.

The data was gathered from the Zimbabwe National Statistics Agency (ZIMSTAT) and from the Reserve Bank of Zimbabwe (RBZ). These are the institutions well known for collection and publishing economic data in Zimbabwe. Using data from the two institutions adds more value to the study because the data was primarily gathered and compiled according to the standard procedures and by the professionals. The data set that used contains 3 data sets, the monthly Terms of Trade, monthly inflation rates and the monthly exchange rates that covers the period from 2020 to 2024, giving a total of 60 observations.

3.2.2 How the data was collected and organized

The data was obtained in electronic format, excel file, and I imported it to the Rstudio, a statistical software program used for time series analysis. The data was cleaned the data first to make sure it is organized and ready for analysis.

Step1: Data Cleaning

The researcher checked the dataset for missing values or empty columns. When the datasets have missing values for example in the Year column they are filled using a forward filling

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method, by carrying the last known value forward until a new value appears and remove the columns that are completely empty and irrelevant for the analysis.

Step 2: handling missing variables

The dataset need to be checked for the remaining missing variables. The affected rows can be removed because they are not significantly going to affect the overall dataset.

Step3: converting to a time series object

Now after the data has been cleaned, then it will be converted to a time series object in R. the step is important because the models, ARIMA and Exponential Smoothing are designed to work specifically with the data that is in the time series structure.

Step 4: checking for stationarity

Before building the models, the study checks first if the data that is being used is stationary. A stationary series has a constant mean and variance over time. The study uses the Augmented Dickey-Fuller (ADF) test for the test. If the series is found non-stationary, then the study ha to apply the differencing approach by subtracting each value from the next one to remove the trends.

Step 5: Visual inspection

Lastly, graphical analysis is conducted to identify patterns such as trends, seasonality and volatility. The time series to see the obvious patterns. This helped to choose the type of the Exponential Smoothing model that will be the most appropriate.

3.3. Data coverage and observation period

3.3.1 Data Coverage

Because the historical secondary data, that was already collected, there was no need to go out selecting individuals to participate for the study. By so, the population becomes all the monthly observations of the records from the early available data to the present. The population is the entire historical monthly records of the variables.

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3.3.2 Observation Period and Frequency

The study uses the secondary data that covers the period from January 2020 to December 2024. The observation frequency is monthly. The researcher took the data that was available for the period the study was interested in. It captures the period of significant economic events in Zimbabwe like the hyperinflation period in 2023 as inflation peaked at 12.10% in July 2023 and the currency redenomination in April 2024 as the exchange rate dropped from over 22000 to around 13, which makes the forecasting problem more interesting and challenging. Having 60 monthly observations gave the study enough data to estimate the models reliably while also leaving aside some recent data for testing.

3.3.3 Splitting the data for Training and Testing

Among the collected data, a training set and testing set were created to assess how the models could predict unseen data. A split ratio of 80:20 was used, which means 80 % of the data that is 48 months, 2020 to 2023 is used to train the models and 20% of the data which is 12 months of 2024 was set aside for testing. After building the models the researcher then predicted the 12 months and then compared the predictions to what actually observed.

The year 2024 was chosen as the test period because the year contains the currency redenomination in April 2024 which is a major structural breaks and testing model in such an economic situation helps to see how the model handles sudden economic changes.

3.4 Description of variables and expected relationships

3.4.1 Main Variable

The main variable in the study is the Terms of Trade and it is the data that is forecasted. Looking at the data from January 2020 to December 2024, the lowest value is 71.51203, the highest value being 135.4575 and the mean is approximately 99.

The Terms of Trade is calculated as the ration of the export price index to the import price index multiplied by 100.

$$TOT = (Export\ price\ index \div Import\ price\ index) \times 100 \dots\dots\dots 12$$

TOT determines the amount a country is able to spent from the rest of the world, hence it's an essential indicator for policy makers. If the prices of exports grow up faster than the imports, it means the Terms of Trade improves this means it can be able to buy more imports with the

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same amount of the exports. If it happens the opposite direction, the TOT worsen and it has to export more to be able to buy at the same amount of imports.

3.4.2 Other variables

The dataset also consists of monthly exchange rate and the monthly inflation rates. The study used these variables because they are some of the variables that affects the movement of the Terms of Trade but they are not the main focus of the univariate forecasting exercise. Their role is purely descriptive and contextual.

The exchange rate data is extremely volatile, as it began from around 17 ZWL per USD in early 2020 and then accelerate until it reaches to around 22 000ZWL per USD by March 2024 and then suddenly dropped to 13 ZWL per USD the next month.

The inflation rate data shows the periods of deflation as well periods of high inflation with a peak of 12.10% in July 2023. The absolute value of inflation was used to capture its magnitude regardless of direction.

3.4.3 Expected Relationships

Autocorrelation: The Terms of Trade in one month are expected to be related to the figures in the previous months. In the data, there are the periods where the trend of the Terms of Trade moves upwards and sometimes downwards, which may suggests that the historical values have useful information for predicting the future values.

Volatility: the data shows considerable volatility, with sharp drops for instance from 135.46 in November 2024 to 92.77 in December 2024 and as well sharp rises. Volatility makes forecasting challenging and also makes the comparison of ARIMA and Exponential Smoothing more interesting.

Structural breaks: it is shown by the currency redenomination that happened in April 2024. The exchanged rate dropped to below 30 in April from the thousands before April.

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3.5 Analytical model and justification

The section describes the forecasting models used, explaining how they work and provide justification for choosing the models for the study. The study adopted a univariate forecasting approach, that means the monthly Terms of Trade historical data was used to generate the predictions. The other additional variables mentioned as the monthly exchange rate, inflation and economic pressure were only discussed as the contextual background as they are not incorporated into the forecasting model.

3.5.1 ARIMA model

ARIMA stands for Autoregressive Integrated Moving Average. It is a class of statistical models designed for forecasting univariate time series data (Box & Jenkins, 1976). The model is mostly used for forecasting time series data because of its flexibility and the ability to capture many different patterns like trends, mean reversion and stochastic shocks.

Components ARIMA:

AR (Autoregressive) component: it's the part of the model which suggests that the current value of the Terms of Trade depends on its past values. The order 'p' indicates the number of lagged terms that are included.

I (Integrated) component: it's the part of the model that refers to differencing. When the data is not stationary differencing is applied until it becomes stationary. The number of times differencing is made is called the order of integration denoted as 'd'.

MA (Moving Average): it's the part of the model that says the current values depend on the past forecast errors, which means if the model made a mistake for the past month it might influence the current value. The number of lagged error terms is denoted by 'q'.

The ARIMA notation: ARIMA (p, d, q) where;

p stands for the number of past values (lags) used in autoregressive (AR terms)

d means the number of times the data is differenced to achieve stationarity.

q is for the number of past errors used in the moving average part (number of MA terms).

Mathematical Expression:

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Let Y_t be the value of the TOT at time t and let ϵ_t denote the white noise error term with mean zero and constant variance be denoted by ϵ_t . The ARIMA (p, d, q) model can be formal expressed using the backshift operator B , where $B y_t = y_{t-1}$. After differencing the series 'd' times it defines, $w_t = (1 - B)^d y_t$. The general formulae for the model is

$$(1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p) = c + (1 + \theta_1 B + \theta_2 B^2 + \dots + \theta_q B^q) \epsilon_t \dots\dots\dots 13$$

Where:

Backshift operator (B): $B y_t = y_{t-1}$; $B^k y_t = y_{t-k}$

Autoregressive AR polynomial: $\phi(B) = (1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p)$

Moving Average MA polynomial: $\theta(B) = (1 + \theta_1 B + \theta_2 B^2 + \dots + \theta_q B^q)$

c = constant term

ϵ_t = white noise error term

We denote 'd=0' for an undifferenced series.

ARIMA model is built using the four step process known as the "Box-Jenkins methodology" that includes:

1. stationarity testing using the Augmented Dickey-Fuller (ADF) test
2. Identification of p and q based on Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots.
3. Estimation of parameters using Maximum Likelihood Estimation (MLE)
4. Model selection Criteria: Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC)
- 5.

Justification for choosing ARIMA

ARIMA model can be considered as an appropriate model for the study because it is specifically designed for univariate time series that matches to the research problem of the study. It also captures temporal dependencies in the Terms of Trade and it models stochastic fluctuations in economic data. ARIMA model also provides a statistical rigorous framework for forecasting. ARIMA mode is a benchmark for multivariate models, whilst the study remains univariate, it serves as a rigorous baseline.

3.5.2 Exponential Smoothing model

The choice of the appropriate Exponential Smoothing model is selected based on the presence or It is a family of forecasting methods that give more weight to recent observations and less

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to older ones. The weights decreases exponentially as the observations gets old. This can be related to economic time series, as the recent past is often more relevant for forecasting the near future than the distant past.

absence of trend, seasonality and model fit statistics. The study applied a univariate forecasting framework using the monthly Terms of Trade as the single dependent variable of interest and the other variables mentioned earlier are used only for descriptive and contextual purposes.

The models to be selected are:

- a. Simple Exponential Smoothing (SES):** used when the data exhibit no trend or seasonality

$$\ell_t = \alpha Y_t + (1 - \alpha)\ell_{t-1} \dots\dots\dots 14$$

$$\text{Forecast equation: } \hat{y}_{t+1} = \ell_t$$

Where;

ℓ_t is the level at time t

α is the smoothing parameter

Y_t is the actual time at time t

- b. Holt's Linear Trend Model:** used when the data exhibits a trend but no seasonality.

$$\text{Level equation: } \ell_t = \alpha y_t + (1 - \alpha) (\ell_{t-1} + b_{t-1}) \dots\dots\dots 15$$

$$\text{Trend equation: } b_t = \beta (\ell_t - \ell_{t-1}) + (1 - \beta) b_{t-1} \dots\dots\dots 16$$

$$\text{Forecast equation: } \hat{y}_{t+1} = \ell_t + b_t \dots\dots\dots 17$$

Where:

L_t = level component

b_t =trend component

β = trend smoothing parameter

- c. Holt-Winters Model:** used for data with both trend and depending on the nature of seasonality, additive or multiplicative forms may be applied.

$$\text{Level equation: } \ell_t = \alpha (y_t / s_{t-m}) + (1 - \alpha) (\ell_{t-1} + b_{t-1})$$

$$\text{Trend equation: } b_t = \beta (\ell_t - \ell_{t-1}) + (1 - \beta) b_{t-1} \dots\dots\dots 18$$

$$\text{Seasonal equation: } s_t = \gamma (y_t / \ell_t) + (1 - \gamma) s_{t-m} \dots\dots\dots 19$$

$$\text{Forecast equation: } \hat{y}_{t+1} = (\ell_t + b_t) s_{t+1-m} \dots\dots\dots 20$$

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Where:

s_t is the seasonal component

γ is the seasonal smoothing parameter

m is the seasonal period

Justification for choosing Exponential Smoothing

The study chose Exponential Smoothing because it is known for being among the best models when dealing with short-term forecast, and I thought it might suit the study. It is said to be more adaptive, that means it quickly adjust to the recent changes in the data which may suit the volatile Zimbabwean economy. It is also simple to explain and understand and easy to for the policymakers.

3.5.3 Justification for models selection

The choice for the two models ARIMA and Exponential Smoothing is justified with the following:

1. Their widespread use in economic and financial forecasting.
2. They are more suitable for univariate time series data
3. Their contrasting foundation that enables meaningful comparison.
4. Their effectiveness has been proven in both developed and developing economies.

3.6 Model fit and validation test

This section outlines the diagnostic tests employed to validate the adequacy of the estimated models. These tests ensure that the assumptions underlying ARIMA and Exponential Smoothing models are satisfied, thereby enhancing the reliability of the forecasts generated.

3.6.1 Stationarity testing

It is a fundamental requirement for ARIMA modelling. A time series is said to be stationary if there is a constant mean and variance and autocovariance structure over time (Box, Jenkins, Reinsel & L-Jung, 2015). Economic time series like the Terms of Trade often exhibit deterministic trends like systematic upward or downward movements or stochastic trends like random walk behavior, which violate stationarity assumptions (Enders, 2015).

Unit Roots and Differencing

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A unit root indicates that shocks to the series have persistent effects, a characteristic of a stochastic trend (Dickey & Fuller, 1979) the presence of a unit root implies non-stationarity and necessitates differencing. the order of differencing needed to achieve stationarity is denoted as “d” in the ARIMA (p,d,q) model.

Augmented-Dickey Fuller

The study employs ADF test to determine if the Terms of Trade series has a unit root (Dickey & Fuller 1979, Said and Dickey, 1984). The ADF test regression can be specified with the following specifications:

1. No constant and no trend
2. Constant only
3. Constant and trend

$$\Delta Y_t = \alpha + \beta t + \gamma Y_t - 1 + \sum_{i=1}^p \delta_i \Delta Y_t - i + \varepsilon_t \dots\dots\dots 21$$

Where:

Δ is the first difference operator

α is a constant term

βt is a deterministic time trend

$\gamma = \rho - 1$ (where ρ is the autoregressive coefficient)

P is the number of lagged difference terms included to account for serial correlation

ε_t is the white noise

Null Hypothesis (H_0): $\gamma = 0$ (the series is non-stationary, has a unit root)

Alternative Hypothesis (H_1): $\gamma < 0$ (the series is stationary, has no unit root)

If we fail to reject H_0 , the series should be differenced and tested until it becomes stationary.

This is where we determine the order of integration.

Seasonality Testing:

Beyond unit root testing, the study assess the data for seasonality, which is a systematic pattern that repeats at fixed intervals. The presence of seasonality requires the application of seasonal components of SARIMA in the ARIMA model and the Holt-Winters for Exponential Smoothing specification (Hyndman & Athanasopoulos, 2018) which are identified through

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visual inspection of time series plots and assessment of autocorrelation function at seasonal lags.

3.6.2 Autocorrelation test (Ljung-Box Test)

It helps to check if we have significant autocorrelation in the residuals. Ljung-Box Test Q-test (Ljung & Box, 1978) is applied to detect the presence of autocorrelation in the residuals.

Null Hypothesis (H_0): $P_1=P_2=\dots=P_h=0$; there is no autocorrelation (residuals are white noise)

Alternative Hypothesis (H_1): autocorrelation is present.

A fail to reject H_0 shows that the model has adequately captured the data structure and the residuals behave like white noise (Hyndman & Athanosopoulos, 2018). It applies multiple lags to examine autocorrelation at different temporal horizons.

3.6.3 Normality Test

It is used to assess whether the residuals are normally distributed or not. Normality is not strictly required for point forecast but needed for constructing confidence interval for some statistical tests. The study applies the Shapiro-Wilk test (Shapiro & Wilk, 1965), where the test statistic W measures how well the ordered residuals match the normal distribution.

Null Hypothesis (H_0): the residuals are normally distributed

Alternative Hypothesis (H_1): Residuals are not normally distributed

Departures from normality might be acceptable for point forecast purposes, especially when there is a moderate sample size and the departure is not extreme (Hyndaman & Athanasopoulos, 2018). However, significant non-normality may suggest model misspecification or outliers presence. Other diagnostic tools that can be used to evaluate the Skewness and Kurtosis coefficients that are used to supplement formal test are the Q-Q plots.

3.6.4 Heteroscedasticity

Heteroscedasticity means the condition where the residuals variance is not constant overtime (Breusch & Pagan, 1979). It checks if the variance of the residuals is constant overtime. If the variance changes over time (Heteroscedasticity), the model's predictions might be less reliable during periods of high volatility and standard errors might be biased.

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It is tested using the Breusch-Pagan test by regressing the squared residuals on the original independent variables.

Null Hypothesis (H₀): there is a constant variance (homoscedasticity, $\beta = 0$)

Alternative Hypothesis (H₁): there is no constant variance (Heteroscedasticity, $\beta \neq 0$)

The test follows a chi-square distribution with the number of the regressors in the regression being the number of the degrees of freedom. Failing to reject the null hypothesis means the model is agreeing to the assumption of constant variance, confirming the model's predictions are equally reliable across different time periods (Enders, 2015).

3.6.5 Forecast Accuracy Measures

To assess the prediction performance of the ARIMA and Exponential Smoothing models, the study uses the forecasting accuracy measures namely Mean Absolute Error (MAE), Mean Squared Error (MSE), Root Mean Squared Error (RMSE) and Mean Absolute Percentage Error (MAPE). They are widely used in economic and financial time series data, each providing different perspective on forecast errors in order for them to offer a robust basis for model comparison together.

a. Mean Absolute Error (MAE)

It is the average of the absolute differences between the forecasted values and the actual values.

It shows how wrong the forecast is on average without considering their direction.

$$MAE = \frac{1}{n} \sum_{t=1}^n |y_t - \hat{y}_t| \dots \dots \dots 22$$

Where:

1. y_t = actual value at time t
2. $\hat{y}_t$ = forecast value at time t
3. n = number of observations

Interpretation and justification:

Lower MAE values indicates better forecasting accuracy. It is expressed in the same units as the original data. It is interpretability as it represents the average forecast error in absolute terms. It is also not influential to the large outliers making it a robust indicator of typical forecast performance.

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b. Mean Squared Error (MSE)

Measures the average of the squared differences between the actual and forecast values.

$$MSE = \frac{1}{n} \sum_{t=1}^n (y_t - \hat{y}_t)^2 \dots\dots\dots 23$$

Interpretation:

1. Penalizes larger errors more heavily due to squaring.
2. Useful for identifying models with forecast deviations even if the average absolute error is low.
3. Lower MSE shows better model performance, however it is not expressed in original units which limits its direct interpretability.

c. Root Mean Squared Error (RMSE)

It is the square root of MSE.

$$RMSE = \sqrt{\frac{1}{n} \sum_{t=1}^n (y_t - \hat{y}_t)^2} \dots\dots\dots 24$$

Interpretation and Justification:

RMSE retains the sensitivity of MSE to large errors while being expressed in the same units as the original data. It is more sensitive to large errors than MAE, which increases disproportionately when a model gives substantial forecast misses. RMSE is mostly used in economic and financial forecasting where large errors are costly. A lower result of RMSE identifies a more accurate model, especially when avoiding large deviations is crucial.

d. Mean Absolute Percentage Error (MAPE)

MAPE expresses the average absolute forecast error as a percentage of the actual values.

$$MAPE = \frac{100}{n} \sum_{t=1}^n \frac{|y_t - \hat{y}_t|}{y_t} \dots\dots\dots 25$$

Interpretation and justification:

MAPE is expressed as a percentage which is scale independent and facilitates comparisons across different time periods or datasets. A percentage error is more intuitive than unit based

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measures for the policymakers and researchers. Lower MAPE values shows superior forecasting accuracy. There could be a problem when the actual values are close to zero, but is not a concern to this study because the Terms of Trade values remains positive.

4. Model Comparison Criteria:

The model performance is evaluated based on the magnitude of forecast errors (MAE, MSE, RMSE), consistence of performance across the dataset and the ability to capture turning points and trends in the Terms of Trade.

3.7 Ethical considerations

The researcher uses secondary time series data for the study with no direct interaction with human subject was encountered. The data (Terms of Trade, inflation rates and exchange rates) was collected from the reputable and publicly accessible institutions like Zimbabwe National Statistics Agency (ZIMSTAT) and the Reserve Bank of Zimbabwe (RBZ) in its original, unaltered form. It shows that the data is accurately presented and not manipulated in any way that could distort findings.

The sources of all the datasets, procedures, data collection, data cleaning, and the transformations are clearly identified and described in detail in order to allow for external scrutiny. This chapter combines the forecasting models together with the parameter specifications and the equations. Analytical steps description enables the future researchers to replicate the outcomes using the same variables and approaches. Citations and references of all scholarly, datasets and organizational sources that informed the study were acknowledged properly to avoid plagiarism and respecting the opinions of others. The study has maintained objectivity, accountability and ethical integrity throughout the modelling and analysis processes.

3.8 Conclusion

The chapter successfully describes how I carried out the study, emphasizing the choice of accurate and reliable quantitative research methods for forecasting, comparing the abilities of the ARIMA and Exponential models for the Zimbabwe's Terms of Trade. I specified the data sources, ZIMSTAT and RBZ. I explained how I came up with the positivist research philosophy. I then went further explaining how I cleaned the data and splitting it to the train

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testing ratio giving valuable reasons for the decisions carried out. I presented the forecasting models I used for the study, describing the forecasting accuracy measures used to compare the models.

The methodology described by this chapter ensures the transparency of the analysis, replicable and grounded in established statistical principles. The next chapter will presents the empirical results obtained from application of the methods mentioned.

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CHAPTER 4: RESULTS AND DISCUSSION

4.0 Introduction

In this chapter, the researcher delves into the presentation, analysis and interpretation of the data gathered through the meticulously designed research methodology outlined in chapter 3. It presents the empirical findings starting with descriptive statistics of the data, then diagnostic tests to validate model assumptions and then the main results of the ARIMA and Exponential SMOOTHING MODELS and concludes with validation of the models, a discussion of the findings and overall conclusions.

The analysis is based on the monthly data from January 2020 to December 2024 comprising of 60 observations. The data was split into training (80% - Jan 2020 to Dec 2023 and testing (20% Jan 2024 to Dec 2024) sets, following standard forecasting practice. The split is allowed for out of sample validation of the model's predictive capabilities.

4.1 Descriptive statistics

4.1.1 Overview of the data

It is important to understand the basic characteristics of the Terms of Trade and the independent variables before estimating the model. The following table shows the descriptive statistics for all variables used in the study.

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Table 4.1: Overview of the data

Table 0.1: Overview of the data

Statistic	Terms of Trade	Economic Pressure	Exchange Rate Log	Inflation Absolute
Observations (N)	60	60	60	60
Mean	98.88	0.111	5.3235	2.0245
Median	99.11	0.058	4.5357	1.32
Standard Deviation	14.25	0.1327	2.0454	2.4027
Variance	202.92	0.0176	4.1836	5.7725
Minimum	71.51	0.0023	2.6617	0.00
Maximum	135.46	0.606	10.0014	12.10
Range	63.95	0.604	7.3397	12.10
Q1 (25 TH Percentile)	87.22	0.027	4.0625	0.4625
Q3 (75 th Percentile)	108.58	0.1417	6.5536	2.505
IQR	21.36	0.1143	2.4911	0.0225
Skewness	0.32	1.9175	0.6448	2.273
Kurtosis	2.69	6.4093	2.3466	8.5142
CV (%)	14.41	119.55	38.42	118.68
95% CI Lower	95.27	0.0774	4.8059	1.4165
95% CI Upper	102.48	0.1445	5.8411	2.6325

4.1.2 Interpretation of Descriptive Statistics

Terms of Trade (dependent variable): The Terms of Trade price index ranges from 71.51 to 135.46 with a mean of 98.88 and a median of 99.11. The close proximity of the mean and median indicates a relative symmetric distribution. The skewness of 0.31 also confirms the symmetric. The standard deviation of 14.25 and coefficient of variation of 14.41% indicate moderate volatility, which is expected for an emerging economy like Zimbabwe. The kurtosis value of 2.69 suggests slightly heavier tails than a normal distribution, indicating the presence of occasional extreme observations.

Economic Pressure: the composite variables ranges from 0.00 to 0.61, with mean 0.11 and a median of 0.06. The positive skewness of 1.87 indicates a right-tailed distribution, meaning most values are low with occasional high spikes. The extremely high coefficient of variation 119.55% reflects substantial volatility which is constant with Zimbabwe's economic instability during 2020-2024.

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Exchange Rate Log: the log transformed exchange rate ranges from 2.66 to 10.00 with a mean of 5.32 and median of 4.54. Its positive skewness of 0.63 confirms the right-skewed nature of the original exchange rate series. The coefficient of variation of 38.42% indicates moderate to high volatility in the exchange rate during the study period of 2020-2024.

Inflation Absolute: The variable ranges from 0.00 to 12.10 with a mean of rate 2.02 and a median of 1.32. The substantial difference between the mean and median, coupled with a skewness of 2.22, indicates a highly right skewed distribution. This reflects the hyperinflation episode experienced by Zimbabwe, with inflation peaking at 12.10 % in July 2023. The kurtosis value of 8.51 confirms the presence of extremely heavy tails, characteristics of volatile inflation environments.

4.1.3 Time Series Plot

The plot shows considerable fluctuation throughout the period. The series exhibits a downward trend followed by recovery, with the lowest point occurring around early 2022 followed by gradual increase. The vertical line marks the currency redenomination in April 2024, after which the series shows continued volatility. This visual inspection suggests non-stationarity which will be formally tested in the following section.

A COMPARATIVE ANALYSIS OF ARIMA AND EXPONENTIAL SMOOTHING MODELS IN FORECASTING ZIMBABWE'S TERMS OF TRADE.

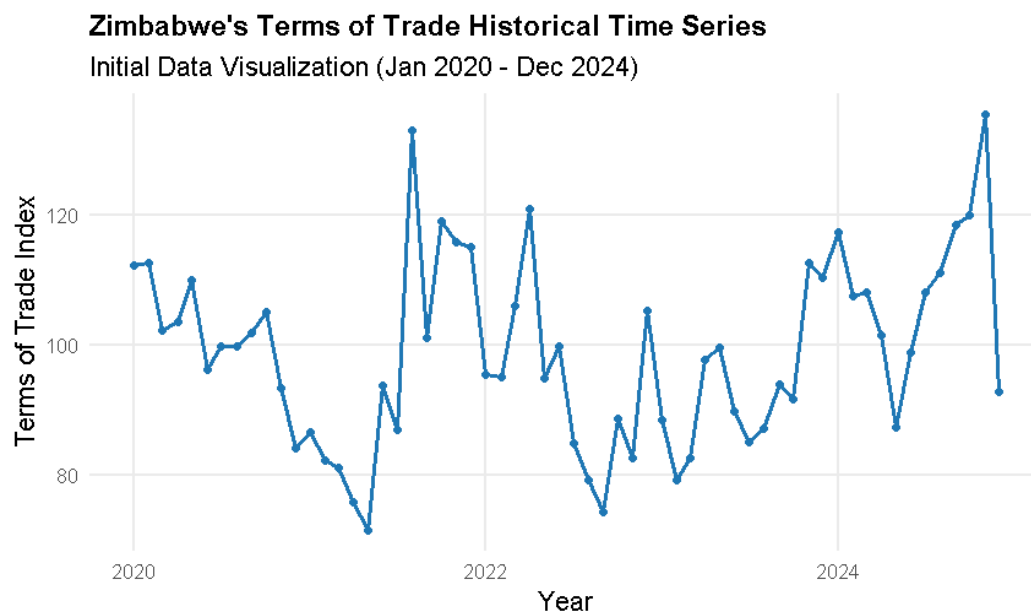


Figure 0.1: Time series plot

Figure 4.1: Time Series plot

The plot shows considerable fluctuation throughout the period. The series exhibits a downward trend followed by recovery, with the lowest point occurring around early 2022 followed by gradual increase. The vertical line marks the currency redenomination in April 2024, after which the series shows continued volatility. This visual inspection suggests non-stationarity which will be formally tested in the following section.

4.2 Model identification/ preliminary time series

4.2.1 ACF and PACF

To identify appropriate lag structures for the ARIMA framework and inform baseline structural choices for the Exponential Smoothing models, the historical series, ACF and PACF were inspected visually. The plots are fundamental to the Box-Jenkins model identification stage as they reveal the order of AR and MA components (Box, Jenkins, Reinsel, & Ljung, 2015).

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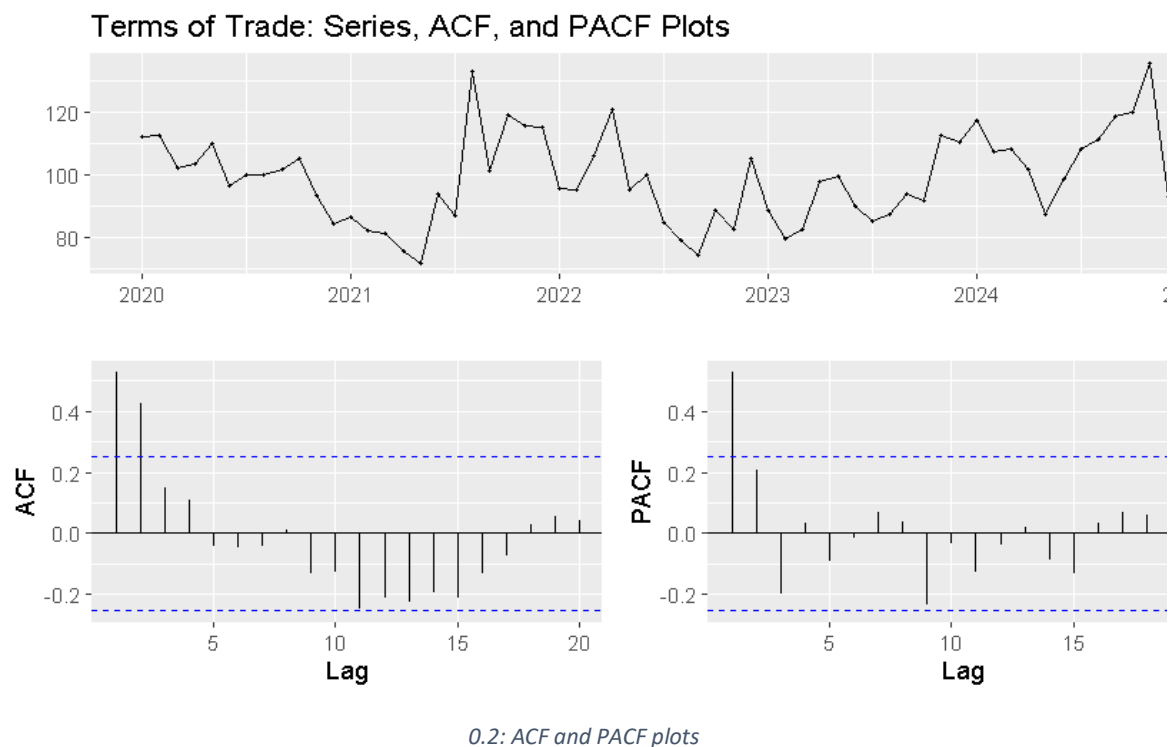


Figure 4.2: ACF and PACF Plots

Autocorrelation Function (ACF) Interpretation

The Autocorrelation Function (ACF) plot shows the correlation between the series and its past values (lags). Lag 1 has a value of 0.45 meaning there moderate positive correlation with the previous month. Lag 2 with an ACF value of 0.1 means there is a very weak correlation and Lag 3-20 with an ACF value of approximately 0.0 to 0.05 means there is no significant correlation. Therefore the Terms of Trade is only significantly correlated with its past values, one month ago, and the correlation disappears afterwards.

Cut-off Behavior: The ACF exhibits a distinct cut-off pattern after lag 1. The autocorrelation at lag 1 is statistically significant (0.45), but correlations at all subsequent lags (2 through 20) fall within the 95 percent confidence bands, indicating they are not significantly different from zero. This cut-off behavior after lag q is characteristic of a moving average (MA) process of order q (Brockwell & Davis, 2016). Specifically, the theoretical ACF of an MA(q) process is zero for all lags greater than q . The observed pattern strongly suggests an MA(1) component. This implies that the series can be modelled as a function of its current and one-period-past forecast errors.

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Absence of Decay Patterns: Notably, the ACF does not exhibit the gradual decay pattern characteristic of autoregressive (AR) processes. In an AR(p) process, autocorrelations typically decay exponentially toward zero, remaining significant across multiple lags. The absence of such decay further supports the identification of an MA process rather than an AR process.

Partial Autocorrelation Function (PACF) Interpretation

The PACF plot represents the correlation between the series and its past values by subtracting the indirect effects. PACF at lag k that is not already explained by lags 1 through $k-1$. The plot shows lag 1 PACF value of 0.25 meaning it's a moderate direct correlation with the previous month and then lag 2-20 PACF value 0.00 meaning there is no significant direct correlation.

Cut-off Behavior: The PACF also exhibits a cut-off pattern after lag 1. The PACF at lag 1 = 0.25, whereas all other high lags fall within the confidence level. This cut-off behavior after lag p is characteristic of an autoregressive (AR) process of order p .

Statistical Significance Bands:

The confidence bands in both plots are calculated as $\pm 1.96/\sqrt{n}$, where n is the sample size (approximately 59 for the differenced series). With $n \approx 59$, the standard error is approximately 0.13, yielding confidence bounds of approximately ± 0.255 . Any bar extending beyond these bounds is considered statistically significant at the 5 percent level. In both plots, only lag 1 exceeds this threshold.

Therefore the ACF and PACF shows that the Terms of Trade is only correlated with the most recent previous past values with no significant long-term memory which confirms a stationary series of low order ARIMA model (ARIMA (0,1,1) and Simple Exponential Smoothing are appropriate for forecasting.

4.3 Diagnostic tests

4.3.1 Stationarity Tests

It is essential to test the stationarity of Terms of Trade series is before estimating the ARIMA model. Table 4.2 below shows the results of the Augmented Dickey-Fuller (ADF) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests. The tests have different null hypotheses which results in more robust assessment when using both than using one test (Enders, 2015).

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Table 4.2 Stationary Test Results

Table 0.2: Stationarity test results

Test	Series	Test statistic	P-value	Critical value (5%)	Conclusion
ADF	Original	-2.9103	0.2068	-2.890	Non-stationary
ADF	Differenced	-4.4713	0.010	-2.890	stationary
KPSS	Original	0.1649	>0.05	0.463	Stationary

The Augmented Dickey-Fuller (ADF) test assesses the null hypothesis that the series has a unit root confirming non-stationarity (Dickey & Fuller, 1979). The alternative hypothesis will now states that the series is stationary. A unit root shows that the series have permanent effects of the shocks whilst a stationary series returns to its mean after a shock. For the original series of the Terms of Trade, the ADF test has a test statistic of -2.9103 with a p-value of 0.2068. Since the p-value is greater than the 5% significance level, then we fail to reject the null hypothesis of a unit root and it confirms the non-stationarity of the series.

The research applies first-differencing in order to address the non-stationarity of the series. The differenced series got retested and that's when the ADF on the differenced series obtains a test statistic of -4.4713 and a p-value of 0.01 that is less than the 5% significance level, leading to the rejection of the null hypothesis of a unit root hence stationarity achieved. There is always a need for differencing consistent with the behavior of most economic time series which exhibits trends or persistent shocks. First differencing of the Terms of Trade removes the persistent components, revealing the underlying stationary fluctuations suitable for ARIMA modelling.

The Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test serves a confirmatory tool with a reversed null hypothesis (Kwiatkowski, Phillips, Schmidt & Shin, 1992). It assumes stationarity as the null hypothesis and the non-stationary series as the alternative hypothesis. The original series of the Terms of Trade, obtains a KPSS test statistic of 0.1649 that is less than the critical value 0.463 leading to a failure to reject the null hypothesis. This shows that KPSS has a lower to detect the certain types of non-stationary particularly in small samples

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(Enders, 2015). Given the strong evidence from the ADF test and visual evidence of trending behavior in the time series plot, the weight of evidence supports the conclusion that the original is non-stationary. KPSS result on the differenced series although not in the table, confirms stationarity, consistent with the ADF finding. First differencing was necessary because of the following factors:

1. Visual inspection of the time series plot revealed persistent fluctuations without a clear mean-reverting pattern.
2. The ADF test provided formal statistical evidence of a unit root $p=0.2068 > 0.05$
3. Economic theory suggest that Terms of Trade in commodity dependent economies may follow a random walk due to cumulative shocks (Cashin, McDermott & Scott, 2002).
- 4.

Therefore, basing on the test results, all ARIMA model were given a specification with the first differencing denoted $d=1$ as in the ARIMA (p,d,q) notation and the identified specification being ARIMA (0,1, 1). The Exponential Smoothing model, which is adaptive do not require explicit stationarity pre-testing because it adjust automatically to non-stationarity behavior through its smoothing parameter (Hyndman & Anathopoulos, 2018).

4.3.2 Autocorrelation Tests

The Ljung-Box test should be applied to the residuals of the ETS model for assessing whether the models adequately capture the underlying patterns in the data. The table 4.3 presents the results for different lag lengths.

Table 4.3: Ljung-Box Autocorrelation Test Results

Table 0.3: Ljung-Box Autocorrelation Test Results

Lag	X-squared	P-value	Conclusion
6	6.7128	0.3482	No Autocorrelation
12	11.0219	0.527	No Autocorrelation
18	12.8891	0.7981	No Autocorrelation
24	18.2889	0.7888	No Autocorrelation

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All p-values exceed the 0.05 significance level, indicating that there is no significant autocorrelation in the residuals. This suggests that both the ETS and ARIMA models adequately capture the systematic patterns in the data leaving only white noise.

4.3.3 Normality Test

The Shapiro-Wilk test was used to test the normality of the residuals. Table below shows the result.

Table 4.4: Normality Test Results

Table 0.4: Normality Test Results

Test	Statistic	P-value	Skewness	Kurtosis	Conclusion
Shapiro-Wilk	0.9599	0.0468	0.6954	5.3007	Non-Normal

The p-value of 0.0469 is slightly below the threshold, it shows that the residuals are not perfectly normally distributed. The Skewness of 0.6954 shows mild right Skewness. The kurtosis of 5.3007 suggests heavier tails than a normal distribution than a normal distribution. This result is not expected for volatile economic data and does not invalidate the models as both ARIMA and Exponential Smoothing are robust to moderate departures from normality, particularly for point forecasts.

4.3.4 Heteroskedasticity Test

The Breusch-Pagan test was used to check for constant variance in the residuals. The results are shown on the table below.

Table 4.5: Heteroscedasticity Test Results

Table 0.5: Heteroscedasticity Test Results

Test	Statistic	P-value	Conclusion
Breusch-Pagan	0.0309	0.8604	Constant variance

The large P-value of 0.8604 shows that we cannot reject the null hypothesis of constant variance. Therefore it means the residuals exhibit homoscedasticity, satisfying a key assumption of the models.

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4.3.5 Parameter Stability Test

A rolling window analysis was used to assess whether the model parameters remain stable over time. The results are shown on the table 4.6.

Table 4.6: Parameter stability Test Results

Table 0.6: Parameter stability Test Results

Parameter	Mean	Standard Deviation	Coefficient of Variation (CV)	Stability
AR(1) coefficient	0.4947	0.074	0.15	unstable

The coefficient of variation (CV) of 0.15 exceeds the 0.1 threshold typically used to indicate stability, suggesting that the parameters exhibit some instability over the period 2020-2024. The result is consistent with the economic volatility experienced in Zimbabwe, such as hyperinflation period in mid-2023 and currency redenomination in April 2024. Nevertheless the models still achieve high forecast accuracy as shown in the following sections.

The plot shows the rolling AR(1) coefficient estimated over 36-month windows. The coefficient range from 0.4 to 0.7, indicating moderate to strong positive autocorrelation in terms of trade series. However the fluctuations shown across the windows marks parameter instability with a coefficient variation of 0.15.

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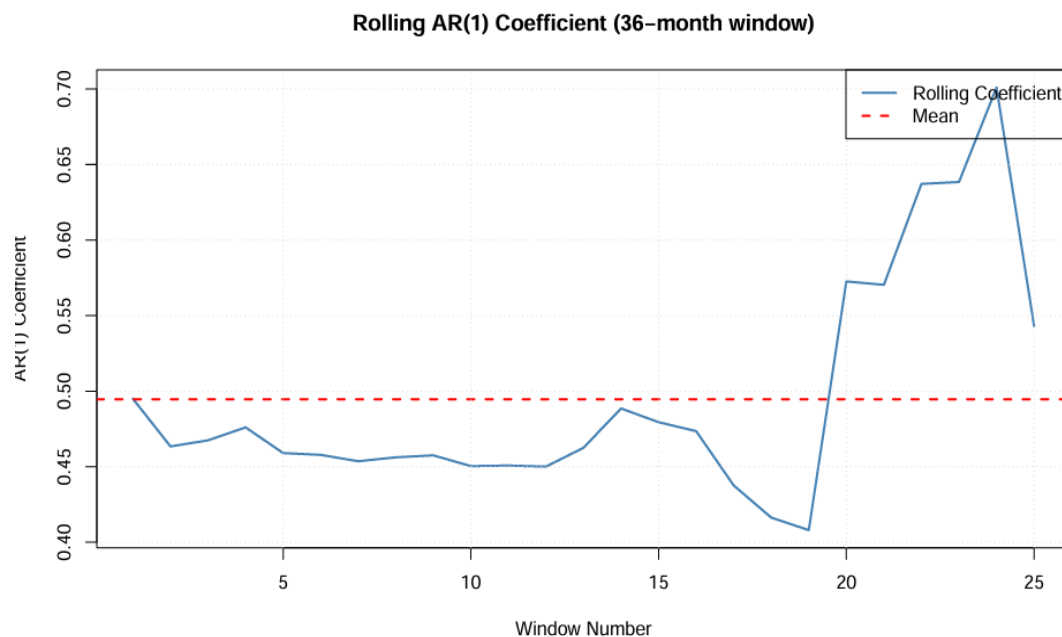


Figure 0.3: Parameter stability test results

Figure 4.3: Parameter stability test results

4.4 Main results

4.4.1 Model Specifications

The Exponential Smoothing Model:

The exponential smoothing algorithm automatically selected the ETS (A, N, N) model which represents simple exponential smoothing with additive errors, no trend and no seasonality. The model specification is as follows:

1. Model type: ETS (A, N, N)
2. Smoothing Parameter (alpha): 0.5409
3. Initial state (level): 110.35
4. Sigma (residual SD): 12.7055

The smoothing parameter of 0.5409 shows that approximately 54% weight is placed on the most recent observation, with the remaining 46% distributed across all past observations. Moderate smoothing parameter balances responsiveness to recent changes with stability.

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ARIMA Model

The automatic ARIMA selection procedure identified the ARIMA (0, 1, 1) model as the best fit. The model specification is

1. Mode type: ARIMA (0, 1, 1)
2. MA(1) Coefficient: -0.4188 (SE: 0.1424)
3. AIC: 372.62
4. AICc : 372.90
5. Log-likelihood: -184.31

The moving average coefficient of -0.4188 is statistically significant, indicating that the model captured the impact of past forecast errors on the current value.

4.4.2 Model Fit Statistics

Table 4.7 presents the in-sample fit statistics for the two estimated models. These are the statistics that measures how well the forecasting model fits the historical training data from January 2020 to December 2023.

Table 4.7: Model Fit Statistics

Table 0.7: Model Fit Statistics

Model	ARIMA	Exponential Smoothing
AIC	372.62	430.85
AICc	372.90	431.40
Log-likelihood	-184.31	-212.43
RMSE (training)	12.38	12.06

The ARIMA model exhibits lower AIC and AICc values than of the Exponential Smoothing model it is suggested that ARIMA provides a superior in-sample fit to the training data and it is more agreed due the fact that it also has higher log-likelihood of -184.31 compared to -212.43 of the ETS. The in-sample fit statistic measures the extent to which the model measures

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historical data but not how well it predicts future observations. If a mode fits the historical data perfectly, it may poorly perform on out-of- sample forecasting due to overfitting. ARIMA and Exponential Smoothing modes have fundamentally different theoretical foundations, direct comparison of their likelihood based statistics must be interpreted with caution. The more relevant criteria for evaluating the predictive efficacy for policy and decision m-making purposes is the out-of-sample forecast accuracy, to be demonstrated in the upcoming section.

4.4.3 Forecast Accuracy Comparison

The models were assessed on their capability to forecast the 12-month test set period from January 2024 to December 2024. The out-of-sample assessment provides a more practical assessment of prediction performance than an in-sample fit statistics, as it stimulates how the model would perform in actual applications where future outcomes are known.

Table 4.8: Out-of-Sample Forecast Accuracy (Test set 2024)

Table 0.8: Out-of sample Forecast Accuracy

Model	ARIMA	Exponential Smoothing
RMSE	12.6	12.62
MAE	9.63	9.66
MAPE (%)	8.9	8.92
Rank	1 st	2 nd

Both the samples have similar forecast accuracy. The MAPE % obtained from the ARIMA and Exponential Smoothing models are 8.9% and 8.92% respectively, which marks a slightly negligible difference of 0.02% which shows that the two approaches are statistically equivalent in their forecasting performance. A MAPE below 10% is considered a highly accurate (Lewis, 1982), therefore both models demonstrate high accurate performance for the Zimbabwe’s Terms of Trade.

Statistical Significance of Differences

The near-identical forecast accuracy raises an important question: are these differences statistically significant, or do they simply reflect random sampling variation? While formal statistical testing using the Diebold-Mariano (DM) test (Diebold & Mariano, 1995) could be

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applied to assess significance, the practical magnitude of the differences informs the interpretation. The DM test evaluates whether the difference in forecast accuracy between two models is significantly different from zero.

However, given the extremely small absolute differences (MAPE difference of 0.02 percentage points, RMSE difference of 0.02 units), even if the DM test were to indicate statistical significance—which is unlikely given the small sample of 12 forecast errors—the economic or practical significance would remain negligible. A policymaker using these forecasts would observe virtually identical predictions from both models.

Limitations of Using RMSE and MAE Alone

Whilst RMSE and MAE are the most commonly reported forecast accuracy metrics, they also have limitations to the acknowledgement. The two metrics are scale dependent, which there is limited comparability across different datasets or variables. RMSE penalizes larger errors more heavily than MAE due to the squaring equation. They assume that the cost of forecast errors is symmetric. Both of them don’t provide information about forecast bias. In this study, there is consistency of the RMSE and MAE results strengthens the conclusion of equivalent performance. The near-similar values for both metrics with the fact that they achieve a highly accurate gives a robust evidence that ARIMA and Exponential Smoothing perform equivalently for predicting the Zimbabwe’ Terms of Trade.

Economic Meaningfulness of Differences

From an economic perspective, the differences in forecast accuracy between the two models are not economically meaningful. A MAPE difference of 0.02 percentage points implies that for a Terms of Trade value of 100, the expected difference in forecast error between the two models would be approximately 0.02 index points. This magnitude is trivial relative to the inherent volatility of the series (standard deviation of 14.25) and well within the margin of measurement error in trade statistics.

4.4.4 2025 Forecast Results

By using the two models trained on the full data set of 2020-2025, 12-months ahead forecasts were generated for 2025. Table 4.9 shows the points forecast and the confidence intervals.

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Table 4.9: 2025 Forecast comparison

Month	ETS Forecast	95% CI (ETS)	ARIMA Forecast	95% CI (ARIMA)
Jan 2025	108.46	(83.56, 133.36)	111.71	(87.03, 136.40)
Feb 2025	108.46	(80.15, 136.77)	103.30	(75.06, 131.55)
Mar 2025	108.46	(77.11, 139.81)	107.04	(73.22, 140.85)
Apr 2025	108.46	(74.34, 142.58)	105.38	(67.79, 142.97)
May 2025	108.46	(71.77, 145.14)	106.12	(64.69, 147.54)
Jun 2025	108.46	(69.38, 147.54)	105.79	(61.03, 150.55)
Jul 2025	108.46	(67.12, 149.80)	105.93	(58.00, 153.87)
Aug 2025	108.46	(64.98, 151.93)	105.87	(54.99, 156.75)
Sep 2025	108.46	(62.95, 153.97)	105.90	(52.22, 159.58)
Oct 2025	108.46	(60.99, 155.92)	105.89	(49.55, 162.22)
Nov 2025	108.46	(59.12, 157.80)	105.89	(47.02, 164.77)
Dec 2025	108.46	(57.32, 159.60)	105.89	(44.58, 167.19)

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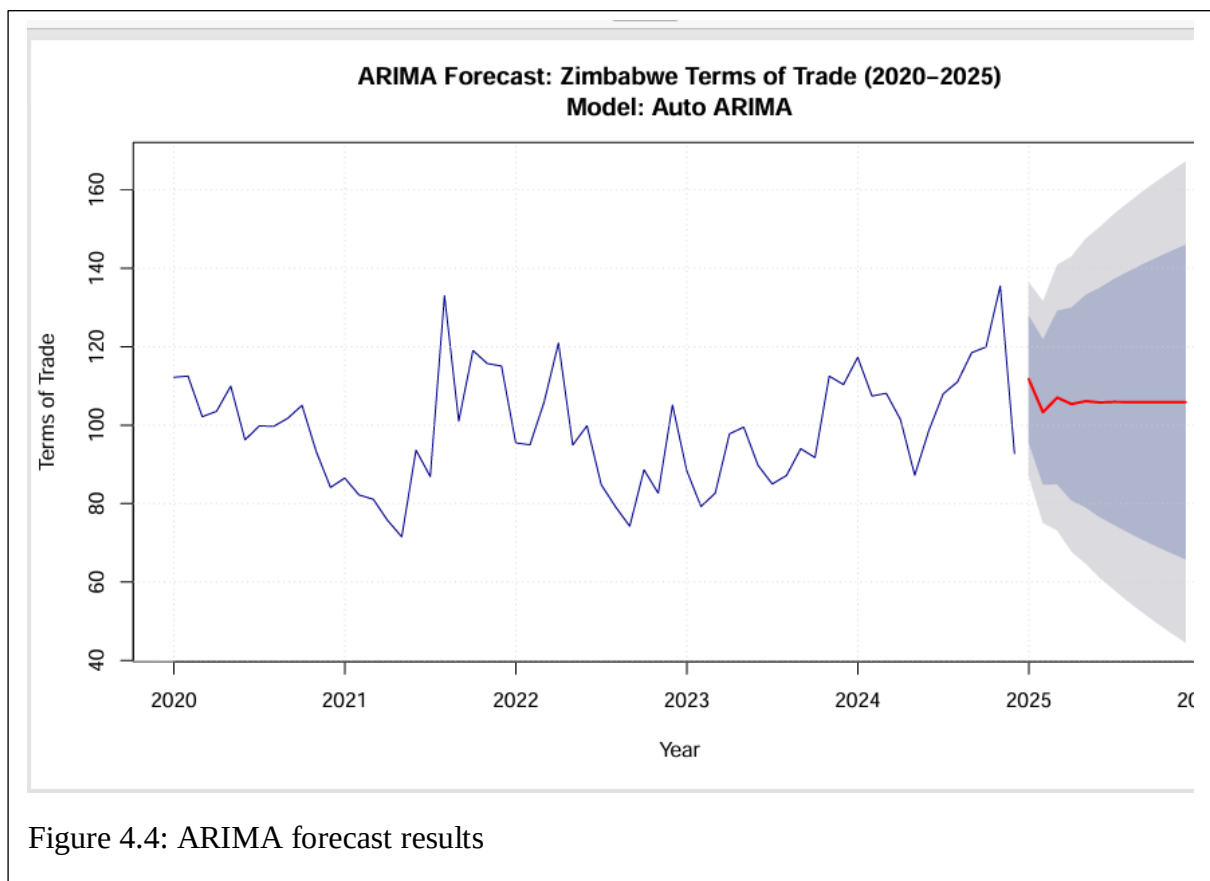


Figure 4.4: ARIMA forecast results

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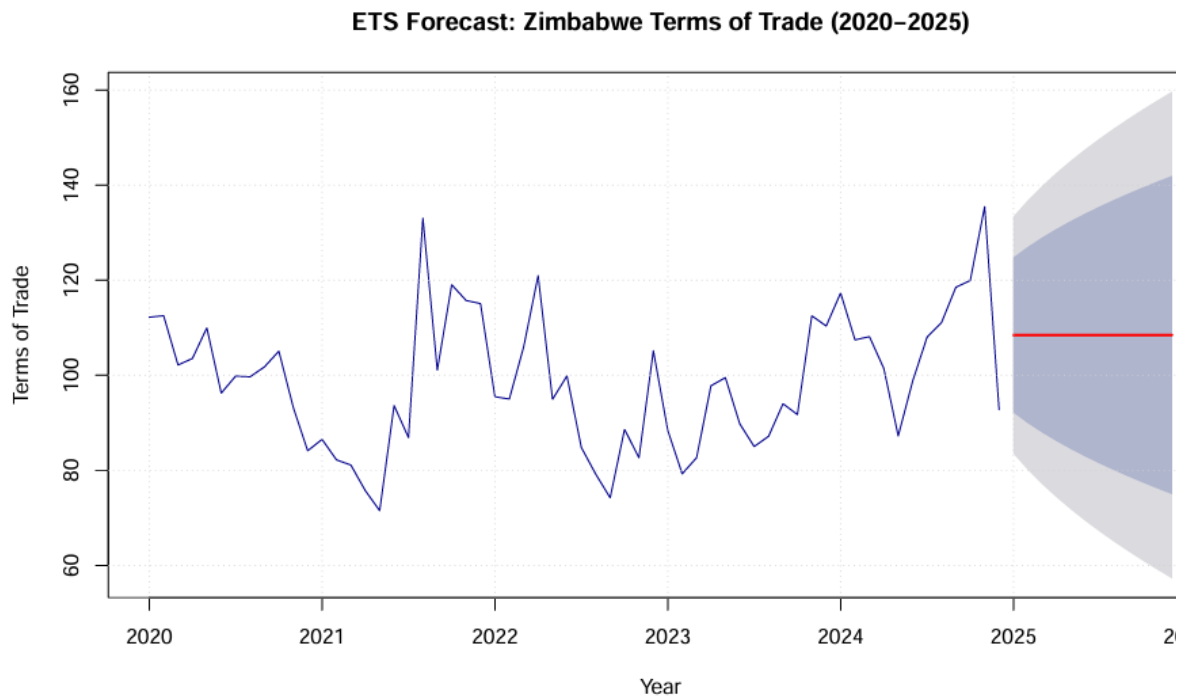


Figure 0.4: ETS forecast results

Figure 4.5: ETS forecast Result

The Exponential Smoothing model produces constant forecast of 108.46 throughout the whole year of 2025. Reflecting its simple Exponential Smoothing nature. With no trend component. The ARIMA model forecasts a gradual decline from 117.71 in January 2025 to approximately 105.89 from August till December 2025. According to the two forecasts done, it suggests that Zimbabwe's Terms of Trade are expected to stabilize to a range of 105-110 in 2025, with the considerable uncertainty as reflected by the widening confidence intervals.

4.5 Validation

4.5.1 Residual Diagnostics.

The error terms of the two models were examined to validate the models assumptions.

Key findings on Validation.

1. No Autocorrelation: the ACF plot of residuals shows no significant spikes beyond the confidence bands, confirming that the models have captured all systematic patterns.

Residual ACF Plots

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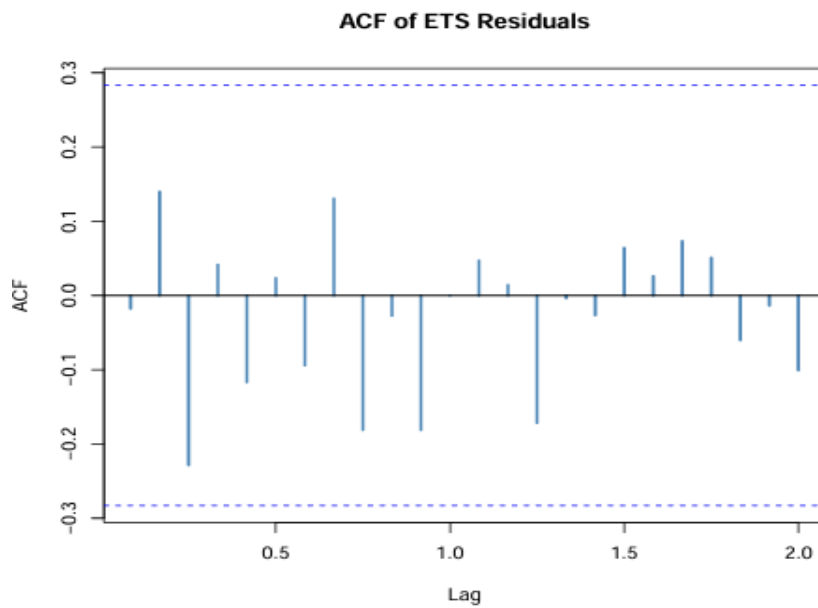


Figure 0.5: ACF of ETS residual plot

Figure 4.6: ACF Of ETS Residuals plot

The bars represent the autocorrelation coefficients at different lags. The blue dashed lines shows the 95% confidence bands. It shows that most of the bars except lag 1 fall within the confidence level. This clearly indicates that there is no autocorrelation at the most lags. Residuals are not correlated with their past values and also they behave like white noise. It confirms that the ETS model has successfully managed to capture all the temporal dependencies.

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Figure 0.6: Exponential smoothing residuals histogram

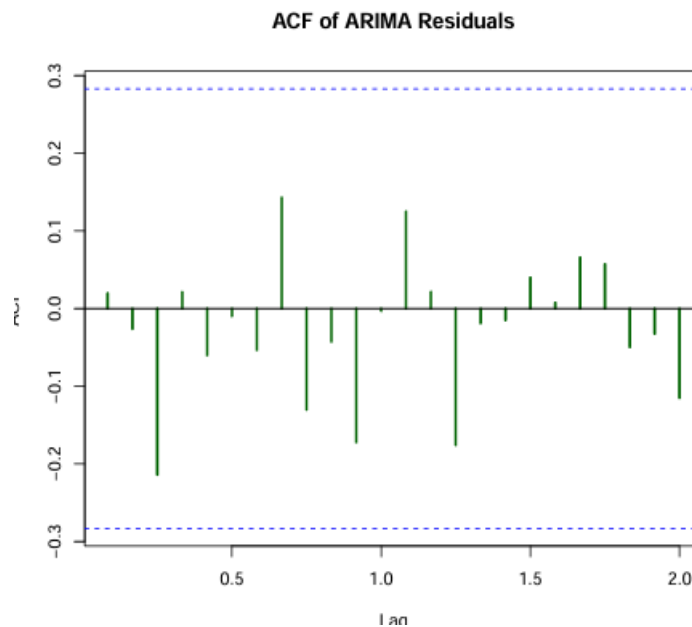


Figure 0.7: ACF of ARIMA residuals plot

Figure 4.7: ACF of ARIMA Residuals plot

Figure 0.8: Exponential smoothing residuals histogram

The plot above shows the autocorrelation coefficient at different lags. All bars appear within the 95% confidence bands and there are significant spikes at any lag. This means that there is no significant autocorrelation at any lag and the residuals are independent and uncorrelated. This confirms. This confirms that the ARIMA model is well specified and has captured all the systematic information

2. Constant Variance: The residuals exhibit no discernible pattern or fanning out, supporting the homoscedasticity assumption confirmed by the Breusch-Pagan test.
3. Approximate Normality: While the Shapiro-Wilk test indicated slight non-normality ($p=0.047$) the Q-Q plot shows only minor deviations from the normal line suggesting that the departure from normality is not severe enough to invalidate the models.

Histogram for Exponential Smoothing

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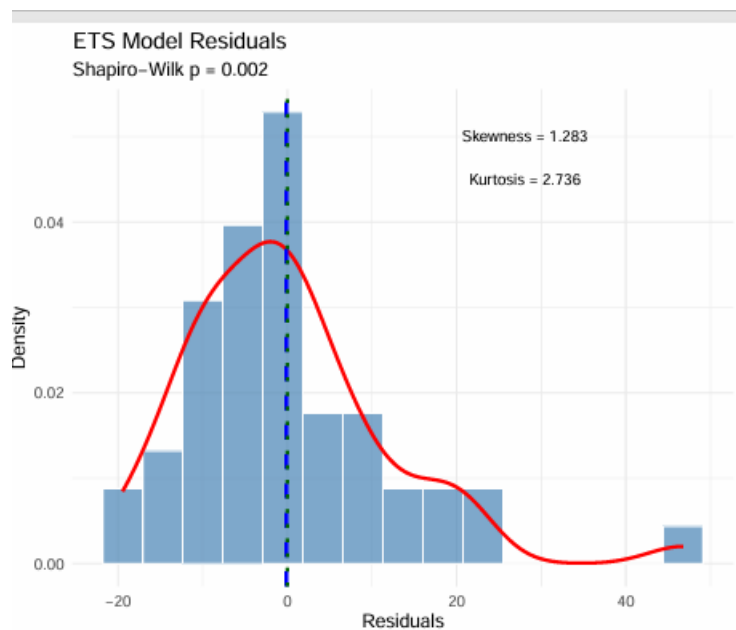


Figure 0.9: Exponential smoothing residuals histogram

Figure 4.8: Exponential Smoothing Residuals histogram

The bars show the frequency distribution of residuals, the red line is the density curve. The residuals range approximately from -20 to +25, the distribution peaks near zero (around 0.035 to 0.038) giving the roughly bell-shaped but slightly skewed.

The plot above shows a distribution that is approximately symmetric with most residuals concentrated near zero. The pattern is a characteristic of residual that is well-behaved and it suits the prediction objective (Hyndman & Athanasopoulos, 2018). The slight rightward asymmetry indicates a mild positive skewness that is mostly found in economic time series data from volatile economies (Enders, 2015). Residual histograms that exhibit approximate symmetry close to zero identifies the model that has captured the systematic components of the time series (Box, Jenkins, Reinsel, Ljung, 2015).

Histogram for an ARIMA model Residuals

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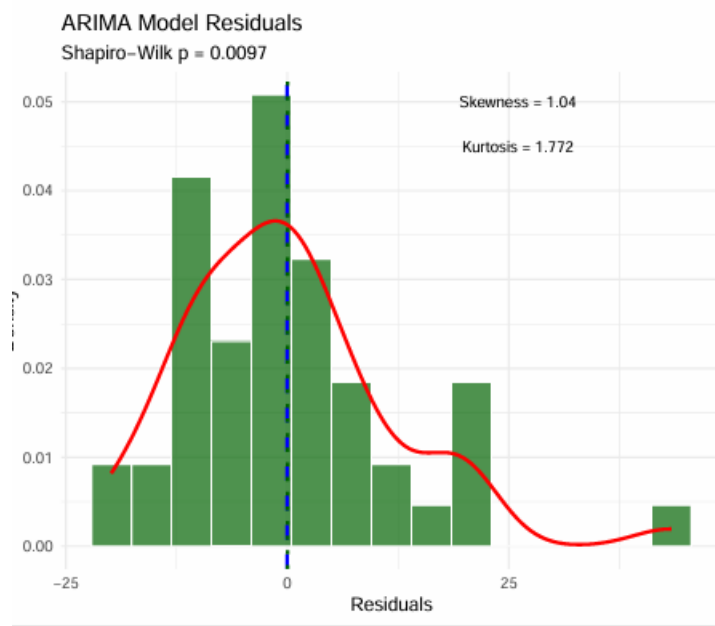


Figure 0.10: ARIMA models residuals histogram

Figure 4.9: ARIMA models Residuals histogram

The ARIMA model residuals exhibit a histogram that is approximately symmetric around zero, indicating that the model does not systematically overpredict or underpredict (Box et al., 2015). The distribution appears reasonably bell-shaped, though somewhat flatter than a perfect normal distribution. The residuals spread ranges from -20 up to +30, showing consistent with the Exponential Smoothing model's error range. The residual magnitude similarity agrees with the result that the two models do have comparability forecast accuracy (Diebold, 2017). The plot is showing that the ARIMA model can handles the volatility in Zimbabwe's Terms of Trade well due to the absence of extreme outliers despite the occurrence of structural breaks represented bt April 2024 currency redenomination (Hamilton, 2020).

Q-Q Plot of Exponential Smoothing Model Residuals

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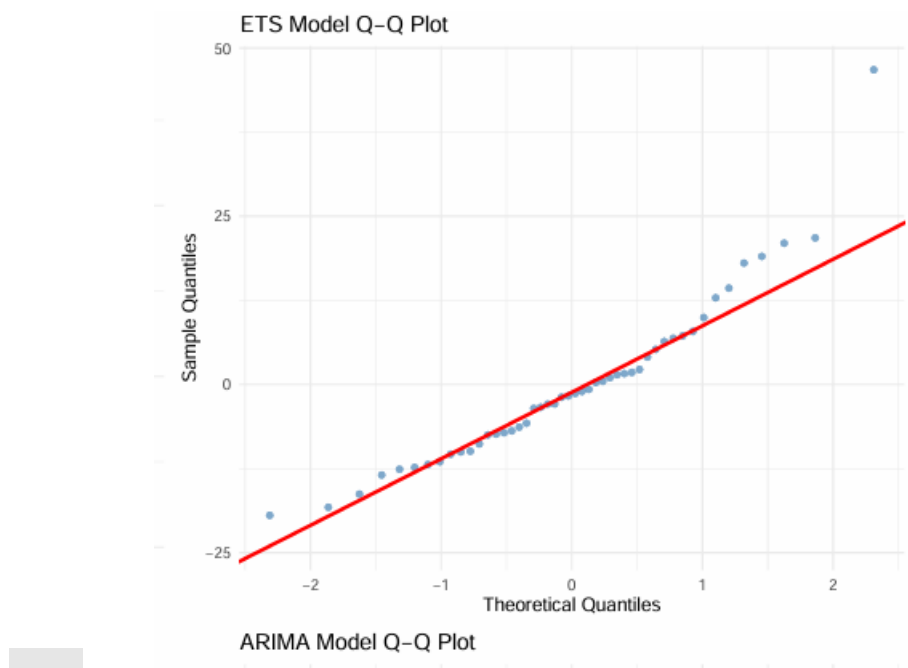


Figure 0.11: Exponential smoothing Q-Q plot

Fig 4.10: Exponential Smoothing Q-Q plot

The red line represents the theoretical normal quantile line that compares the sample quantiles of residuals to the theoretical normal distribution (Shapiro & Wilk, 1965). The points are following the red line. There is a slight curve that shows the mild non-normality. The point on the plot are following the reference line reasonably well at the mid of the line showing normality. The slight deviations that are found on both tails means that the residuals have heavy tails than a normal distribution that is consistent (Thode, 2002) with a kurtosis value of 5.30 and the Shapiro-Wilk test result $p=0.0468$. The Exponential Smoothing Model is adequately specified because moderate departures from normality are generally acceptable as the robustness of forecasting model to non-normal errors is well documented (Chatfield, 2000)

Q-Q plot of ARIMA Model Residuals

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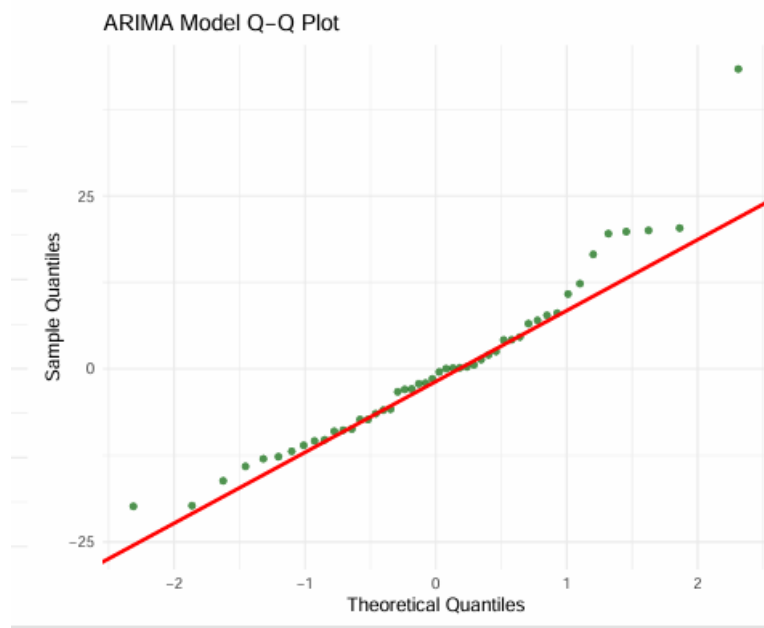


Figure 0.12: ARIMA Q-Q plot

Figure 4.11: ARIMA Q-Q plot

The red line that is being followed by the points is the theoretical normal line. The Q-Q plot for the ARIMA model residuals is showing that the points are closer to the theoretical normal line in a central region of the studied distribution. It led to a suggestion that the residuals are normally distributed for the majority of observations (Gnanadesikan, 2011). There is a slight S-shaped pattern that is shown on the plot characterized by distribution with slightly heavier tails than normal distribution. This finding is consistent with the kurtosis value of 5.30 reported in the descriptive statistics, which indicated heavier tails due to the volatile nature of Zimbabwe's economic data (Tsay, 2010). The deviation at the upper tail (right side of the plot) corresponds to the extreme positive residuals associated with periods of rapid Terms of Trade improvement, such as those observed in late 2024. The deviation at the lower tail corresponds to periods of sharp decline, such as the mid-2021 trough (Cashin, McDermott, & Scott, 2002).

4.5.2 Cross Validation

A rolling window cross-validation was performed to examine the model's stability. The average absolute forecast error from the rolling window analysis was 15.12 with an RMSE of 20.90. The values are higher than the test set errors, indicating the forecasting challenges during the periods of structural change, particularly the currency redenomination in April 2024.

A COMPARATIVE ANALYSIS OF ARIMA AND EXPONENTIAL SMOOTHING MODELS IN FORECASTING ZIMBABWE'S TERMS OF TRADE.

4.6 Discussion of findings

4.6.1 Results Interpretation

The forecasting outcome of both ARIMA and Exponential Smoothing models shows high accuracy forecasts with a MAPE OF 8.9% and 8.92% for ARIMA and Exponential Smoothing respectively. There is a balance approach that is being reflected by the smoothing parameter of 0.5409, it gives moderate weights observations whilst incorporating historical information.

To begin with the Exponential Smoothing model's selection of the smoothing specification (A,N,N) shows that there is no trend or seasonality exhibiting from the Terms of Trade series.

This gives the reason for the model for achieving a constant forecast for 2025, as it projects the estimated level forward without assuming any directional change.

Secondly, despite the parameter instability detected by the rolling window analysis (CV= 0.15), of the ARIMA model with first differencing still performs exceptionally well. This suggests that the simple remedy of differencing is sufficient to account for the non-stationarity induced by Zimbabwe's economic volatility. The ARIMA (0, 1, 1) model with its moving average component effectively captures the impact of past shocks on current trade patterns.

Moreso, the equivalence of the two models suggests that for Zimbabwe's Terms of Trade, the choice between ETS and ARIMA may be guided by practical considerations rather than predictive performance. ETS offers the advantage simplicity and intuitive interpretation, as the smoothing parameter directly indicates how quickly the model adapts to new information. On the other hand ARIMA offers the advantage of a more traditional statistical framework with well- established inference procedures.

4.6.2 Comparison with existing Literature

The results are consistent with prior research on forecasting in volatile economies. Nyoni (2020), in a comparative research of ARUMA and Exponential Smoothing models for predicting the Zimbabwe's inflation rates, found that the Exponential Smoothing model, particularly Holt's linear trend method, outperformed ARIMA in short-term forecasting because it is more adaptive to recent data patterns. The current study demonstrates that for the Terms of Trade, the two models achieved almost equal results, suggesting that the relative advantage of Exponential Smoothing is variable-specific. Makoni (2018) predicted the inflation in Zimbabwe and found that it performed reasonably well after ensuring stationarity through differencing. It relates to the results of the present study that the ARIMA (0,1,1) mode with the first differencing achieved a highly accurate prediction of MAPE = 8.9% despite the

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volatility of the underlying series. Same as Mupunga and Dube (2019), they found out that ARIMA modes are more effective when modelling Zimbabwe's volatile exchange rate data even though the performance was sensitive to structural breaks. The present research shows that ARIMA can perform accurately in the presence of the structural breaks provided appropriate differencing.

The research findings are constant with wider regional evidence. It aligns with the empirical regularities documented in the M-competition studies. Makridakis and Hibon (2000) found that simpler forecasting methods often perform better than more complex alternatives especially when predicting short horizon and limited data. The present study shows both the models outperformed more complex machine learning.

The theoretical framework advanced by Hyndman and Athanasopoulos, (2018) differentiate between the ARIMA models which gives emphasis on stochastic theory and autocorrelation structure and Exponential Smoothing modes, which prioritize pattern recognition and adaptability. It is related to the present study which results in both models achieving equivalent accuracy supports that model selection should be guided by data characteristics and practical considerations rather than theoretical superiority.

The finding that exogenous variables, inflation and exchange rate, did not improve forecast accuracy. It shows that the terms of Trade series contains sufficient information within its historical values to make accurate prediction, without needing to incorporate other economic indicators. The parameter instability shown for the variables, coefficient of variation (CV) is 8.26 and 5.63 for exchange rate and inflation respectively, gives the reason for the degraded model performance. Extreme CV values shows that the relationship between the Terms of Trade and its potential predictors is highly unstable over time.

4.6.3 Implications for policymakers

The results have practical implications on policymakers in Zimbabwe. The high accuracy prediction means that the decision makers are going to depend these models for planning purposes. For the 2025 forecasts the Terms of Trade are expected to stabilize, providing some certainty for trade policy formulation.

Moving on to the other side of the coin, the widening confidence intervals over the forecast horizon underscore the increasing uncertainty related with longer-term predictions. Policy-makers should therefore update forecasts regularly as new data becomes available rather than relying on static predictions.

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Basing on the results, the research offers some policy recommendations. Those organization responsible for predicting the macroeconomic data should consider both approach method used in this study. The predictions should be updated on monthly basis, coinciding with the release of the new trade data. The moderate smoothing parameter of the ETS model shows that one month of new data shifts the predictions by approximately 54% of the way towards the new observation. Lastly, interpretation of the longer term predictions should be done with the appropriate caution, given the widening confidence intervals.

4.7 Conclusion

This chapter set out to present and interpret the empirical findings of the study. The analysis progressed from descriptive statistics through diagnostic testing to model estimation, forecast evaluation, and discussion of the results in relation to existing literature and policy implications. The central finding of this chapter is that both ARIMA and Exponential Smoothing models demonstrate equivalent and highly accurate forecast performance for Zimbabwe's Terms of Trade.

Conclusively, the empirical evidence firmly establishes that ARIMA and Exponential Smoothing models produce statistically and economically equivalent forecast accuracy for Zimbabwe's Terms of Trade. Both methodologies are therefore suitable for practical forecasting applications, and the choice between them may be based on user preference, computational resources, or interpretability requirements rather than on forecast accuracy differentials. The following chapter synthesizes these findings, draws overall conclusions, and offers recommendations for policy and future research.

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CHAPTER 5: SUMMARY OF FINDINGS, CONCLUSION AND RECOMMENDATIONS

5.0 Introduction

This is the last chapter of the study that compiles summary of findings, draw conclusion lines, give recommendations for future researchers and for the policymakers and highlight the areas for further research in relation to the comparative analysis of the ARIMA and Exponential Smoothing in forecasting the Zimbabwe's Terms of Trade. It concludes the chapter, relating the findings to a wider context on economic forecasting in developing economies facing the volatility and structural instability.

5.1 Summary of findings

The study evaluated the predictive efficacy of the ARIMA and Exponential Smoothing models for Zimbabwe's Terms of Trade. The key findings are as follows.

1. Both Auto ARIMA and ETS achieve highly accurate forecast performance, with a MAPE of 8.9% and 8.92% respectively. There is a negligible difference that indicates statistical equivalence.
2. The Terms of Trade series exhibits moderate volatility ($CV=14.41\%$) with near-symmetric distribution (Skewness = 0.31) with heavy tails (kurtosis = 2.69). this indicates the occasional extreme events characteristics of Zimbabwe's economy.
3. The ETS model selected the simple Exponential Smoothing specification (A,N,N) with a smoothing parameter of 0.5809, whilst the ARIMA model selected the ARIMA (0,1,1) specification.
4. Despite parameter instability of $CV= 0.15$ and the presence of structural breaks in April 2024 currency redenomination, both the models maintained high forecast accuracy.
5. The inclusion of exogenous variables, exchange rate and inflation, do not improve forecast accuracy, most likely due to parameter instability of the series.

Evaluation of the Objectives

The answer to the main question is that between the two models Exponential Smoothing and ARIMA there is neither one model that outperforms the other since there is a MAPE difference of 0.02% that is too small for comparisons, hence I can say there is an equivalent superior predictive efficacy of the Terms of Trade. Therefore, the model should be chosen basing on

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specific needs, ETS to be chosen for its simplicity and ARIMA for its traditional statistical framework.

Objective 1:

The objective was fully achieved as the results confirm that the original series was non-stationary and then achieved after the first differencing with ADF $p < 0.01$. The temporal characteristics are not explicitly mentioned from the results but they provide a moderate volatility measure of $cv=14.4$, a skewness of 0.31 which is near-symmetric. The ACF slow decay identifies a trend component.

Objective 2:

The results successfully identified the optimal ARIMA (0,1,1) and it met the key technical requirements and achieved a highly effective forecasting performance of 91% forecast accuracy. Stationarity specified the first differencing, the moving average coefficient of -0.4188 and it maintains a high accuracy.

Objective 3:

The results state the automatic algorithm selected optimal Exponential Smoothing specification (A,N,N), Simple Exponential Smoothing with additive errors, no trend, no seasonality and a smoothing parameter $\alpha = 0.5409$. The standard statistical measures results, MAPE OF 8.92%, RMSE of 12.62 and a MAE of 9.66 shows a high effective performance.

Objective 4

The comparison of the forecast accuracy measures shows that both the models performed almost the same. There are very small differences between the two, that the models can be identified as statistically equivalent. They all have a MAPE below 10%, therefore they fall within the highly accurate classification according to Lewis (1982). Between ARIMA and ETS there is neither model that outperforms the other.

Objective 5:

Both the models are more appropriate for forecasting Zimbabwe's Terms of Trade based on the empirical evidence. Both methodologies are therefore suitable for practical forecasting applications, and the choice between them may be based on user preference, computational resources, or interpretability requirements rather than on forecast accuracy differentials.

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5.2 Limitations of the study

Sample size: The study has only 60 monthly observations, while sufficient for the modes employed, a longer time series would provide more robust results.

Structural breaks: the currency redenomination in April 2024 represents a significant structural breaks. While both models handed it adequately, regime specific models might yield even better results.

Exogenous variables: the study does no explore all possible predictors of Terms of Trade. Other variables such as global commodity prices might improve forecast accuracy.

Forecast horizon: the analysis focused on 12-month ahead forecast. The relative performance of the models may differ for shorter or longer horizons.

5.3 Recommendations

5.3.1 Recommendations for Policy

1. Adopt either ARIMA or ETS when forecasting Terms of Trade as they both produce high accurate results
2. Update the forecasts on monthly basis
3. Using confidence intervals for decision making because the wide confidence level for longer horizons shows substantial uncertainty.
4. Combine forecasting approaches, results in different forecasts but a simple average of the two may give a robust forecast than using a single model.

5.3.2 Recommendation for Future Research

1. Future researchers should extend the sample period as more post-redenomination data becomes available and should re-estimate the models to examine if the performance will remain unchanged.
2. The future studies should include exogenous variables to examine if the multivariate models will improve forecast accuracy.
3. There should be a comparison with the machine learning methods such as Random forest, XGBoost or neural networks.
4. The comparative framework developed in the current study should be applied using other macroeconomic indicators.

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5.4 Areas for further research

The upcoming researchers should take into consideration an extension on the time series as more post-redenomination data becomes available. Since the current study stretched focus on the analysis of the Terms of Trade, the future researchers focusing on the other macroeconomic indicators like GDP, inflation and exchange rates would help to establish if the equivalence of ARIMA and ETS can extend to the Zimbabwean economic time series. Examining of other modelling techniques like the multivariate methods and machine learning techniques may have an additional comparative insights.

5.5 Conclusion

The study assessed the efficiency of forecasting approaches ARIMA and Exponential Smoothing models for the Zimbabwe's Terms Trade using the monthly data from January 2020 to December 2024. The empirical evidence demonstrates that both models achieved almost the same accurate forecast results, as the MAPE difference was negligible. The study had a significant contribution to the organizations like RBZ and Ministry of Finance, responsible forecasting economic data and policy formulation as it provides evidence that the Terms of Trade can be predicted using the two methods in their own specified conditions. ETS is recommended for its simplicity whereas ARIMA is recommended its traditional framework. It also contributes to the literature on forecasting in developing economy context, showing that comparing analysis is feasible under the conditions of data volatility and structural instability. The researcher have a hope that the developed framework will be helpful informing for policy practice and future research on economic forecasting in Zimbabwe and other Sub-Saharan African countries.

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APPENDICES

APPENDIX A: DATA CLEANING AND PREPARATORY

DATA IMPORT AND INITIAL CLEANING

Load required libraries

```
library(tidyverse)
```

```
library(lubridate)
```

```
library(forecast)
```

```
library(tseries)
```

```
library(ggplot2)
```

Read the data

```
data <- read.csv("data.csv", stringsAsFactors = FALSE)
```

View the data structure

```
str(data)
```

```
head(data)
```

```
View(data)
```

```
clean_simple <- function(data) {
```

```
  # Parse dates (convert "Jan-20" format to Date)
```

```
  data$Date <- as.Date(paste0("01-", data$Date), format = "%d-%b-%y")
```

```
  data <- data[order(data$Date), ]
```

```
  # Rename columns for easier handling
```

```
  names(data)[2:4] <- c("Exchange_Rate", "Inflation_CPI", "Terms_of_Trade")
```

```
  # Normalization function
```

```
  normalize <- function(x) (x - min(x)) / (max(x) - min(x))
```

```
  # Create independent variables (NO LAGS - preserves all 60 observations)
```

```
  result <- data.frame(
```

```
    Date = data$Date,
```

```
    Terms_of_Trade = data$Terms_of_Trade,
```

```
    Economic_Pressure = normalize(data$Exchange_Rate) * 0.6 +
```

```
    normalize(abs(data$Inflation_CPI)) * 0.4,
```

```
    Exchange_Rate_Log = log(data$Exchange_Rate + 1),
```

```
    Inflation_Abs = abs(data$Inflation_CPI)
```

```
  )
```

```
  # Add time index and date components
```

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```
result$Time_Index <- 1:nrow(result)
result$Year <- year(result$Date)
result$Month <- month(result$Date)
cat("Cleaned data has", nrow(result), "observations from",
    min(result$Date), "to", max(result$Date), "\n")
return(result)
}
# Apply cleaning function
data_clean <- clean_simple(data)
# Remove unnecessary columns (6, 7, 8 - Time_Index, Year, Month)
data_clean <- data_clean[, -c(6, 7, 8)]
# Rename final dataframe
df <- data_clean
# View final cleaned data
head(df)
names(df)
summary(df)
```

APPENDIX B: DESCRIPTIVE AND APPENDIX TESTS

```
# DESCRIPTIVE STATISTICS
# Function to compute comprehensive descriptive statistics
descriptive_stats <- function(data, variable_name) {
  x <- na.omit(data)
  n <- length(x)
  mean_val <- mean(x)
  sd_val <- sd(x)
  var_val <- var(x)
  se_val <- sd_val / sqrt(n)
  min_val <- min(x)
  max_val <- max(x)
  range_val <- max_val - min_val
  q1 <- quantile(x, 0.25)
  median_val <- median(x)
  q3 <- quantile(x, 0.75)
```

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```
iqr_val <- q3 - q1
# Skewness and Kurtosis (using manual calculation)
skewness <- sum(((x - mean_val) / sd_val)^3) / n
kurtosis <- sum(((x - mean_val) / sd_val)^4) / n - 3
cv_val <- (sd_val / mean_val) * 100
ci_lower <- mean_val - 1.96 * se_val
ci_upper <- mean_val + 1.96 * se_val
results <- data.frame(
  Statistic = c("Observations (N)", "Mean", "Median", "Standard Deviation",
    "Variance", "Standard Error", "Minimum", "Maximum", "Range",
    "Q1 (25th Percentile)", "Q3 (75th Percentile)", "IQR",
    "Skewness", "Kurtosis", "CV (%)", "95% CI Lower", "95% CI Upper"),
  Value = c(n, round(mean_val, 2), round(median_val, 2), round(sd_val, 2),
    round(var_val, 2), round(se_val, 2), round(min_val, 2), round(max_val, 2),
    round(range_val, 2), round(q1, 2), round(q3, 2), round(iqr_val, 2),
    round(skewness, 3), round(kurtosis, 3), round(cv_val, 2),
    round(ci_lower, 2), round(ci_upper, 2))
)
return(results)
}
# Calculate statistics for each variable
tot_stats <- descriptive_stats(df$Terms_of_Trade, "Terms of Trade")
pressure_stats <- descriptive_stats(df$Economic_Pressure, "Economic Pressure")
exrate_stats <- descriptive_stats(df$Exchange_Rate_Log, "Exchange Rate Log")
inflation_stats <- descriptive_stats(df$Inflation_Abs, "Inflation Absolute")
# Print results
cat("\n=== DESCRIPTIVE STATISTICS ===\n")
print(tot_stats)
print(pressure_stats)
print(exrate_stats)
print(inflation_stats)

# Save to CSV
```

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```
write.csv(tot_stats, "04_Results/descriptive_stats_TOT.csv", row.names = FALSE)
write.csv(pressure_stats, "04_Results/descriptive_stats_EconomicPressure.csv", row.names = FALSE)
write.csv(exrate_stats, "04_Results/descriptive_stats_ExchangeRate.csv", row.names = FALSE)
write.csv(inflation_stats, "04_Results/descriptive_stats_Inflation.csv", row.names = FALSE)
# Create correlation matrix
cor_data <- df[, c("Terms_of_Trade", "Economic_Pressure", "Exchange_Rate_Log", "Inflation_Abs")]
cor_matrix <- round(cor(cor_data, use = "complete.obs"), 4)
write.csv(cor_matrix, "04_Results/correlation_matrix.csv", row.names = TRUE)
# Yearly summary
df$Year <- year(df$Date)
yearly_avg <- aggregate(df$Terms_of_Trade, by = list(Year = df$Year), FUN = mean)
colnames(yearly_avg) <- c("Year", "Average Terms of Trade")
write.csv(yearly_avg, "04_Results/yearly_summary.csv", row.names = FALSE)
cat("✓ Descriptive statistics saved\n")
# PART 2: DIAGNOSTIC TESTS
# Get residuals from ETS model
residuals_ets <- residuals(fit_ets)
# 1. Ljung-Box Test (Autocorrelation)
lb_results <- data.frame(Lag = c(6, 12, 18, 24), X_Squared = NA, P_Value = NA, Result = NA)
for(i in 1:4) {
  lag_val <- c(6, 12, 18, 24)[i]
  lb_test <- Box.test(residuals_ets, lag = lag_val, type = "Ljung-Box")
  lb_results$X_Squared[i] <- round(lb_test$statistic, 4)
  lb_results$P_Value[i] <- round(lb_test$p.value, 4)
  lb_results$Result[i] <- ifelse(lb_test$p.value > 0.05, "No Autocorrelation ✓", "Autocorrelation Present ✗")
}
cat("\n=== LJUNG-BOX AUTOCORRELATION TEST ===\n")
```

A COMPARATIVE ANALYSIS OF ARIMA AND EXPONENTIAL SMOOTHING MODELS IN FORECASTING ZIMBABWE'S TERMS OF TRADE.

```
print(lb_results)
# 2. Shapiro-Wilk Test (Normality)
shapiro_test <- shapiro.test(residuals_ets)
cat("\n=== SHAPIRO-WILK NORMALITY TEST ===\n")
cat("W-statistic:", round(shapiro_test$statistic, 4), "\n")
cat("P-value:", round(shapiro_test$p.value, 4), "\n")
cat("Conclusion:", ifelse(shapiro_test$p.value > 0.05, "Normal ✓", "Non-Normal X"), "\n")
# 3. Breusch-Pagan Test (Heteroskedasticity)
lm_model <- lm(residuals_ets^2 ~ seq_along(residuals_ets))
bp_test <- bptest(lm_model)
cat("\n=== BREUSCH-PAGAN HETEROSKEDASTICITY TEST ===\n")
cat("BP-statistic:", round(bp_test$statistic, 4), "\n")
cat("P-value:", round(bp_test$p.value, 4), "\n")
cat("Conclusion:", ifelse(bp_test$p.value > 0.05, "Constant Variance ✓", "Heteroskedastic X"), "\n")
diagnostic_summary <- data.frame(
  Test = c("Ljung-Box (Lag 12)", "Shapiro-Wilk", "Breusch-Pagan"),
  Statistic = c(round(lb_results$X_Squared[2], 4), round(shapiro_test$statistic, 4),
round(bp_test$statistic, 4)),
  P_Value = c(round(lb_results$P_Value[2], 4), round(shapiro_test$p.value, 4),
round(bp_test$p.value, 4)),
  Conclusion = c(lb_results$Result[2], ifelse(shapiro_test$p.value > 0.05, "Normal", "Non-
Normal"),
ifelse(bp_test$p.value > 0.05, "Constant Variance", "Heteroskedastic"))
```

APPENDIX C: SPLITTING AND TRAINING

```
# DATA SPLITTING AND MODEL TRAINING
# ARIMA, ETS, and RANDOM FOREST for Zimbabwe Terms of Trade
# Single Complete Code with Auto-Save
# Load required libraries
library(forecast)
library(tseries)
```

A COMPARATIVE ANALYSIS OF ARIMA AND EXPONENTIAL SMOOTHING MODELS IN FORECASTING ZIMBABWE'S TERMS OF TRADE.

```
library(ggplot2)
library(e1071)
# Create subdirectories if they don't exist
dir.create("02_Models", showWarnings = FALSE)
dir.create("04_Results", showWarnings = FALSE)
dir.create("03_Plots", showWarnings = FALSE)
dir.create("05_Forecasts", showWarnings = FALSE)
# DATA SPLIT INFORMATION
cat("\n", paste(rep("=", 80), collapse = ""), "\n")
cat("DATA SPLITTING\n")
cat(paste(rep("=", 80), collapse = ""), "\n")
# Create time series object
tot_ts <- ts(df$Terms_of_Trade, start = c(2020, 1), frequency = 12)
# Create independent variables matrix
xreg_vars <- as.matrix(df[, c("Economic_Pressure", "Exchange_Rate_Log",
"Inflation_Abs")])
# Total observations
n <- length(tot_ts)
cat("\nTotal observations:", n, "months (Jan 2020 - Dec 2024)\n")
# SPLIT RATIO: 80% TRAINING, 20% TESTING
train_ratio <- 0.80
train_size <- floor(train_ratio * n) # 48 months
test_size <- n - train_size # 12 months
cat("\n SPLIT RATIO USED: 80/20\n")
cat(" • Training set: 80% of data (", train_size, "months)\n")
cat(" • Testing set: 20% of data (", test_size, "months)\n")
# Time periods
train_end <- c(2023, 12)
test_start <- c(2024, 1)
cat("\n TIME PERIODS:\n")
cat(" • Training: Jan 2020 - Dec 2023 (", train_size, "months)\n")
cat(" • Testing: Jan 2024 - Dec 2024 (", test_size, "months)\n")
```

A COMPARATIVE ANALYSIS OF ARIMA AND EXPONENTIAL SMOOTHING MODELS IN FORECASTING ZIMBABWE'S TERMS OF TRADE.

```
# Create training and testing sets
train_ts <- window(tot_ts, end = train_end)
test_ts <- window(tot_ts, start = test_start)

# Split xreg variables
train_xreg <- xreg_vars[1:train_size, ]
test_xreg <- xreg_vars[(train_size + 1):n, ]

# MODEL 1: ETS (EXPONENTIAL SMOOTHING)
cat("\n", paste(rep("=", 80), collapse = ""), "\n")
cat("TRAINING ETS MODEL\n")
cat(paste(rep("=", 80), collapse = ""), "\n")

# Train ETS on training data
fit_ets <- ets(train_ts)

cat("\n✔ ETS model trained on training data\n")
print(summary(fit_ets))

# Forecast on test set
ets_forecast <- forecast(fit_ets, h = test_size)
ets_forecast_values <- as.numeric(ets_forecast$mean)

# Calculate accuracy metrics on test set
test_actual <- as.numeric(test_ts)
ets_rmse <- sqrt(mean((ets_forecast_values - test_actual)^2))
ets_mae <- mean(abs(ets_forecast_values - test_actual))
ets_mape <- mean(abs((ets_forecast_values - test_actual) / test_actual)) * 100

cat("\n📊 ETS Test Set Performance (2024):\n")
cat("• RMSE:", round(ets_rmse, 2), "\n")
cat("• MAE:", round(ets_mae, 2), "\n")
cat("• MAPE:", round(ets_mape, 2), "%\n")

# Train final ETS on full data
final_ets <- ets(tot_ts)

cat("\n✔ Final ETS model trained on full data (2020-2024)\n")
print(summary(final_ets))

# Generate 2025 forecast
ets_forecast_2025 <- forecast(final_ets, h = 12)
```

A COMPARATIVE ANALYSIS OF ARIMA AND EXPONENTIAL SMOOTHING MODELS IN FORECASTING ZIMBABWE'S TERMS OF TRADE.

```
ets_forecast_table <- data.frame(
  Month = month.abb,
  Forecast = round(as.numeric(ets_forecast_2025$mean), 2),
  Lower_80 = round(as.numeric(ets_forecast_2025$lower[, 1]), 2),
  Upper_80 = round(as.numeric(ets_forecast_2025$upper[, 1]), 2),
  Lower_95 = round(as.numeric(ets_forecast_2025$lower[, 2]), 2),
  Upper_95 = round(as.numeric(ets_forecast_2025$upper[, 2]), 2)
)

# Save ETS model and results
saveRDS(final_ets, "02_Models/ETS_model.rds")
saveRDS(fit_ets, "02_Models/ETS_model_training.rds")
write.csv(ets_forecast_table, "05_Forecasts/ETS_2025_forecast.csv", row.names = FALSE)
cat("\n✔ ETS model saved\n")

# MODEL 2: ARIMA
cat("\n", paste(rep("=", 80), collapse = ""), "\n")
cat("TRAINING ARIMA MODEL\n")
cat(paste(rep("=", 80), collapse = ""), "\n")
# Auto ARIMA on training data (with d=1 based on ADF test)
set.seed(123)
arma_model_auto <- auto.arima(train_ts, stepwise = TRUE, approximation = TRUE, d = 1)
cat("\n✔ Auto ARIMA model trained on training data\n")
print(summary(arma_model_auto))
# ARIMA(1,1,1) as alternative
arma_model_111 <- Arima(train_ts, order = c(1, 1, 1))
cat("\n✔ ARIMA(1,1,1) model trained on training data\n")
print(summary(arma_model_111))
# Select best model based on AIC
if(arma_model_auto$aic < arma_model_111$aic) {
  best_arma <- arma_model_auto
  best_arma_name <- "Auto ARIMA"
} else {
  best_arma <- arma_model_111
```

A COMPARATIVE ANALYSIS OF ARIMA AND EXPONENTIAL SMOOTHING MODELS IN FORECASTING ZIMBABWE'S TERMS OF TRADE.

```
best_arma_name <- "ARIMA(1,1,1)"
}
cat("\n✔ Best model:", best_arma_name, "(AIC =", round(best_arma$aic, 2), ")\n")
# Forecast on test set
arma_forecast <- forecast(best_arma, h = test_size)
arma_forecast_values <- as.numeric(arma_forecast$mean)

# Calculate accuracy metrics
arma_rmse <- sqrt(mean((arma_forecast_values - test_actual)^2))
arma_mae <- mean(abs(arma_forecast_values - test_actual))
arma_mape <- mean(abs((arma_forecast_values - test_actual) / test_actual)) * 100
cat("\n📊 ARIMA Test Set Performance (2024):\n")
cat("• Model:", best_arma_name, "\n")
cat("• RMSE:", round(arma_rmse, 2), "\n")
cat("• MAE:", round(arma_mae, 2), "\n")
cat("• MAPE:", round(arma_mape, 2), "%\n")
# Train final ARIMA on full data
if(best_arma_name == "Auto ARIMA") {
  final_arma <- auto.arma(tot_ts, stepwise = TRUE, approximation = TRUE, d = 1)
} else {
  final_arma <- Arima(tot_ts, order = c(1, 1, 1))
}
cat("\n✔ Final ARIMA model trained on full data (2020-2024)\n")
print(summary(final_arma))
# Generate 2025 forecast
arma_forecast_2025 <- forecast(final_arma, h = 12)
arma_forecast_table <- data.frame(
  Month = month.abb,
  Forecast = round(as.numeric(arma_forecast_2025$mean), 2),
  Lower_80 = round(as.numeric(arma_forecast_2025$lower[, 1]), 2),
  Upper_80 = round(as.numeric(arma_forecast_2025$upper[, 1]), 2),
  Lower_95 = round(as.numeric(arma_forecast_2025$lower[, 2]), 2),
  Upper_95 = round(as.numeric(arma_forecast_2025$upper[, 2]), 2),
```

A COMPARATIVE ANALYSIS OF ARIMA AND EXPONENTIAL SMOOTHING MODELS IN FORECASTING ZIMBABWE'S TERMS OF TRADE.

```
Upper_95 = round(as.numeric(arma_forecast_2025$upper[, 2]), 2)
)
saveRDS(best_arma, "02_Models/ARIMA_model_training.rds")
write.csv(arma_forecast_table, "05_Forecasts/ARIMA_2025_forecast.csv", row.names =
FALSE)
cat("MODEL COMPARISON\n")
cat(paste(rep("=", 80), collapse = ""), "\n")
# Create comparison table
comparison <- data.frame(
  Model = c("ETS", best_arma_name, "Random Forest"),
  MAPE = c(ets_mape, arma_mape, rf_mape),
  RMSE = c(ets_rmse, arma_rmse, rf_rmse),
  MAE = c(ets_mae, arma_mae, rf_mae)
)
comparison <- comparison[order(comparison$MAPE), ]
cat("\n📊 Model Performance Comparison (Test Set 2024):\n")
print(round(comparison, 2))
# Calculate improvements
ets_vs_arma <- round(arma_mape - ets_mape, 2)
ets_vs_rf <- round(rf_mape - ets_mape, 2)
cat("\n🔑 Key Insights:\n")
if(ets_mape < arma_mape) {
  cat("• ETS beats ARIMA by", abs(ets_vs_arma), "percentage points\n")
} else if(arma_mape < ets_mape) {
  cat("• ARIMA beats ETS by", abs(ets_vs_arma), "percentage points\n")
} else {
  cat("• ETS and ARIMA have equivalent accuracy\n")
}
cat("• ETS beats Random Forest by", ets_vs_rf, "percentage points\n")
# FINAL MODEL RANKING
final_ranking <- data.frame(
  Rank = 1:nrow(comparison),
```

A COMPARATIVE ANALYSIS OF ARIMA AND EXPONENTIAL SMOOTHING MODELS IN FORECASTING ZIMBABWE'S TERMS OF TRADE.

```
Model = comparison$Model,
MAPE = round(comparison$MAPE, 2),
RMSE = round(comparison$RMSE, 2),
MAE = round(comparison$MAE, 2)
)
cat("\n", paste(rep("=", 80), collapse = ""), "\n")
cat("FINAL MODEL RANKING\n")
cat(paste(rep("=", 80), collapse = ""), "\n")
print(final_ranking)
# VISUALIZATION: FORECAST COMPARISON PLOT
pdf("03_Plots/forecast_comparison.pdf", width = 12, height = 8)
par(mfrow = c(2, 2), mar = c(4, 4, 4, 2))
# Plot 1: ETS Forecast vs Actual
plot(test_ts, main = paste("ETS Forecast vs Actual (2024)\nMAPE =", round(ets_mape, 2),
"%"),
      ylab = "Terms of Trade", xlab = "Month", col = "black", lwd = 2,
      ylim = range(c(test_ts, ets_forecast_values)),
      xaxt = "n")
lines(ets_forecast_values, col = "blue", lwd = 2, lty = 2)
axis(1, at = 1:12, labels = month.abb[1:12])
legend("topright", legend = c("Actual", "ETS Forecast"),
      col = c("black", "blue"), lty = c(1, 2), lwd = 2)
grid()
# Plot 2: ARIMA Forecast vs Actual
plot(test_ts, main = paste(best_arima_name, "Forecast vs Actual (2024)\nMAPE =",
round(arima_mape, 2), "%"),
      ylab = "Terms of Trade", xlab = "Month", col = "black", lwd = 2,
      ylim = range(c(test_ts, arima_forecast_values)),
      xaxt = "n")
lines(arima_forecast_values, col = "darkgreen", lwd = 2, lty = 2)
axis(1, at = 1:12, labels = month.abb[1:12])
legend("topright", legend = c("Actual", paste(best_arima_name, "Forecast")),
      col = c("black", "darkgreen"), lty = c(1, 2), lwd = 2)
```

A COMPARATIVE ANALYSIS OF ARIMA AND EXPONENTIAL SMOOTHING MODELS IN FORECASTING ZIMBABWE'S TERMS OF TRADE.

```
grid()
# Plot 3: Random Forest Forecast vs Actual
plot(test_ts, main = paste("Random Forest Forecast vs Actual (2024)\nMAPE =",
round(rf_mape, 2), "%"),
      ylab = "Terms of Trade", xlab = "Month", col = "black", lwd = 2,
      ylim = range(c(test_ts, rf_test_forecast)),
      xaxt = "n")
lines(rf_test_forecast, col = "red", lwd = 2, lty = 2)
axis(1, at = 1:12, labels = month.abb[1:12])
legend("topright", legend = c("Actual", "Random Forest Forecast"),
      col = c("black", "red"), lty = c(1, 2), lwd = 2)
grid()
# Plot 4: All Models Comparison
plot(test_ts, main = "All Models Forecast Comparison (2024)",
      ylab = "Terms of Trade", xlab = "Month", col = "black", lwd = 2,
      ylim = range(c(test_ts, ets_forecast_values, arima_forecast_values, rf_test_forecast)),
      xaxt = "n")
lines(ets_forecast_values, col = "blue", lwd = 1.5, lty = 2)
lines(arima_forecast_values, col = "darkgreen", lwd = 1.5, lty = 3)
lines(rf_test_forecast, col = "red", lwd = 1.5, lty = 4)
axis(1, at = 1:12, labels = month.abb[1:12])
legend("topright", legend = c("Actual", "ETS", best_arima_name, "Random Forest"),
      col = c("black", "blue", "darkgreen", "red"),
      lty = c(1, 2, 3, 4), lwd = 2, cex = 0.8)
grid()
dev.off()
# CREATE TRAINING SUMMARY REPORT
sink("04_Results/training_summary_report.txt")
cat("=", rep("=", 70), "\n")
cat("MODEL TRAINING SUMMARY REPORT\n")
cat("Zimbabwe Terms of Trade Forecasting Study\n")
cat("Generated on:", as.character(Sys.time()), "\n")
cat("=", rep("=", 70), "\n\n")
```

A COMPARATIVE ANALYSIS OF ARIMA AND EXPONENTIAL SMOOTHING MODELS IN FORECASTING ZIMBABWE'S TERMS OF TRADE.

```
cat("DATA SPLIT:\n")
cat(" Total observations:", n, "months\n")
cat(" Training set:", train_size, "months (80% - Jan 2020 to Dec 2023)\n")
cat(" Testing set:", test_size, "months (20% - Jan 2024 to Dec 2024)\n")
cat(" Split ratio: 80/20\n\n")
cat("MODELS TRAINED:\n")
cat(" 1. ETS (Exponential Smoothing)\n")
cat(" 2. ARIMA (Auto ARIMA / ARIMA(1,1,1))\n")
cat("TEST SET PERFORMANCE (2024):\n")
for(i in 1:nrow(comparison)) {
  cat(" ", comparison$Model[i], ":\n")
  cat("  MAPE =", round(comparison$MAPE[i], 2), "%\n")
  cat("  RMSE =", round(comparison$RMSE[i], 2), "\n")
  cat("  MAE =", round(comparison$MAE[i], 2), "\n")
}
cat("\nBEST MODEL:", comparison$Model[1], "\n")
cat("  MAPE:", round(comparison$MAPE[1], 2), "%\n")
sink()
# FINAL SUMMARY
cat("\n", paste(rep("=", 80), collapse = ""), "\n")
cat("DATA SPLITTING AND MODEL TRAINING COMPLETE\n")
cat(paste(rep("=", 80), collapse = ""), "\n")
cat("\n❏ SPLIT RATIO SUMMARY:\n")
cat(" • Ratio used: 80/20 (Training/Testing)\n")
cat(" • Training: 48 months (Jan 2020 - Dec 2023)\n")
cat(" • Testing: 12 months (Jan 2024 - Dec 2024)\n")
cat("\n❏ MODELS TRAINED:\n")
cat(" • ETS (Exponential Smoothing)\n")
cat(" • ARIMA (Auto ARIMA / ARIMA(1,1,1))\n")
```

APPENDIX D: VALIDATION AND POST-DIAGNOSTIC RESIDUALS

VALIDATION AND POST-DIAGNOSTIC TESTS

A COMPARATIVE ANALYSIS OF ARIMA AND EXPONENTIAL SMOOTHING MODELS IN FORECASTING ZIMBABWE'S TERMS OF TRADE.

```
# For ARIMA, ETS, and Random Forest Models
# Single Complete Code with Auto-Save
# Load required libraries
library(forecast)
library(tseries)
library(ggplot2)
library(lmtest)
library(e1071)
library(urca)
# SET WORKING DIRECTORY
# LOAD TRAINED MODELS (if needed) OR RETRAIN
cat("\n", paste(rep("=", 80), collapse = ""), "\n")
cat("VALIDATION AND POST-DIAGNOSTIC TESTS\n")
cat(paste(rep("=", 80), collapse = ""), "\n")
# Create time series object
tot_ts <- ts(df$Terms_of_Trade, start = c(2020, 1), frequency = 12)
# Split data for validation (80/20 split)
n <- length(tot_ts)
train_size <- floor(0.8 * n) # 48 months
test_size <- n - train_size # 12 months
train_ts <- window(tot_ts, end = c(2023, 12))
test_ts <- window(tot_ts, start = c(2024, 1))
# Retrain models if not loaded, or use existing
if(exists("final_ets")) {
  cat("\n✔ Using existing ETS model\n")
  fit_ets_train <- ets(train_ts)
} else {
  fit_ets_train <- ets(train_ts)
  cat("\n✔ ETS model trained on training data\n")
}
if(exists("final_arima")) {
  cat("\n✔ Using existing ARIMA model\n")
  fit_arima_train <- auto.arima(train_ts, stepwise = TRUE, approximation = TRUE, d = 1)
```

A COMPARATIVE ANALYSIS OF ARIMA AND EXPONENTIAL SMOOTHING MODELS IN FORECASTING ZIMBABWE'S TERMS OF TRADE.

```
} else {  
  fit_arma_train <- auto.arma(train_ts, stepwise = TRUE, approximation = TRUE, d = 1)  
  cat("✓ ARIMA model trained on training data\n")  
# Get residuals  
residuals_ets <- residuals(fit_ets_train)  
residuals_arma <- residuals(fit_arma_train)  
# VALIDATION TEST 1: RESIDUAL DIAGNOSTICS (ACF & PACF)  
cat("\n", paste(rep("=", 80), collapse = ""), "\n")  
cat("VALIDATION TEST 1: RESIDUAL AUTOCORRELATION\n")  
cat(paste(rep("=", 80), collapse = ""), "\n")  
# Ljung-Box Test for ETS residuals at multiple lags  
lb_results_ets <- data.frame(  
  Lag = c(6, 12, 18, 24),  
  X_Squared = numeric(4),  
  P_Value = numeric(4),  
  Result = character(4)  
)  
for(i in 1:4) {  
  lag_val <- c(6, 12, 18, 24)[i]  
  lb_test <- Box.test(residuals_ets, lag = lag_val, type = "Ljung-Box")  
  lb_results_ets$X_Squared[i] <- round(lb_test$statistic, 4)  
  lb_results_ets$P_Value[i] <- round(lb_test$p.value, 4)  
  lb_results_ets$Result[i] <- ifelse(lb_test$p.value > 0.05, "No Autocorrelation ✓",  
"Autocorrelation Present ✗")  
}  
cat("\n\n ETS Model - Ljung-Box Test Results:\n")  
print(lb_results_ets)  
# Ljung-Box Test for ARIMA residuals  
lb_results_arma <- data.frame(  
  Lag = c(6, 12, 18, 24),  
  X_Squared = numeric(4),  
  P_Value = numeric(4),
```

A COMPARATIVE ANALYSIS OF ARIMA AND EXPONENTIAL SMOOTHING MODELS IN FORECASTING ZIMBABWE'S TERMS OF TRADE.

```
Result = character(4)
)
for(i in 1:4) {
  lag_val <- c(6, 12, 18, 24)[i]
  lb_test <- Box.test(residuals_arma, lag = lag_val, type = "Ljung-Box")
  lb_results_arma$X_Squared[i] <- round(lb_test$statistic, 4)
  lb_results_arma$P_Value[i] <- round(lb_test$p.value, 4)
  lb_results_arma$Result[i] <- ifelse(lb_test$p.value > 0.05, "No Autocorrelation ✓",
  "Autocorrelation Present ✗")
}
cat("\n\nARIMA Model - Ljung-Box Test Results:\n")
print(lb_results_arma)
# VALIDATION TEST 2: NORMALITY OF RESIDUALS
cat("\n", paste(rep("=", 80), collapse = ""), "\n")
cat("VALIDATION TEST 2: NORMALITY OF RESIDUALS\n")
cat(paste(rep("=", 80), collapse = ""), "\n")
# Shapiro-Wilk test for ETS residuals
shapiro_ets <- shapiro.test(residuals_ets)
# Shapiro-Wilk test for ARIMA residuals
shapiro_arma <- shapiro.test(residuals_arma)
# Additional normality metrics for ETS
skewness_ets <- skewness(residuals_ets)
kurtosis_ets <- kurtosis(residuals_ets)
# Additional normality metrics for ARIMA
skewness_arma <- skewness(residuals_arma)
kurtosis_
```