BINDURA UNIVERSITY OF SCIENCE EDUCATION

Integrating Argumentation in Teaching Probability: Exploring the Pedagogic Impact on Culturally Diverse Secondary School Learners in Bulawayo, Zimbabwe

BY

JOSHUA CHITERA
B1646699

A THESIS SUBMITTED IN FULFILMENT OF THE REQUIREMENTS OF THE BINDURA UNIVERSITY OF SCIENCE EDUCATION FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN CURRICULUM STUDIES, FACULTY OF SCIENCE EDUCATION

2026

Abstract

The integration of argumentation into mathematics education is increasingly perceived as a crucial pedagogical strategy that enhances learners' engagement and understanding of challenging concepts such as probability. Probability helps learners cope with real scenarios that entail risk and uncertainty. Although argumentation is globally acknowledged, empirical studies evaluating its efficacy within culturally diverse contexts such as among secondary school learners in Bulawayo, remain limited. The lack of such studies calls for further research into how effectively argumentation can be incorporated into the teaching of probability in order to enhance learner achievements. Hence, this study sought to explore the effectiveness of incorporating argumentation into the teaching of probability to the culturally diverse secondary school learners in Bulawayo Central district, focusing on its impact on learners' academic performances, attitudes, motivation, and critical thinking. The use of the pragmatic paradigm provided the philosophical flexibility to integrate both quantitative measures and qualitative learner experiences thereby ensuring a practically relevant evaluation of the argumentation-based intervention. Because there were only two secondary school mathematics classes at the selected school, a census approach was adopted, encompassing both intact classes ($N = 40$, with 20 learners each). Cluster randomisation was then applied to designate one intact class as the experimental group, which received argumentation-based instruction, while the counterpart class served as the control group, taught through traditional methods. Quantitative data were collected through pre-tests and post-tests while qualitative data were collected through focus group discussions, and semi-structured questionnaires. The quantitative data were analysed using Mann Whitney U test, and the Difference in Difference (DID) analysis. The pre-test results yielded no statistically significant difference ($U = 197.00$, $p = 0.925$, $p > 0.05$; with a Cohen's d of 0.015) in performance between the experimental and control groups. However, the post-test results ($U = 28.000$, $p = 0.015$, $p < 0.05$; with Cohen's $d = 0.855$) indicated a very large and significant improvement in learners' performances. The Difference in Difference (DID) analysis further highlighted an average improvement of 11.05 points in academic performance for the experimental group, demonstrating the effectiveness of the argumentation-based instruction. The thematically analysed qualitative data revealed that the argumentation intervention significantly improved the learners' attitudes, motivation, engagement, enthusiasm, and a deeper understanding of probability concepts among the experimental group learners. Insights from the qualitative results support the quantitative success as shown by increased confidence and motivation among the learners. Learners reported that a shift in their attitudes towards probability and their participation in argumentation activities improved their conceptual understanding of probability. This combination of the quantitative and qualitative data paints a comprehensive picture of the intervention's effects, highlighting that when learners feel more engaged and confident, their performance reflects the change. The findings suggest that integrating argumentation in teaching probability is an effective instructional strategy that improves the performance of culturally diverse secondary school learners. This study, therefore, recommends the adoption of argumentation as a fundamental approach in mathematics and statistics education, emphasising its potential to foster critical thinking and deeper conceptual understanding among learners. This can be achieved through leveraging the learners' cultural backgrounds as a sense-making resource.

Keywords: Integrating instructional methodology, argumentation, probability teaching, culturally diverse secondary school learners, cultural diversity, mathematics teaching

Declaration

I, Joshua Chitera, hereby declare that this thesis: **Integrating Argumentation in Teaching Probability: Exploring the Pedagogic Impact on Culturally Diverse Secondary School Learners in Bulawayo, Zimbabwe,** is my original work, except where explicitly indicated through citations. The Enhanced Argumentation Probability Instructional (E-API) Model presented in Chapter 5 is a conceptual product of my independent research, developed iteratively from the theoretical synthesis in Chapter 2 and the empirical findings of the intervention described in Chapter 4. Independent corroboration of the E-API model's developmental trajectory is provided in the study drafts archives shown in Annexure O. This model was published through Zenodo in 2024 and can be accessed through: https://doi.org/10.5281/zenodo.19874036. Furthermore, I acknowledge the scholarly guidance of my supervisors, Dr. D. Dziva and Dr. Z. Ndemo regarding two publications: **Investigating the impact of argumentation on high school students' achievement in probability** and **Cultivating reasoning: Developing socio-mathematical norms for argumentation in form one algebra lessons in Mzilikazi District, Zimbabwe**, included as Annexure T, I acknowledge the erroneous omission of Dr. Z. Ndemo as a co-author of the second paper.

Signed

Date: 04 August 2026

BINDURA UNIVERSITY OF SCIENCE EDUCATION

RELEASE FORM

NAME OF STUDENT: JOSHUA CHITERA

DISSERTATION TITLE: Integrating Argumentation in Teaching Probability: Exploring the Pedagogic Impact on Culturally Diverse Secondary School Learners in Bulawayo, Zimbabwe

DEGREE TITLE: Doctor of Philosophy in Curriculum Studies

YEAR GRANTED: 2026

PERMANENT ADDRESS: 13813 Cowdray Park, Bulawayo

SIGNED:

DATE: 04 August 2026

Acknowledgements

I would like to express my heartfelt gratitude to my supervisors, Dr. Daimond Dziva and Dr. Zakaria Ndemo, for their invaluable guidance and support throughout my research journey. Their expertise, encouragement, and constructive feedback were instrumental in shaping this thesis.

I also extend my appreciation to the members of the University community and the Department of Curriculum Studies and Educational Management. Their constructive feedback and insights greatly enriched my academic experience. Furthermore, I am deeply thankful to my friends and family, whose unwavering support and encouragement provided me with the strength and motivation to persevere. Special thanks go to Dr. C. Makuvire for proofreading this document.

Dedication

In memory of my mother, Emmah Kumo (1942-2020), to whom I dedicate this thesis.

Research is to see what everybody else has seen and to think what nobody else thought

(Albert Szent-Gyorgyi)

Table of Contents

List of Tables

List of Figures

List of Annexures

List of Acronyms and Abbreviations

ABSL	Argumentation-Based Science Learning
ADI	Argument Driven Inquiry
AEI	Assertion-Evidence-Interpretation
CAT	Contiguity Argumentation Theory
CERCA	Claim, Evidence, Reasoning, Counterargument, and Audience
DID	Difference in Difference
DSI	District Schools' Inspector
E-API	Enhanced Argumentation-based Probability Instruction
E-Toulmin + CP	Enhanced Toulmin's model that incorporates Cultural Context, and Probability-Specific considerations
FGD	Focus Group Discussion
H0	Null Hypothesis
H1	Alternative Hypothesis
IQR	Interquartile Range
JD	Jamaican Dialects
MDT	Mathematics Diagnostic Test
MoPSE	Ministry of Primary and Secondary Education
PED	Provincial Education Director
SE	Standard English
SPSS	Statistical Package for the Social Sciences
TAP	Toulmin's Argumentation Pattern
ZIMSEC	Zimbabwe School Examination Council

Approval Form

The undersigned certify that they have supervised, have read and recommend to the University for acceptance and examination a research project entitled:

Integrating Argumentation in Teaching Probability: Exploring the Pedagogic Impact on Culturally Diverse Secondary School Learners in Bulawayo, Zimbabwe, submitted by **JOSHUA CHITERA** in partial fulfilment of the requirements for the award of the degree of **Doctor of Philosophy**.

Supervisor: Dr Daimond Dziva **Signature**: **Date**: 04-08-2026

Co-Supervisor: Dr Zakaria Ndemo **Signature**: **Date**: 04-08-2026

.

Chairperson: Dr Young Mudavanhu **Signature**: **Date**: 04-08-2026

CHAPTER 1: THE PROBLEM AND ITS SETTING

1.0 Introduction

This chapter articulates the underlying motivation for this study while concurrently delineating contributions from numerous scholars pertaining to argumentation, cultural diversity, and probability. A thorough examination of the contextual framework for the research is undertaken, alongside the formulation of the research questions and objectives pertinent to the identified research problem. In addition, the chapter addresses the delimitations, and underlying assumptions guiding the study. The chapter highlights the gaps in the way argumentation is used in teaching probability in different cultures. Also, the chapter gives precise meanings of important words used, so that the terms are understood in the study context. The structure of the thesis is described, and a summary of the main discussions is given at the end of the chapter.

1.1 The background to the study

The integration of cultural perspectives into probability studies has become increasingly critical in the twenty-first century. When learners from diverse cultures are present in a classroom, teachers must deal with the different ways learners perceive and understand probability concepts. Because these cultural views are not considered, there is a gap between the way probability is taught at school and how learners understand it, which can hinder their learning experiences and academic performance (Dede, 2018).

This chapter examines how using argumentation helps improve learning probability in mathematics, especially by considering the influence of cultural background on learners in Bulawayo, Zimbabwe. In education, cultural diversity encompasses different cultures such as peer, school and mathematics cultures interacting, which greatly affect learners' learning experiences.

In their study, Nicol et al. (2013) explain that if a learner's culture does not match the school culture, it may create obstacles to effective education. The authors believe that promoting cross-cultural exchanges helps learners connect with different perspectives, which teachers can use to relate mathematical content to the learners' lived experiences. Often, when culturally diverse secondary school learners interact, it results in arguments about how to explain certain concepts (Aguirre & del Rosario, 2013).

The authors consider argumentation as a powerful practice in the teaching of probability for secondary school learners from diverse backgrounds in Bulawayo, suggesting that it promotes critical thinking (DeJarnette & González, 2017), logical reasoning and the resolution of cognitive conflicts. Argumentation, or the process of constructing and analysing arguments, is believed to assist learners understand probability better (Kuhn, 2011). By using this approach, learners can discuss their thoughts on probability and interact with each other which can result in teamwork and acceptance of various cultural viewpoints on probability (Aikenhead & Jegede, 1999).

Many scholars discuss argumentation by emphasising its role in dialogue and its teaching applications. According to Rapanta (2024), argument-based instruction is designed to support learners by involving them in discussions that develop their argumentative communication and critical thinking skills. This is supported by Iwuanyanwu (2022), who asserts that argumentation is core to science learning. The author posits that science learners should be given opportunities to develop their ability to think and act in ways that support argumentation. This means they should be able to find multiple solutions to problems, gather and evaluate evidence on multiple issues, and communicate scientific knowledge. This view agrees with Hacıeminoğlu and Yıldız (2022), who believe that argumentation in the classroom encourages learners to be involved and helps them develop their logical thinking skills.

Fasha (2024) emphasises that, in supporting argumentation in the classroom, teachers should use materials that meet different levels of argumentative ability. The author argues that this method is necessary for differentiated instruction. As a result, the current study is based on two theories that focus on how argumentation plays a role and how learners' knowledge is shaped by their cultural background.

According to Sociocultural Theory Vygotsky (1978), the process of learning is influenced by social interaction and culture, since knowledge is developed through communication and teamwork among learners. The constructivist perspective also plays a part, claiming that learners build knowledge by being active and social with others (Arends, 1998). Evidence shows that argumentation can support and improve how probability is taught in the classroom. According to Hughes (2021), using both types of argumentation instruction, embedded and explicit, gives learners better understanding than simply listening to lectures. Because of this, including argumentation in the teaching of probability could help secondary school learners grasp mathematical concepts better.

Moreover, using argumentation in education supports a setting where learners want to discuss their own thoughts and evaluate the arguments of their classmates. According to Ibraim and Justi (2022), good argumentation assists by making sure evidence backs up claims, which improves class discussions. Using argumentation in probability lessons helps learners manage uncertainty and make decisions supported by probabilistic reasoning (Meral et al., 2021). By introducing argumentation in probability teaching, Bulawayo teachers can strengthen critical thinking, reasoning and collaboration among their learners from different cultural backgrounds, as well as make discussion and different viewpoints important in the classroom.

The way learners understand probability is strongly affected by their cultural background, according to Sharma (2012). The study by Amir and Williams (1999) in England confirms this, demonstrating that beliefs, language and experience play a major role in learners' performance in probability. When taught in a way that suited their religious backgrounds, the Asian learners showed better understanding of probability than when taught through traditional methods. Such evidence supports the need to include cultural knowledge in the teaching of probability.

In addition, Chassapis and Chatzirasileiou (2008) discovered that cultural backgrounds play a major role in shaping how learners perceive random events. In their research, devout Muslim learners considered random events to be guided by God instead of chance, which points to the need for teachers to use instruction that values cultural beliefs to improve learners' perceptions about probability. According to Young (2010) and Sharma (2012), including cultural practices in teaching helps learners perform better.

In a study by Morris (2014), both language and cultural practices influence how Tongan learners understand probability, as language shapes their thinking about probability and different cultures impact their understanding of it. The study points out that teachers should recognise the language and cultural background of their learners when teaching topics in probability as these have impact in learning mathematics.

Besides, Ramírez-Uclés and Ruiz-Hidalgo (2022) point out that what learners learn about probability at school can contradict their cultural understanding, so educational programmes should try to link these two areas (Chassapis & Chatzirasileiou, 2008). In their work, Castro and Toro (2023) support activities that relate to learners' backgrounds, believing that this approach makes understanding probability concepts easier. In support, Khazanov and Prado (2010) acknowledge that cultural views of probability are sometimes different from what is taught at school, so teachers should consider these cultural views instead of neglecting them.

Can and Isleyen (2016) conducted an experiment in Turkey to determine how different cultures impacted learners' grasp of probability. Although the experimental group did better than the control group on overall learning, the study did not investigate how their new knowledge of probability was used in everyday situations or how it changed their opinion of alternative cultures. The study mainly explained findings based on quantitative data but did not explore the richer qualitative aspects that might provide additional understanding. Because of this, the current study uses both qualitative and quantitative techniques to fill these gaps. Even though there is a growing amount of research on cultural diversity in mathematics education, it is still not clear what teaching practices are effective in different cultures. One major disadvantage is that there is not enough research showing that teaching argumentation improves how learners understand probability. Even though Tristanti and Nusantara (2022) describe the benefits of argumentation in building critical thinking, little research has been done on this topic in culturally diverse classrooms. Because Bulawayo is a multicultural context, the way education is delivered should reflect this multiplicity (Mapuranga & Bukaliya, 2014).

Special approaches to teaching probability are needed since cultural diversity brings different types of challenges and opportunities. The purpose of this study is to address gaps found in research such as the need for teacher strategies that help culturally diverse secondary school learners understand probability. There is not enough use of argumentation in teaching probability, though research shows that it helps learners in multicultural contexts. By suggesting strategies for teaching probability in the classroom that are sensitive to learners' cultural backgrounds, this research seeks to improve the teaching and learning of probability and to support teachers working with culturally diverse groups of learners. While argumentation-based instruction has been explored in various educational settings and globally, this study is situated within the secondary school mathematics classrooms of Bulawayo, Zimbabwe.

1.2 Statement of the problem

Integrating argumentation into the teaching of probability presents an opportunity to assist culturally diverse secondary school learners in Bulawayo, Zimbabwe. The 2022 ZIMSEC Examiner Report highlights this application gap, noting that learners demonstrate procedural knowledge but fail to solve complex problems or engage in multi-stage probabilistic reasoning (ZIMSEC Examiner-report, 2022).

Current pedagogical approaches inadequately incorporate the cultural backgrounds of learners, including those from Ndebele, Shona, Kalanga, Batonga, and mixed descent communities (Mapuranga & Bukaliya, 2014). The predominant study method among these learners is rote memorisation characterised by a lack of culturally relevant examples, impeding effective learning. Consequently, learners experience cognitive dissonance when principles of probability theory conflict with cultural beliefs and interpretations (Cooper & Feldman, 2019). Specifically, the belief among many learners that divine intervention governs life events complicates their comprehension of probability, which is typically defined through empirical events rather than narratives. This challenge in understanding probability adversely affects their academic performance and interest in learning.

While there is substantial evidence that argumentation enhances critical thinking and reasoning, its application in probability education for diverse learners in Bulawayo remains underexplored. Addressing this issue aligns the study with global relevance and supports Sustainable Development Goal No. 4, which advocates for inclusive and equitable quality education for all. Chiwiye (2013) contends that the pedagogical strategies employed in Zimbabwean schools are responsible for learners' unsatisfactory results, as they fail to recognise learners as active participants in their educational journey. The argument supports the notion that learners are more inclined to engage with mathematics when they perceive its relevance in their lives, thereby boosting motivation and positively influencing understanding of probability concepts. Brown et al. (2019) posit that argumentation can form part of the teaching of probability to help eradicate the difficulties such as those faced by culturally diverse secondary school learners in Bulawayo when learning probability. Although argumentation has been recommended as an effective strategy in teaching, its implementation in teaching probability still needs more study, especially in culturally diverse contexts. Studies suggest that some teachers are not fully aware of how argumentation could be used in teaching, which may keep them from practising it effectively (Kuki et al., 2023). It is still unclear from the literature, despite the known positive role of argumentation in learning, how argumentation could be adjusted for use in diverse classrooms (Nussbaum, 2011). This is an important gap since being able to form and evaluate arguments helps learners improve their ability to think and reason (Hendratmoko, 2023). In addition, modern ways of teaching probability usually emphasise understanding probability more than they prioritise having learners argue their reasoning. Sampson and Blanchard (2012) admit that there is only limited information available about how well these models work in practice.

1.3 Main objective

This study investigated the pedagogical impact of incorporating structured argumentation on culturally diverse secondary school learners' academic performance in probability. Specifically, the research evaluated the effects of this instructional approach on conceptual understanding, critical thinking competencies, learner attitudes, and motivational engagement.

*1.3.1 **Research objectives***

To address the main objective, the following sub-objectives were formulated to:

a) Establish the understanding of probability concepts and argumentation of culturally diverse secondary school learners in Bulawayo prior to the intervention.
b) Assess the impact of the argumentation-based instructional approach on the development of probabilistic reasoning and justification by secondary school learners.
c) Describe changes in attitudes and perceptions of culturally diverse learners towards probability following the implementation of an argumentation-based teaching approach.
d) Assess the impact of argumentation-based probability instruction on critical thinking, collaboration, and active engagement among culturally diverse learners.

1.4 Main research question

How does the use of an argumentation-based teaching approach impact the teaching and learning of probability of culturally diverse secondary school learners?

*1.4.1 **Sub-research questions***

To address the main research question, the following sub-questions were explored:

a) How do the culturally diverse secondary school learners demonstrate their baseline conceptual understanding of probability and argumentation prior to the pedagogical intervention?
b) How does the implementation of an argumentation-based instructional approach impact the development of probabilistic reasoning and justification skills among secondary school learners?
c) How does the argumentation teaching approach impact learners' attitudes and motivation towards learning probability?
d) How much did the argumentation-based probability instruction promote critical thinking, collaboration, and active engagement among culturally diverse secondary school learners in learning probability?

The research questions and objectives aim to furnish empirical evidence pertaining to the efficacy of argumentation as a pedagogical instrument in the teaching of probability. This study seeks to foster a mathematics education and responsive to diverse cultural backgrounds of the learners. The resultant data could offer significant guidance to teachers as they address the complexities inherent in teaching probability, particularly to learners whose cultural experiences play a pivotal role in shaping their understanding of mathematical concepts.

To ensure methodological alignment, the first two research questions about the academic performance are treated as quantitative inquiries. They are addressed through hypothesis testing and inferential statistics to provide a measurable baseline and a post-intervention impact measurement. In contrast, the third and fourth research questions seek to establish the intervention impact on learners' motivation, attitude, critical thinking, collaboration, and active engagement. These two questions are exploratory. This phase of the study seeks the rich, descriptive data necessary to refine and validate the E-API model within a real-world classroom context.

1.5 Hypotheses

The primary hypotheses for the academic performance are:

1. **Null Hypothesis (*H*0):** There is no significant difference in the probability achievement scores between secondary school learners taught using the Argumentation-based approach and those taught using traditional methods over time.
2. **Alternative Hypothesis (*H*1):** Secondary school learners taught using the Argumentation-based approach will demonstrate significantly higher performance scores in probability compared to those taught using traditional methods.

1.6 Scope of the study

This study was conducted at a secondary school located in the Bulawayo Central district of Zimbabwe. It focused on the implementation of an argumentation-based instructional approach aimed at teaching probability to learners from diverse cultural backgrounds. Two secondary school classes were chosen so that the strategy could be studied in detail. The aims of the research were to understand how being taught through argumentation changes learners' perceptions about probability, to explore how knowledge of their culture affects how much they are involved in the subject and their academic performance and to find out about the challenges learners face when their cultural views and formal education are in conflict on the notion of probability. It also analysed the usefulness of argumentation as a tool for learning and teaching in this context. The study was carried during one academic term to closely examine the direct effects of the instructional intervention on learners' understanding and involvement with probability. Because Bulawayo is known for its cultural diversity, it was suitable context to study how cultural factors influence the learning of probability. The independent variable was the argumentation-based instructional intervention, and the dependent variables were how learners perceived probability, how motivated they were in probability learning and their academic performance as measured by assessments in probability. The research was carried out according to pragmatism which involved gathering both numerical and qualitative data. Quantitative data were analysed through statistical approaches and qualitative data were analysed using thematic analysis (Olivier, 2017), to find trends that supported or contradicted the quantitative results. By using this method, the strength of the outcomes was enhanced, making it easier to assess the impact of the instructional intervention on learners in probability learning.

1.7 Significance of the study

This research adds value outside the immediate setting and has potential to influence National educational policies formulation. These findings on the role of argumentation in education could inform the Ministry of Primary and Secondary Education (MOPSE) and the Curriculum Development and Technical Services Division in Zimbabwe. Also, it can support policymakers in designing curricula that fit different cultures and support a range of learner backgrounds. Emphasising critical thinking and teamwork through new teaching approaches is very important. This study can help to meet the learning needs of learners from different cultures which ensures that education practices cater for all.

The research is expected to influence how teachers are trained in professional programmes that could use argumentation methods in their teaching. It also aims to encourage teaching practices that value and use cultural diversity to support fairness, higher engagement and better learning results for everyone in the classroom.

In this way, the study recognises that evidence-based strategies are vital for developing policies and courses that fit the unique needs of various learners. Its purpose is to help teachers in Zimbabwe teach mathematics more effectively, so all learners have equal opportunities in education.

The focus on culturally diverse learners in Bulawayo emanates from the observation that probability often contradicts indigenous ontological beliefs regarding luck, fate, or divine intervention. The study of this demographic shows how learners handle the border crossing between their cultural and Western mathematical perspectives. This study therefore, sought to move away from viewing cultural diversity as a barrier but to reclaim it as a cognitive resource. The study asserts that for mathematical logic to truly take root, it must be reconciled with the diverse knowledge systems brought into the classrooms by learners.

1.8 Assumptions of the study

The current study believes that learners with different cultural backgrounds often experience challenges when learning about probability. This occurs because the topics studied by learners in school are defined differently than the experiences from their culture. Prahmana and D’Ambrosio (2020) posit that cultural knowledge greatly influences people’s views which can shape the field of mathematics education. For instance, it has been found that some learners’ cultural views about chance and probability differ from what is stated in mathematics. As a result, these differences create cognitive dissonance, causing learners to be confused and make mistakes when learning about probability.

To handle this challenge, the study suggests that argumentation can help bring together the different concepts. The purpose is to make it easier for learners to think critically, exchange ideas and combine culturally relevant learning with the study of probability. It is expected that combining these concepts will lead to a better understanding that may benefit performance in probabilistic activities. It is believed that learners’ cultural background sometimes disagrees with the ideas presented in probability, so it is necessary to teach using methods that bring their cultural knowledge and probability ideas together.

While cultural knowledge can support learning by giving meaning to new information, it is also necessary to recognise that a learner’s cultural background might cause difficulties with understanding probability. Some learners might understand probability as being influenced by fate or divine will instead of just randomness. By considering these counterarguments, we see that teaching mathematics in a systematic way allows learners to examine their values and learn mathematics to success.

The link between cognitive dissonance and culture influences the choice of research questions, mainly when the goal is to better teach probability in culturally diverse societies. Moreover, the study intends to show how assessing learners' cognitive dissonance can help in using arguments to integrate cultural aspects into their understanding of probability. The main aim of this study is to overcome conflicts between academic and cultural knowledge which supports the development of better teaching methods.

1.9 Delimitation of the study

The researcher sets up delimitations as an important way to outline the limits of the research. Delimitations refer to the deliberate methodological boundaries set by the researcher to keep the investigation manageable and focused. Scope refers to the substantive breadth of the study, what phenomena are examined and in which population (Leedy & Ormrod, 2019). Though related, scope is descriptive of what the study encompasses while delimitations are prescriptive decisions made by the researcher about what lies beyond the study's boundaries. By setting limits, researchers can focus on important issues and reduce possible confusion (Dimitrios, & Antigoni, 2019). Besides, setting limits the authors assert that it helps keep research organised which makes it easier to complete and stay in line with the set goals.

For the current research, it was important to set limits or delimitations, to explore the topic in a more targeted way. Only learners from one particular school were chosen for the study and no learners from other schools were included. Even though this approach may limit how general the findings are, studying just one setting helps better understand how argumentation shapes probability teaching. In addition, this research only covered probability learning and left out other important mathematical topics.

To ensure a thorough understanding of the data from this study, a mixed-methods approach was used. Therefore, the study used pre- and post-tests, semi-structured questionnaires and group discussions, and did not apply methods such as long-term studies or experiments. Because of these defined boundaries, research remained focused on one educational context and selected sample. Because of this strategy, the researcher managed to carry out an in-depth study on how argumentation and cultural knowledge are used in teaching probability.

1.10 Definition of key terms

*1.10.1 **Integrating***

Integrating involves the process of combining different components to make a coherent system in academic discourse, focusing on the unification of different elements (Drake & Reid, 2020). It emphasises blending of the probability theory and argumentation to make it understandable and usable. Integrating within the current study involves combining the process of teaching argumentation and probability to derive a coherent learning process, allowing the learners to link the topic of probability with cultural perceptions of the same.

1.10.2 ***Argumentation***

Walton (2009) describes argumentation as a type of reasoning that entails laying out premises to establish a conclusion, highlighting the dialogical exchange of ideas by learners. Vandoulakis (2018) expands on the dialogical notion by describing argumentation in the form of a dialog between agents in which counterarguments and arguments are exchanged in order to make clear claims of truths, primarily in mathematical settings. For the purposes of this study, argumentation is a systematic process of reasoning wherein a learner or group of learners pose and criticise claims based on evidence and logical reasoning through systematic exchange of counterarguments and arguments.

1.10.3 ***Probability***

Halmos (1944) defines probability as a mathematical structure that measures uncertainty and the chance of occurrences within a known sample space. Probability defines events with values from 0, which represents the notion of impossibility, to 1, which represents certainty. Probability is regarded as a measure of likelihood. The definition reflects the mathematical strictness and axiomatic nature of probability theory. Probability here is perceived as a measurable determination of the event's likelihood of occurrence through theoretical axioms and empirical evidence. Probability has the mathematical rules of random occurrences and the subjective judgements that individuals make based on beliefs and experience. The duality of probability allows it to find application across disciplines, capturing objective measurements and individual judgment of uncertainty.

1.10.4 ***Probability teaching***

Probabilities instruction entails educating learners on the concepts of chance, randomness, and uncertainty, with a balance of theoretical and application-based aspects. Successful probabilities teaching enables learners to develop the ability to analyse uncertain situations and make sound decisions.

1.10.5 ***Culture***

Culture is a system of knowledge, beliefs, behaviours, and customs shared by members of an interacting group that serves as a foundation for further interaction (Göncü & Gauvain, 2012). It encompasses language, customs, morals, institutions, knowledge, ideas, beliefs, and belief systems that influence a society's worldview, shaping perceptions and interpretations of natural phenomena and events (Ogunniyi & Hewson, 2008). In this study, culture is defined as the social heritage or way of life of society members that is transmitted socially.

1.10.6 ***Cultural diversity***

Cultural diversity refers to a system of beliefs and behaviours that acknowledges, accepts, and respects diverse cultural groups within an organisation or society, recognising and valuing socio-cultural differences. Promoting cultural diversity promotes contributions in culturally inclusive environments, empowering all individuals within the organisation or society. In this study, cultural diversity specifically relates to learners' cultural backgrounds and how their heritage influences their understanding of probability concepts. Culturally diverse education refers to educational strategies and resources developed to assist teachers in addressing the challenges presented by learners' diverse cultural backgrounds, as outlined by (Sudhi, 2016).

1.11 Structure of the thesis chapters

Chapter One serves as an introductory framework for the study, providing an overview of the research background while outlining the research problem, significance of the study, research questions, objectives, delimitations, assumptions, and definitions of key terms of the study. Chapter Two presents an overview of relevant literature concerning argumentation and the teaching of probability within culturally diverse contexts, emphasising the role of argumentation in this process. This chapter outlines and elaborates on the theoretical and conceptual frameworks guiding the study. Chapter Three details the methodology employed in the study, including research paradigm, design, participant selection, data collection methods, data analysis procedures, and ethical considerations. Chapter Four presents, discusses, and analyses the gathered data, while Chapter Five summarises the study's findings, highlights conclusions drawn from the results, discusses implications of the findings, limitations and offers recommendations for various stakeholders, including mathematics teachers and curriculum advisors, alongside suggestions for future research directions.

1.12 Chapter summary

This chapter articulates the justification for the integration of argumentation into the teaching of probability among culturally diverse secondary school learners in Bulawayo Central. Highlighting the essential role of probability within the educational context, the chapter addresses specific challenges experienced by secondary school learners, including inadequate performance, prevalent misconceptions, and insufficient engagement within the context of probability learning. The purpose of the research is to assess how argumentation influences learner involvement, performances and understanding which are the main objectives of the study.

Identifying deficiencies in other studies in the use of argumentation and culturally responsive teaching to teach probability provides a basis for the design of this study. Because there is little empirical research on how argumentation affects probability learning among culturally diverse groups such as those in Bulawayo, this study addresses that gap. The research uses both qualitative and quantitative approaches to collect vast information about what learners go through and achieve. Because of the sample size limit, the research has focused more on detailed qualitative group discussions along with quantitative findings. The use of both qualitative and quantitative methods makes it possible to fully understand the effects of the instructional approach.

The following chapter builds on this chapter by organising research related to argumentation, probability and culturally responsive teaching. This section discusses the important theories related to probability teaching and argumentation, to support the study’s objectives. By reviewing the literature, this section ensures that the importance of argumentation and culturally sensitive teaching that forms the backbone of mathematics education is clearly discussed

CHAPTER 2: REVIEW OF RELATED LITERATURE

2.0 Introduction

The chapter details existing research about probability, cultural diversity and argumentation. Four essential areas that include probability challenges in probability teaching, cultural boundary transcendence, probability learning through diverse knowledge systems and argumentation in probability are detailed. The chapter covers the contributions of the different authors and how they have handled probability teaching issues. The conceptual frameworks and methodological approaches including research findings and data analysis techniques used are analysed. The research draws its findings from the past studies while explaining their practical applications in teaching and learning of probability. Both the advantages and disadvantages of argumentation teaching in probability and mathematics in general are evaluated. The chapter also explains the theoretical frameworks which guide the current study.

2.1 Theoretical frameworks

This study is informed by four main interconnected theories: the Sociocultural Theory, Constructivist Theory, Culturally Responsive Pedagogy and Argumentation Theory. These theories jointly assist in understanding the research questions and objectives.

*2.1.1 **Sociocultural theory***

According to Vygotsky's (1978) sociocultural theory, the role of social interaction and culture is very important in learning. Hughes (2021) and Newman (2018) assert that learners build knowledge together by discussing and sharing ideas. Because learners in Bulawayo come from various cultural backgrounds, this theoretical framework is especially appropriate, as diverse cultural groups work collaboratively to learn probability. By arguing, learners will be able to express their reasoning and learn to relate probability ideas to their background. For this reason, the connection is necessary to help learners understand and take interest in their learning (Nifriza, 2021). Allowing discussions to be based on learners' cultural backgrounds ensures the classroom is welcoming to everyone. Researchers have found that using culturally responsive ways of teaching can have a strong impact on how learners feel and act in class (Thomas & Berry, 2019). As an example, Gay (2010) found that using these teaching methods can help schools better support diverse learners, making learners more engaged in their learning (Miller, 2011). Using argumentation in probability teaching also allows learners to discuss their ideas and relate mathematics to their cultural experiences (Ratner, 2015). By accepting and valuing learners' backgrounds, this approach also helps learners learn and retain knowledge together as a group (Zhou & Chen, 2014). Using Vygotsky's theory of learning in classrooms, by emphasising argumentation and supporting learners from various cultures, greatly improves how learners learn. If teachers bring up perspectives that relate to learners' cultural knowledge, they create inclusive contexts that help learners engage and understand concepts related to probability.

2.1.2 Constructivist theory

According to constructivist theory, learners build their own knowledge by actively creating and interacting with different theories and life experiences (Arends, 1998). Thus, using argumentation in probability classes encourages learners to work together and use their critical thinking skills in solving problems. Furthermore, Elliott et al. (2000) elucidate the significance of incorporating cultural knowledge into mathematics education via culturally pertinent activities. Such activities are designed to promote active engagement and enhance understanding among learners. In this study, principles of constructivism were instrumental in the design of activities based on argumentation. These activities facilitated the exploration of probabilistic concepts within the context of real-world scenarios that are relevant to the cultural backgrounds of the learners. This method not only contributes to a deeper understanding of the subject matter but also encourages learners to take greater ownership of their educational journey.

In Figure 2.1 below, Prawat and Floden (1994) illustrate how knowledge is created and shared through interaction among group members.

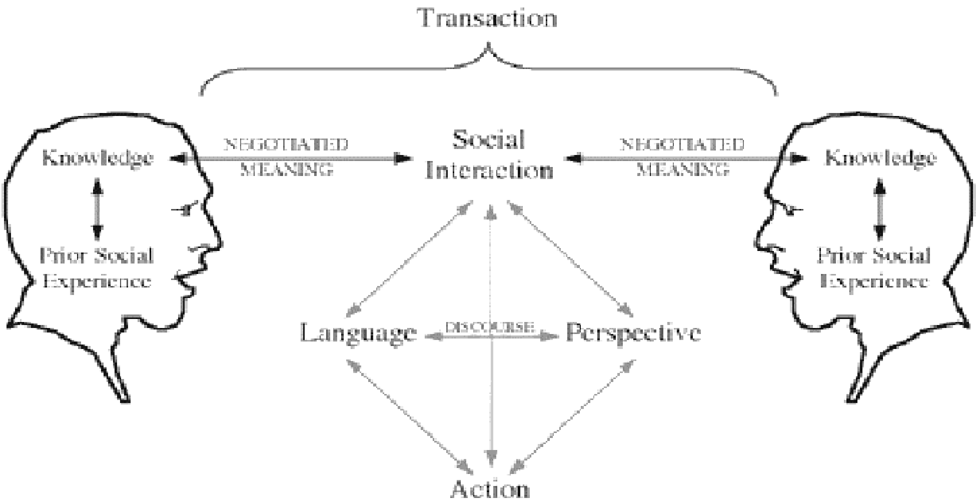


Figure 2.1: Diagrammatic representation of social constructivism by Prawat and Floden (1994). Adapted from Doolittle, (2001, p. 509)

Learning is characterised as an active engagement rather than a mere receptacle process, whereby individuals are recognised as passive learners. This perspective, as articulated by Dewey (1938) and Doolittle (2001), posits that meaningful learning occurs predominantly through interpersonal interactions, wherein knowledge is co-constructed through the sharing and negotiation of meanings within a social context. A fundamental aspect of this paradigm is the recognition of the subjective nature of knowledge, which indicates that learners cultivate unique perceptions shaped by their individual experiences. The variability in these perceptions is frequently influenced by cultural backgrounds, thereby impacting the distinct learning processes of individuals. Such cultural influences necessitate a critical examination of the instructional methods employed in educational contexts.

According to constructivism, the role of the teacher is to help learners collaborate by ensuring a suitable environment. Here, learners are urged to collaborate on solving problems which helps them guide their learning process. As research shows, learners sometimes face differences between their culture and the content taught in schools, so it is important to involve everyone in discussions. This process ensures learners have opportunities to take part in discussions allowing them to agree or to question well-established concepts.

2.1.3 ***Culturally responsive pedagogy***

Ladson-Billings (1995) introduced Culturally Responsive Pedagogy to motivate teachers to acknowledge and include learners' cultural backgrounds in their teaching. Such pedagogy is particularly useful in contexts such as Bulawayo, where learners' backgrounds and experiences are very diverse. Including examples from different cultures in probability instruction helps teachers make every learner feel included and appreciated. With this method, learners are more involved, and it helps explain mathematics concepts in ways that relate to what learners encounter in daily life. By using culturally responsive techniques, more learners are represented and accepted which supports better learning results. When these practices are stressed, everyone in the classroom is treated fairly and appreciated for their own skills.

2.1.4 ***Argumentation theory***

Toulmin (1958) developed the argumentation theory that explains the importance of reasoning and discussions in educational contexts. With probability teaching, argumentation application provides a structured process for learners to explain their views and analyse their arguments and explanations. Learners use this pedagogical approach to form and defend claims. The approach also promotes receptivity to divergent perspectives.

In addition, directing discussions and arguments about probability topics to learners, strengthens the discursive and communication abilities of the learners. The use of this theoretical framework helps learners understand difficult mathematics concepts as learners become more involved in learning. Learners benefit more in classrooms with a mix of cultures by being active participants. Toulmin’s Model of Argumentation is a central concept in this study - it offers a framework for presenting and analysing arguments. This model identifies and explains the formulation of - claims, evidence, warrants as well as rebuttals - which facilitates learners’ abilities to evaluate and articulate ideas critically (Hmelo-Silver, 2014). Teachers using this model have a structured tool to guide learners in effectively developing argumentative skills.

2.1.4.1 Toulmin’s Argumentation Pattern (TAP)

Toulmin’s argumentation model provides a dependable method to examine pedagogical arguments. With the model, teachers can see the structure and standard of arguments in class and refine how they instruct learners in argumentation. Toulmin (1958) asserts that arguments deserve more understanding than formal logic, which mostly addresses premises and conclusions. According to Erduran and Jiménez-Aleixandre (2015) the framework puts an argument in order by addressing several parts:

a) Related aspects of a claim

b) Data supporting that claim

c) Warrants linking the data and the claim

d) Backings that support the warrants

e) Rebuttals highlighting circumstances that invalidate the claim

This study employed TAP as a mechanism for the analysis of arguments, particularly in relation to logical reasoning and the controversial socio-cultural dimensions that accompany it. Ogunniyi (2008a) asserts that the existence of two separate knowledge systems, mathematics and cultural knowledge, helps form optimal cognitive states because of their complementary nature. The author claims that this relationship helps to combine different elements based on the connections found in the process of forming ideas.

In addition, they are required to make sense of the connections between the two types of knowledge. Negotiations show that different ways of thinking can be brought together and merged. Ogunniyi (2008a) demonstrates the ways in which mental states change when a stance is communicated. Many things can affect these mental states, including logic, religious ideas, culture or a combination of these.

2.1.4.2 Contiguity Argumentation Theory (CAT)

Ogunniyi (2007a) states in his Contiguity Argumentation Theory (CAT) that changes in thinking occur for individuals when their initial beliefs conflict with what they learn in school. The theory points out that the ways learners see and understand the world through schooling are often very different from how they understand it through their culture. This framework argues that being close in space and time to other concepts and knowledge greatly improves argumentation and understanding during learning. In addition, CAT emphasises the connections among different subjects, especially in science, where it encourages the mixing of indigenous knowledge and standard mathematics language.

Much research demonstrated that CAT has value for use in educational settings. Ghebru and Ogunniyi (2017) posit that CAT helps with understanding changes in reasoning about science among pre-service teachers. It showed CAT can help combine different ways of thinking. This finding agrees with Mukwambo's (2023) point that cultural as well as background factors have a key role in scientific discourse. Bang and Medin (2010) present evidence in a similar way. The manner events are presented in time has a great effect on human performance in tasks that involve actions plus their outcomes. It suggests the timing and context of information have great importance for learning.

In their study, Nhalevilo and Ogunniyi (2014a) pointed out how the Contiguity Argumentation Theory (CAT) has a key role in studying subjects that include both scientific and cultural knowledge. They hold that CAT can explain how different cognitive processes relate to each other. Msimanga and Lelliott (2012) posit that argumentation is key to making science concepts understandable during instructional dialogues. This becomes especially true with CAT.

CAT supports understanding the ways arguments are formed and used in educational settings, mainly around integrating different types of knowledge. Emphasising the way ideas are related in space and time highlights how environment and timing impact learning and understanding. Thus, CAT proves useful for teaching argumentation in different settings for culturally diverse learners in Bulawayo.

CAT principles point out that the mix of different worldviews can lead to learners feeling attracted or repelled, depending on the situation, as discussed by Ogunniyi and Hewson (2008). Ogunniyi (2007a) describes five cognitive categories of CAT, showing how arguments and dialogues interact in the minds of learners. In this case, learners use ideas about probability and their own culture to aim for a short-term balance that suits them. When learners encounter cultural and school views about probability, they must deal with new concepts which may result in either personal reflection or dialogues with fellow learners (Ogunniyi & Hewson, 2008).

The five cognitive categories that Ogunniyi (2007a) describes through CAT operate in learner cognition: (1) Dominance, scientific knowledge overrides cultural knowledge; (2) Equipollence, scientific and cultural knowledge coexist with equal explanatory force; (3) Suppression, one knowledge form is temporarily set aside; (4) Subordination, one knowledge form serves as a resource to explain the other; and (5) Recontextualisation, indigenous knowledge is transformed through formal argumentation into a school-compatible form. To illustrate these categories in the Bulawayo probability context: a Tonga learner who initially explains a coin-toss outcome as “luck determined by ancestors” (Dominance of cultural frame) and, through structured argumentation, arrives at “P(Heads) = $1/2$ because each outcome is equally likely” (Recontextualisation), demonstrates the cognitive trajectory CAT predicts. This model of learner transformation underpins the design of the E-API intervention in this study.

Using the knowledge from earlier studies, the current study aims to help learners engage with probability concepts, by mixing their own cultural beliefs about probability with normal school approaches. Mixing argumentation with knowledge about multiple cultures adds new energy, involves learners more and encourages acceptance of many ways of thinking. Emphasising argumentation can motivate learners to provide explanations backed by evidence and rationality for diverse cultural perspectives, revealing the true learning from learners' social interactions and leading to lively discussions.

2.2 The conceptual framework

The conceptual framework represents a synthesis of existing literature undertaken by a researcher, with the objective of elucidating a specific phenomenon (Imenda, 2014). The framework states how a researcher conceptualises their variables and the relations that these variables have with each other. As a result, it directs the progress of the research.

A narrative describing the connections among the variables and how they apply to this study is given after Figure 2.2. This framework outlines the conflict between cultural and school conceptions of probability, how learners navigate that conflict through code switching, the proposed argumentation intervention, and its anticipated outcomes.

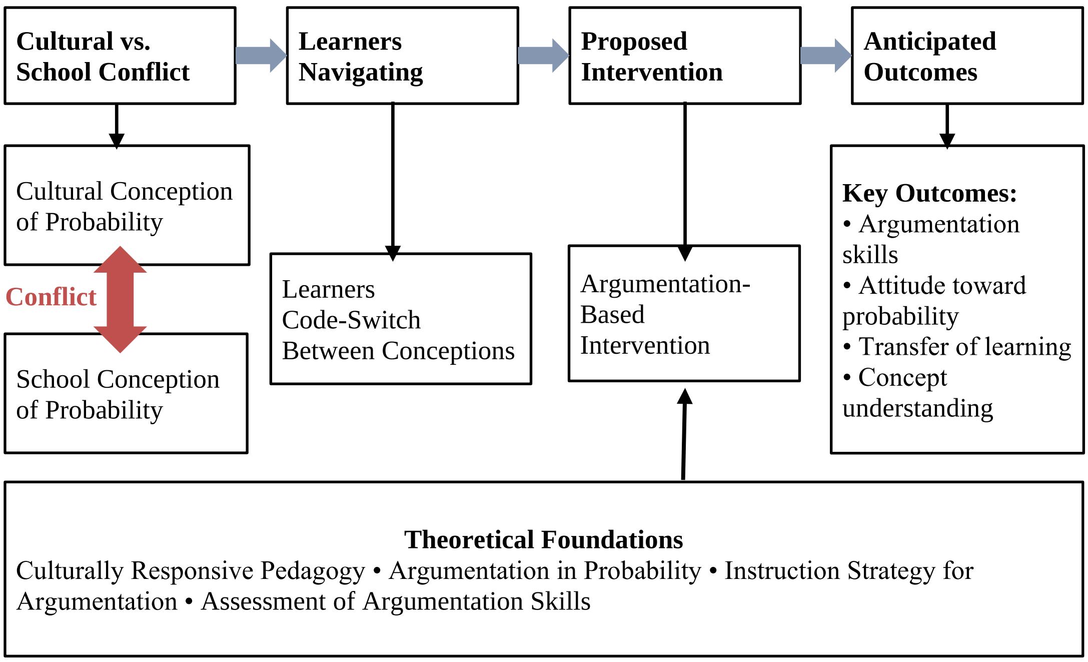


Figure 2.2: The conceptual framework of the study

This framework illustrates two distinct and opposing conceptions of probability: cultural and school interpretations, as shown by the arrows. Learners from different backgrounds usually bring their own perceptions about probability to school and these often differ from those taught in school (Elbehary, 2022). Learners often code switch to make sense of probability in schools, by translating school concepts to their own cultural perspectives which often leads to misunderstandings (Planas & Civil, 2013). Moyo (2012) suggests the use of argumentation to resolve the conflicts between the two interpretations. The goal of this study is to clarify how argumentation can help learners grasp probability which could benefit their performance in school. The research also seeks to find out if argumentation changes the learners' perceptions about probability and how much the intervention assists in building argumentation skills. Besides, it considers whether learners are able to use their argumentation skills in a range of different situations.

The framework points out how theory, culture, teaching methods, strategies and assessment connect. The aim is to help learners learn more about probability and improve their reasoning skills and critical thinking by advocating for the use of argumentation in mediating the conflict between the two perspectives about probability. It promotes success in school and helps learners develop the skills to take part in discussions about probability in real life.

2.3 Conceptualisation of argumentation

2.3.1 ***Definition and importance of argumentation***

Argumentation is a basic mental activity that consists of developing, expressing and defending arguments based on logic and evidence (Hmelo-Silver, 2014). Learning how to use language well is crucial in schools, helping learners voice their opinions clearly, analyse problems carefully and communicate well with peers. Argumentation in the teaching of mathematics is helpful, as it helps learners gain a deeper understanding, challenge their own beliefs and build the reasoning skills needed for effective problem-solving (Savard et al., 2014). Such dialogic interactions create a group of learners who accept different opinions and become involved in meaningful conversations (Hmelo-Silver, 2014). According to Hmelo-Silver (2014) in mathematics, where logical thinking and explanations matter most, argumentation assists learners to better understand and remember difficult content.

In teaching, argumentation is used to either confirm or reject claims and then assess the strength of the arguments formed (Ghebru & Ogunniyi, 2017). Argumentation prompts learners to present their arguments and take part in discourse that helps them review their thoughts leading to better understanding (Marranghello, 2026). Empirical evidence shows that argumentation boosts learners' ability to think and reason critically. Marranghello (2026) argues that teaching scientific argumentation is now essential in science education, showing that modern science teaching now stresses using higher order thinking instead of just remembering facts. By learning this way, learners not only grasp scientific concepts but also develop the ability to handle difficult problems by building strong arguments (Ghebru & Ogunniyi, 2017).

Valencia (2022) recommends that probability is taught so that learners can use it to deal with uncertainty and variability in every-day situations. Using probability concepts in the context of argumentation encourages learners to discuss, interact and share their thoughts making learning more enjoyable and effective for all (McNeill & Knight, 2013). Argumentation theories such as Toulmin's model specifically focus on the components of an argument such as claims, grounds, warrants and backing (Metaxas et al., 2016). The authors describe how this model can be used in teaching to analyse and develop learners' ability to argue, improving the whole educational process. Using this method helps learners develop their reasoning clearer and encourages them to discuss their arguments which is necessary for becoming skilled in argumentation.

Also, when arguing in mathematics, learners construct and support claims and at the same time, they collaborate with peers creating meaningful social exchanges (Berland & Reiser, 2011). This goes with the idea that argumentation is not limited to learners thinking on their own but involves collaborating to better understand each other. The social aspect of argumentation matters a lot in classrooms with learners of diverse cultural backgrounds, as different perspectives can improve discussions and help everyone understand better. Macagno and Konstantinidou (2013) argue that learning can be understood as engaging in argumentative discourse. The authors focus on helping learners become better at arguing by taking part in organised activities that let them practice and improve their arguments. This framework helps especially in teaching probability by letting learners discuss misconceptions and increase their understanding.

Demir and Isleyen (2015) further explain the idea of dialogic argumentation, pointing out that it helps learners work together and hone their critical thinking and reasoning abilities. Using this instructional approach assists learners in learning more effectively, as it encourages them to communicate with each other and practice using their minds and social skills. When argumentation is emphasised, it matches culturally responsive teaching methods by accepting and including all learners in the classroom.

This study acknowledges that the conceptual contradiction between cultural and school-based probability understandings is not universal across all learners or all probability topics. Some learners may already hold culturally compatible intuitions, for example, those familiar with traditional games of chance. In such cases cultural-academic apposition may be minimal or absent. The role of argumentation-based instruction in these cases, therefore, shifts from reconciling conflicting frameworks to consolidating and formalising pre-existing probabilistic intuitions. This study treats cultural apposition as a learner-specific and context-dependent phenomenon rather than a universally applicable assumption (Nhalevilo & Ogunniyi, 2014b).

2.3.2 Argumentation as a tool for teaching and learning

Because of the problems involved in cultural diversity, using argumentation in mathematics lessons can help learners understand difficult topics such as probability. Castro and Toro (2023), Fitriani et al. (2019) and Kartika et al. (2024) are among those who suggest that learning to argue helps learners think critically and speak clearly which is very helpful in contexts characterised by cultural diversity. When learners engage in argumentation, they use their different backgrounds to discuss and collaborate about the subject matter. This viewpoint agrees with what Zhou and Fischer (2013) described, which emphasises that teaching should include and welcome learners from various cultures to promote inclusion and involvement in the classroom. Further, Manz (2014) describes argumentation as a way of developing, supporting and criticising claims so that individuals can gain shared understanding, emphasising its collaborative nature.

Through interacting in this way, learners can help each other improve and better understand what they are learning. In addition, Llewellyn (2013) argues that engaging in argumentation activities builds strong critical thinking abilities which are important for dealing with important topics. If teachers incorporate argumentation to the teaching of probability, learners can feel confident to discuss their thoughts and question each other which helps them grasp mathematical ideas more deeply. Cynar and Bayraktar (2014) also state that argumentation supports the development of learners' thinking, promoting a common understanding of ideas.

By working together, learners benefit from their own progress and feel connected to the group which is very important in classrooms with learners from different cultures. Cynar and Bayraktar (2014) show that argumentation plays a major role in improving learners' academic performance, better than standard teaching methods. They argue that argumentation is an effective way to help learners from different cultures to work together and build a more inclusive classroom. By identifying argumentation as a way of learning, Freeman et al. (2014) point out that involving learners in discourse helps them focus on learning rather than just listening to lectures.

It is especially important for learners from various backgrounds to be involved in this way, since it helps them explore mathematics by working with others. At the same time, Gillet, Vallerand and Lafreniere (2012) point out that while dialogic methods are important, they must be balanced with authoritative discourse to help learners understand and follow probability rules. To work towards this equilibrium is key so that learners can discuss important topics and receive the help they need to fully understand mathematics. With this approach, the teacher assists learners by guiding and letting them assume an active role in their learning.

2.3.3 ***Relevance of argumentation in teaching and learning of probability***

Can and Isleyen (2016) carried out a study on the importance of argumentation in probability teaching through a quasi-experimental study. The researchers assigned 41 pre-service teachers into two groups: one group was taught through the Argumentation-Based Science Learning (ABSL) approach, while the second group received standard teaching. The analysis found that the experimental group showed a significant improvement in probability skills which shows that ABSL was more effective than common teaching methods.

In addition, this shows that argumentation can help learners grasp probability better, suggesting that mixing different ways of teaching can be beneficial for learners. The research method by Can and Isleyen (2016) informed the choice of the methodological approach of the current study especially the design of the quasi-experimental design. The authors assert that probability plays an important role in many fields such as games of luck, genetics and weather. They highlight that most traditional teaching practices do not effectively promote teaching and learning that encourage collaborative learning and culturally relevant pedagogy.

The authors further explain that teaching probability is made difficult mainly because teachers do not use varied instructional methods, lack of sufficient teaching materials and a lot of common misunderstandings among learners. Similarly, Koirala, Gurung and Wagle (2020) point out that inadequate probabilistic training for teachers causes difficulties for learners. The situation demonstrates that teacher education programmes are not giving teachers the knowledge they need to effectively teach probability through instructional approaches such as argumentation.

To investigate the difficulties of learning probabilistic problem-solving, Arum, Kusmayadi and Pramudya (2018) carried out a case study involving ten ninth-grade learners. They found that learners had trouble figuring out problems, selecting the best approach and using the necessary computing skills. Nevertheless, the study did not include cultural aspects or elements from argumentation which may have lowered its ability to help learners understand the concept well. This provided a gap to be filled by the current research.

Researchers have shown that research that aligns cultural aspects with argumentation improves learners' grasp of probability (van Eemeren & Grootendorst, 1992; Simons, Morreale & Gronbeck, 2001). These studies point to the importance of teaching approaches that address learning challenges and make use of learners' cultural backgrounds and argumentative techniques.

2.3.4 ***Benefits of argumentation in mathematics teaching and learning***

Argumentation in mathematics education has many acknowledged benefits which research has proven. According to studies, learners who argue about mathematics tend to grasp concepts better, think more logically and actively participate in classroom activities (Cynar & Bayraktar, 2014). The study by Chitera et al. (2025) adds that argumentation serves as a corrective mechanism for common stochastic misconceptions. By requiring learners to articulate their reasoning, the instructional approach encourages them to confront and resolve logical inconsistencies that are often masked during passive learning.

Moreover, engaging in argumentation encourages learners to analyse their arguments and those made by others.

Also, the practice of arguing adds to the friendly and supportive nature of the classroom. When everyone in the classroom feels safe to express their thoughts and differ in opinions, learning becomes a collaborative experience that values everyone's opinions. For learners from various cultural backgrounds such an environment is important as it reflects the approach to learning that they might be used to (Hmelo-Silver, 2014; Savard et al., 2014). In addition, what people achieve through argumentation is extremely useful when tackling real-world problems. The skill to build and support a good argument is necessary for application in many professions (Hmelo-Silver, 2014; Savard et al., 2014).

*2.3.5 **Challenges associated with argumentation***

Though argumentation is useful in education, many challenges need to be overcome for it to be used successfully. Many learners in these cultures are reluctant to speak in class discussions which is a significant challenge. Some feel uncomfortable when disagreeing with their classmates as their cultures disapprove of it (Hmelo-Silver, 2014; Savard et al., 2024). Teachers are expected to help create an atmosphere where all learners can collaborate with peers and share different ideas.

There is also a challenge related to helping teachers improve their argumentative skills. If teachers are not familiar with argumentation and are not sure how to guide conversations, it can be difficult for them to excel in this area. So, it is important to provide teachers with training and resources that support them in using argumentation in their teaching practices (Sunzuma & Maharaj, 2020; Hmelo-Silver, 2014). Effective argumentation-based instruction depends on both teachers and learners being well prepared and open to changes (Hewson & Ogunniyi, 2011). According to the authors, argumentation activities take a lot of effort to plan and organise, and teachers must put in a lot of time and energy to develop the necessary teaching resources. In addition, learners need time to prepare for discussions and do presentations which can be challenging when the curriculum is already packed. The perspective implies that schools should make changes to their practices to support the use of argumentation.

Because teachers must consider language and mathematical skills, presenting probability arguments can be hard in mixed cultural classrooms. The words and structures found in argument texts might reduce learners' capacity to express their arguments. Several studies (Sharma, 2019; Planas and Setati-Phakeng, 2014; Hine, Blackley, & Cooke, 2019) indicate that understanding mathematical ideas is greatly affected by language. For example, Tonga and Ndebele learners confuse the meaning of *random* in everyday usage. To them it means anything unplanned or haphazard yet in probability it implies a specific mathematical distribution. In a study, Sharma (2019) posit that teachers have difficulties incorporating Pasifika learners' native languages and everyday situations in their teaching. This finding points out that dealing with language barriers is important for assisting learners grasp probability concepts and improve their arguments.

According to Sharma (2019), immigration and migration result in different language situations which can be difficult for both teachers and learners when the language of teaching does not match the learners' native language. Instead of seeing language as a barrier, Planas and Setati-Phakeng (2014) proposed that making use of learners' first languages can improve their learning. According to Hine, Blackley and Cooke (2019), the many ways that people use probabilistic terms in everyday speech can make it difficult for learners to understand science concepts. To deal with the difficulties associated with language in mathematics education, a thoughtful teaching strategy for probability is necessary. According to Daly and Sharma (2018), using home language and academic language can be helpful, but there is potential for miscommunications if the meaning is not well translated. The implementation of such models in diverse classrooms is not without systemic contradictions. Chitera et al. (2025) identified that while argumentation fosters critical thinking, it can also induce cognitive overload if the mathematical register used in the classroom remains divorced from the learners' primary cultural discourse. Consequently, the integration of argumentation requires pedagogical translanguaging, where learners are encouraged to use their full linguistic abilities to justify probabilistic claims before formalising them in standard mathematical English.

In the field of probability, different ways of defining specialised vocabulary can confuse those who are learning the subject. Despite the problems that arise from code-switching, encouraging learners to switch between languages can make arguments more engaging and help learners discuss about which meanings for probabilistic terms are acceptable. Code switching was used for example when resolving lexical ambiguity such as of the word **Fair**, when used in English it can mean *equal* or *average*. However, it has a strict mathematical definition in probability regarding equal outcomes. In discussing its meaning, the following conversation ensued showing how learners code switched as they explained it.

Teacher: *Why do we call this a fair die?*

Learners A (in L1/L2 mix): *It is fair because onke ama-side alomthwalo ofanayo (all sides have the same weight). If one side was heavier, it would be biased.*

Teacher: *Correct. So, in our probability argument, what does fair mean?*

Learner A (in English): *It means every outcome has an equally likely chance of occurring, $P(x) = 1/6$.*

Smith et al. (2018) also support code switching. In their study the authors assert that using learners' native language in class enhances their ability to do well in mathematics. Results from their Mathematics Diagnostic Test (MDT) were compared for learners learning in Jamaican Dialects (JD) and Standard English (SE) and JD learners were found to have a clear advantage. This means that when learners are taught linguistic and cultural details, they relate these to their own experiences and therefore perform better academically.

2.3.6 ***The role of argumentation in teaching and learning processes***

The act of learning is mainly social, and argumentation supports the engagement of learners with each other. People use argumentation to clarify their own thinking, discuss what they know and develop their mental models (Elbehary, 2022). This is particularly noticeable in learning mathematics, where abstract topics such as probability can be difficult for learners to understand (Hmelo-Silver, 2014). When learners argue, they are required to review both their ideas and the ideas of others which stimulates their cognitive skills. As such, learners can consider different opinions which supports the building of strong and logical arguments (Hmelo-Silver, 2014). Besides helping people think, argumentation is essential for growing metacognitive awareness. When learners explain their reasoning and analyse their arguments, they learn more about how they think and solve problems. As a result of this self-reflection, people may become better at solving problems and feel more independent while learning (Hmelo-Silver, 2014). The strong connection between argumentation and metacognition allows learners to develop the skills required to handle the difficulties in mathematical tasks.

*2.3.7 **Argumentation in the context of mathematics education***

Batanero et al. (2016) propose reasons why argumentation plays a key role in mathematical learning. The authors assert that people view mathematics as a subject where rules and methods are predetermined and unchanging. Incorporating argumentation into mathematics teaching assists learners to realise the experimental and thoughtful aspects of this subject. This method helps learners realise why different mental strategies in mathematics are used, rather than only looking for right answers (Hmelo-Silver, 2014).

Besides changing perspectives on mathematics, active discussion helps learners to study mathematical topics in detail. When learners engage in discussions and debates, they can learn the main ideas behind different mathematical concepts which helps them understand mathematics better. For example, when learners learn about probability, they might discuss about different ways to solve problems. Discussing such ideas not only shows the different ways to reason but also helps them to understand probability theory (Hmelo-Silver, 2014; Savard et al., 2014).

*2.3.8 **Culturally responsive argumentation***

When teachers use argumentation in classrooms with learners from various cultures, they must consider the cultural backgrounds of their learners. Culturally responsive pedagogy makes it clear that understanding and appreciating the backgrounds and cultures of learners is very important (Hmelo-Silver, 2014). Teachers can improve how well learners connect with argumentation activities by choosing scenarios and illustrations that fit the backgrounds of all learners.

Making learners from diverse backgrounds feel included is extremely important. Teachers can build a setting where everyone is respectful and open which allows learners to share their views confidently. This way of teaching develops learners' argumentation abilities and helps them create a supportive group to learn with (Ferrari & Saccoletto, 2023).

Argumentation plays a key role, especially when learning mathematics. Its importance lies in its emphasis on critical thinking, increasing understanding and bringing people together in learning a subject. While using argumentation in teaching, it is necessary to be aware of any obstacles and cultural issues that might occur and impede teaching and learning. If teachers are trained and encouraged to include everyone's viewpoints in class and if they receive specialised training in argumentation, everyone in the class can benefit from such instructional approach.

2.3.9 ***The role of argumentation in mathematical reasoning***

Argumentation is necessary for people to develop strong mathematical reasoning skills. It supports learners in using their minds more creatively, going past just memorising facts and following steps to understanding the concepts. Some of the main steps of this educational mechanism are:

2.3.9.1 Justification of claims

The act of argumentation encourages learners to back up their claims with facts which supports more careful thinking in mathematics. When learners are asked to explain their solutions or thinking, they must analyse how they think and explain their ideas clearly.

2.3.9.2 Engagement with counterarguments

Counterarguments are an important part of developing an argument. Learners should analyse others' opinions and defend what they believe in when different perspectives are introduced to them. Interactions like these help learners develop critical thinking and better grasp mathematical concepts (Yackel & Cobb, 1996).

2.3.9.3 Collaborative learning

When learners work together, they often argue to solve problems and explain how they think. This type of environment allows them to exchange many ideas which encourages interesting discussions and leads to better learning. According to research, taking part in group argumentation supports improvement in solving problems and gaining a better understanding of mathematics (Johnson, 2017).

2.3.9.4 Implementing argumentation in mathematics classrooms

It is important to focus on argumentation in the classroom to make sure this method is successfully implemented in teaching mathematics (Brown, 2017). It encourages learners to state their thoughts and respectfully disagree with one another. Setting out the rules for respectful discussion and giving guidelines for helpful feedback can help create an environment where arguments can be made safely.

Moreover, using tasks that support argumentation has been shown to raise learner involvement. Teachers could use activities in which learners have to develop and explain their mathematical arguments. Having debates, role plays, and small-group discussions lets learners practice arguing and receive support from others (Francisco, 2022).

Furthermore, using open-ended problems inspires learners to look for different ways to solve the problem and discuss the methods they used. Because these problems have no single right answer, they encourage learners to try different approaches and talk about their ideas. Since the methods are flexible, they stimulate thinking skills and help learners explain how they solve problems (Solar & Piquet, 2016).

Mayweg-Paus et al. (2020) assert that the use of technology can greatly help learners in discussing mathematical topics. Online forums, online laboratories for mathematics and interactive simulations make it possible for learners to actively argue, exchange ideas and join forces to solve problems. They make online learning easier and enable conversations that continue outside the classroom. These can also be used in teaching probability.

Teachers can effectively demonstrate how to make and present arguments in the classroom. With examples of strong and weak arguments, learners can learn how to recognise the factors of good reasoning. Using think-aloud approaches during problems encourages learners to express what they are thinking and explains their reasoning steps (Lin, 2018).

Understanding how learners reason is possible through checking their argumentation skills. Rubrics, peer reviews and reflections in journals give useful insights into what learners know. Setting up rubrics with the criteria for grading arguments clarifies what is expected of everyone and having learners assess each other's arguments helps them take responsibility for their own work (Ayalon & Hershkowitz, 2018). Moreover, by using reflective journals, learners practice analysing their own argumentation which leads to deeper knowledge of the topic (Uzun, 2024).

All in all, to include argumentation well in mathematics, a mix of different strategies is needed. When teachers encourage argumentation, plan structured activities, include open-ended problems, demonstrate good practices and use various ways to assess learners, they greatly improve learners' abilities to think critically, solve problems and communicate in mathematics.

2.3.9.5 Methodologies used in probability argumentation

Probability education research investigates argumentation using several techniques, which points to the broadness of the topic. Most commonly, qualitative case study methods are chosen to investigate the process of argumentation in schools. One example is the study done by Yackel and Cobb (1996) which looked at how the use of argumentation during small group talks influenced learners' understanding of probability.

Along with qualitative approaches, there are studies that apply quantitative as well as qualitative tools to better understand discussions about probability education. For example, Andam et al. (2025) used surveys to measure learners' opinions about probability and argumentation, and they also conducted interviews to explore learners' experiences during activities involving argumentation. Using both methods helps researchers get a full understanding of the subject.

There are many useful teaching strategies discussed in the literature for teaching probability. Andam et al. (2025) point out that involving learners in studies is very important. As shown in the studies, using innovative approaches in probability helps learners learn and perform better. Besides, the knowledge gained in these studies can guide research efforts now and can form the basis for future teaching methods, therefore enhancing the teaching of probability.

2.4 Conceptualisation of probability

*2.4.1 **History of probability***

The historical development of probability theory can be traced back to the 16th century, originating primarily from the exploration of games of chance and gambling activities. The formalisation of probability as a mathematical discipline was significantly influenced by the correspondence initiated in 1654 between Blaise Pascal and Pierre de Fermat, stemming from an inquiry posed by the gambler Chevalier de Méré concerning the odds in a dice game (Song & Song, 2013). This exchange of ideas established the foundational principles of probability, marking a transition from speculative reasoning to a rigorous mathematical framework. Before the contributions of Pascal and Fermat, Girolamo Cardano had already made notable advancements in the 15th and 16th centuries by analysing games of chance and articulating the concept of odds as the ratio of favourable outcomes to unfavourable ones. Nevertheless, it was the collaboration of Pascal and Fermat that resulted in the formulation of a structured theory of probability. This theoretical foundation facilitated the development of systematic methodologies for addressing uncertainty and decision-making processes rooted in numerical analysis rather than superstitious beliefs. The implications of this foundational work have subsequently permeated various domains, including statistics, finance, and risk assessment, thereby shaping modern perspectives on uncertainty management (Large, 2013).

The 18th century marked a period of substantial progress in the field, as articulated by (Devlin & Samarawickrema, 2010). The contributions of Pierre Simon Laplace in 1812 built significantly upon the earlier concepts introduced by Pascal and Fermat, applying probability theory to the mathematical examination of games of chance. Furthermore, Laplace’s extension of probability theory to address scientific challenges in the 19th century encompassed a wide array of applications, including the development of life insurance tables and advancements in the theories of errors and actuarial mathematics, thereby demonstrating the extensive utility of probability in multiple practical contexts.

2.4.2 ***Real-life applications of probability***

The field of probability theory has attained significant recognition across various disciplines, including, but not limited to, insurance, medical research, engineering, genetic analysis, quality control, and finance (Devlin & Samarawickrema, 2010). In the medical domain, healthcare practitioners utilise probabilistic concepts to evaluate the probabilities of disease outbreaks and the efficacy of specific treatments. The absence of probabilistic reasoning in such decision-making processes would create considerable challenges. Similarly, in the financial sector, investors rely on probability to evaluate risks and returns tied to diverse investment strategies. The formulation of informed predictions regarding future performance relies on an analysis of historical data and market trends, thereby facilitating efficient portfolio management. Additionally, in the context of meteorological predictions, probability assumes a significant role; meteorologists apply probabilistic models to notify individuals and organisations of imminent weather conditions, thereby enabling the implementation of necessary precautions.

In different cultures, probability may be seen and understood in very different ways. This point is made particularly clear in the study by Savard et al. (2014), who examined how the Nunavik Inuit community understands probability. They stress that teaching probabilistic reasoning should include cultural elements to highlight the effects of cultural contexts on mathematics and a person’s sense of self. In addition, Du, Stevens, Ahern and Shigaev (2016) point out that similar probabilistic phrases in accounting rules might have different meanings depending on where people come from. These studies emphasise the need to consider cultural differences in the way people think about probability, to ensure that education and workplaces are accessible and relevant to everyone.

How individuals see risks and uncertainty is deeply affected by their cultural beliefs and practices. In some places, it is typical for decisions to be made by a group, so the way probabilistic information is shared is also affected. Also, if learners share cultural beliefs that luck or chance play a big role, it might affect their willingness to accept standard teaching approaches in probability. Recognising these finer details allows teachers to set up environments that focus on different ways of viewing probability.

Learners usually encounter some obstacles and misunderstandings while trying to grasp probabilistic ideas which might hinder their ability to make use of them. Frequently, learners think that the principle of independence in probability applies to mutually exclusive events. Besides, people may misunderstand the impact of sample size on probability, as most learners do not realise how increasing the sample size leads to better estimates. Thus, it is important to use teaching methods that both outline the mathematics behind probability and explain the problems learners might experience. For this reason, having several instructional methods and tools is important to help learners grasp probability and improve their ability to solve problems and think critically.

The daily relevance of probability in different areas such as in medicine, finance and weather forecasting has been demonstrated. Yet, the way probability is presented and understood in society has an impact on how interested learners become in the subject. Research by Wilson et al. (2018) expounds that learners from different cultures have different ways of understanding safety risks which then affects how they respond to and deal with such problems in real life. Furthermore, Francisca et al. (2021) explain that medical decisions rely on probability and cultural background can help decide how people view and accept these probabilities which influences patient care and treatment compliance. Given the strong effects that culture has on how learners understand and use probability, these studies stress the importance of including cultural contexts in teaching and using the subject. Exploring widely held misconceptions and obstacles connected with probability can help teachers improve their strategies which can benefit all learners from different cultural backgrounds.

*2.4.3 **Probability in multicultural contexts***

The argument presented by Naresh et al. (2014) points out how mathematical concepts are found in many cultures. The authors stress that teachers should give importance to cultural context and choose a variety of ways to teach. Croom (1997), in Naresh et al. (2014), argues that teachers can achieve their goals in multicultural education by considering both mathematical and sociocultural factors at the same time. Moreover, this way of thinking builds a greater awareness of the roles different cultures have played in shaping mathematics.

Naresh et al. (2014) assert that understanding probability is not achieved through maturity; it requires intentional learning experiences that focus on probability. Integrating mathematics, for example probability, with cultural aspects determines how and what learners are taught in schools. Naresh et al. (2014) also stress that it is essential to understand cultural differences for multicultural education, as such factors strongly affect how mathematical perspectives are understood and used, mostly when assessing risks and probabilities.

Also, cultural beliefs and values often affect gambling behaviours, since gambling depends on assessing probabilities. It is clear from this phenomenon that the way people view and understand probability is greatly influenced by their cultural background since others may associate gambling outcomes with cultural influences (Venuleo, Salvatore, & Mossi, 2014). This combination of ideas allows teachers to create strategies that work for learners with different cultural backgrounds. They, in the end, help them truly grasp probability and use it in practical ways.

*2.4.4 **Teaching and learning of probability at secondary school***

At higher levels, teaching probability requires bringing together both the theories and the practical uses. As pointed out by Groth (2010), the main problem in teaching probability is that concepts are often presented in abstract ways, cutting off links between probability and daily life. Groth (2010) asserts that this gap in knowledge prevents teachers from understanding probability themselves and points out that the challenges teachers face as they teach conditional probability and independence come from their own lack of knowledge, so they need to learn more. There are a variety of forms of probabilistic thinking (largely culturally influenced). Probability in the context of this study refers to probability in the formal mathematics discipline.

In addition, Batanero et al. (2016) stress the value of relating probability to real-life situations. They emphasise that if teachers include real-world cases in teaching, learners will be more engaged and learn more effectively. Because of this, teachers are encouraged to use effective teaching practices that help learners realise how the concepts relate to their lives. In addition, Abrahamson (2014) promotes using experiments as part of classroom teaching. The author clarifies that it makes it possible for learners to discover and correct their misunderstandings about probabilistic events which leads to a deeper knowledge of a concept.

Zhou and Fischer (2013) mention that challenges with probability terms can hinder learners' understanding. The authors believe the use of games and similar activities can be useful in instructional strategies to help make learning easier. This concurs with modern teaching theories that focus on relevance and usefulness of materials that learners are familiar with, hence learners become more engaged and understand concepts better. Moreover, Zhou and Fischer (2013) discuss the many challenges encountered in culturally diverse classrooms and suggest teachers should use techniques that recognise and include the learners' cultures. It is thought that such an inclusive strategy will improve knowledge and involvement in learning probability.

2.5 Data analysis in mixed-methods educational research

*2.5.1 **Non-parametric statistical tests in educational intervention research***

Non-parametric tests such as the Mann-Whitney U test are employed widely in educational quasi-experiments where sample sizes are small and normality assumptions cannot be guaranteed (Field, 2018). Several argumentation interventions studies have used Mann-Whitney U comparisons of pre- and post-test scores to evaluate intervention effects (Can & Isleyen, 2016; Tristanti & Nusantara, 2022).

2.5.2 ***Difference-in-Differences (DID) analysis***

DID analysis has been used in educational policy research to isolate the causal effect of an intervention by comparing changes in outcomes over time between a treatment and control group (Gertler et al., 2016). Its application in classroom-based intervention studies allows researchers to control for time-invariant confounders (Angrist & Pischke, 2009).

2.5.3 ***Thematic analysis***

Braun and Clarke’s (2006) reflexive thematic analysis provides a flexible yet rigorous framework for identifying, analysing and reporting patterns within qualitative data. Its application in mixed-methods educational research has been well documented (Nowell et al., 2017).

2.6 Culture and cultural diversity

Understanding culture and cultural diversity is very important in schools, especially for addressing learners from different backgrounds. Culture is a group’s unique set of beliefs, values and practices (Jairrels, Brazil & Patton, 1999). On the other hand, cultural diversity means there are many different cultures or ethnic groups in a certain society. Having diversity in culture is significant in education because learners contribute their personal experiences to the classroom which act as a starting point for their learning (Athanases & De Oliveira, 2008). According to Gatua (2014), learners’ development is greatly affected by their cultural backgrounds, so teachers should tailor their teaching practices to fit the variety of cultures in the classroom to help learners achieve understanding.

Recognising and using learners' cultural contexts helps teachers create a learning environment that fits their learners' lives and thoughts. This theory is in line with what Li and Wang (2015) advocate. The authors point out that appreciating diverse cultural backgrounds is important in teaching and that understanding various learning styles can benefit learners from diverse cultures. The use of these principles by teachers leads to a school environment where everyone feels included.

Including cultural knowledge in probability teaching is achieved through using games and knowledge from various cultural groups. Indigenous games that have aspects of probability can help learners relate more to what they are learning in mathematics. Culturally relevant teaching can also be used by teachers to explain the probability concepts which can be difficult for some to grasp. These strategies can make learning effective and help affirm learners' identities which supports diversity in schools.

In contrast to those who support cultural integration, Du et al. (2016) explain that combining several cultural perspectives in education can cause disunity and make education hard to understand. Authors stress that including different cultural perspectives is necessary but also address the risk of letting cultural content be integrated in a random way which could negatively impact the structure of the curriculum. According to Law (2014), when the curriculum only tells a single cultural perspective, this can interfere with the objectives of teaching. Identifying these difficulties supports the need for education policies that respect cultural differences and encourage inclusivity.

Sharma (2012) points out that the need to include many opinions in lesson planning could confuse or overload teachers. Because of this, training teachers in specialised skills is viewed as necessary in this framework. Teachers must be able to handle cultural diversity well and should have the knowledge and abilities to build classrooms that acknowledge, respect and embrace differences. According to Sharma (2012), it is vital to recognise the cultural backgrounds of learners so that teaching practices are relevant for them all.

Linking cultural knowledge and probability concepts can bring about a lot of complexity (Diress et al., 2023). Culture plays a role in shaping a learner's understanding which can change their view of mathematics and lead to mistakes if not handled properly (Diress et al., 2023). Therefore, in-depth discussions on the problems and benefits of cultural diversity in schools need to be considered. Although there are challenges to mixing learners with different backgrounds, promoting a place where everyone's experiences are respected often improves education and leads to better outcomes (Bećirović & Bešlija, 2018). Promoting cultural diversity in schools improves learners' performance and at the same time equips them to take part in a world where people from different cultures interact.

*2.6.1 **Influence of socio-cultural backgrounds on learning***

Assessing learners' cultural backgrounds and their influence on probability knowledge is an important research topic (Ogunniyi, 2011; Okere & Keraro, 2002). The authors argue that what learners have gone through and the culture they come from play a major role in their learning progress. According to the authors, that’s why it is necessary to design teaching strategies that assist learners in connecting their cultural traditions with what is taught in school.

People learn better and remember more when what they study in school connects with their real-life experiences. The authors assert that while some cultural aspects are often misunderstood as misconceptions, the way a person’s socio-cultural background influences the way they think about probability is significant. This implies that a person’s learning is strongly influenced by their past experiences and the culture in which they live.

In addition, Ogunniyi (2011) suggests that understandings rooted in culture which are sometimes labelled as misconceptions, can offer valuable lessons. To value cultural diversity, teachers must change their instructional methods to respond to the different needs of their learners. Hence, it is important to include many cultural perspectives in a way that supports effective learning environments.

*2.6.2 **Cultural knowledge and its importance in teaching probability***

Scholars have become more interested in including cultural knowledge in probability courses in mathematics classes in recent years. Sudhi (2016) states that including cultural diversity in mathematics teaching helps learners understand the traditions and performances of different communities. This way of teaching enables people to understand their culture and examine their thoughts which helps to create a tolerant environment and reduce biases, as Sharma (2014) points out. In addition, places where learners interact with others from various cultures help them develop important skills in listening and understanding others.

For learners to interpret probabilities in many different situations, it is essential to use effective teaching methods. Batanero et al. (2016) emphasise that it is important to properly train teachers so they can help learners build strong knowledge of probability. Cultural games have been found to be one of the best ways to help learners grasp probability concepts and reflect on how probability is related to culture, as explained by Naresh et al. (2014). It helps learners think beyond basic probability, making the topic more friendly and interesting for them.

Even so, studies specifically addressing how cultural knowledge can be used in probability education are not yet common. Leavy and Hourigan (2014) state that teacher preparation programmes are often unable to equip teachers properly to teach probability. Batanero et al. (2016) go on to argue that not enough games are used to teach probability, even though games are known to be interesting educational tools. Games as a form of active learning have become more accepted in education (Koparan, 2019).

2.6.3 ***Cultural diversity and its importance in teaching and learning***

According to Horencio and Dennis (2025), ethnomathematics is about including culture in the way mathematics is taught. With this approach, learners' cultural experiences can be linked to mathematics which helps them find the subject more interesting. Studies suggest that using learners' cultural knowledge leads to their personal, mental and political growth, makes them feel included and helps them perform well in school.

With demographic changes in classrooms becoming more pronounced, Sharma (2014) promotes adopting a culturally responsive mathematical curriculum. Such a change is necessary to support inclusivity and to make sure mathematics education is tailored to learners from different backgrounds. Ethnomathematics can assist learners to discover and appreciate the importance of mathematics in their personal lives (Sharma, 2014; D'Ambrosio, 1990).

According to D'Ambrosio (1990) there is a big difference between cultural mathematics and traditional school mathematics and understanding this distinction can help learners engage better with standardised mathematics concepts. Horencio and Dennis (2025) stress the importance of including and preserving the cultural backgrounds of learners in mathematics education. When learners realise that their teachers consider their cultural views, it makes learning mathematics more relatable and shows them how it is useful in life. Hence, learners will feel more motivated to learn.

However, it is also recognised that research is needed to confirm that these educational approaches really improve mathematical learning in learners. According to Sharma (2012), there has been a historical mistake in ignoring how learners' cultural backgrounds are linked to school mathematics which has usually encouraged giving more attention to mathematics itself as taught in school instead of considering cultural understandings. Because of this gap, learners consider mathematics as having no connection to what they do in everyday life.

As a result, it becomes clear that a more integrated style of teaching mathematics is needed which is likely to increase learners' motivation and comprehension of mathematical ideas. Bang and Medin (2010) advise against relying on cultural knowledge systems to judge school mathematics. To emphasise a broad approach that honours all types of knowledge, they support a school environment where many ways of knowing are respected and considered.

In their works, Haring-Smith (2012) and Gollnick and Chinn (2012) have stressed the importance of teaching learners about various cultures in relation to mathematics. The view aligns with the main educational goal of teaching learners to think critically and respect different viewpoints. Therefore, including cultural knowledge in mathematics lessons at school is important as it values and honours what every learner has to contribute.

*2.6.4 **Challenges of cultural diversity in teaching***

Dealing with diverse cultures within schools is a problem that teachers need to solve to ensure learners are guided in positive ways. As explained by Hashim and Hoover (2010), several differences in culture, beliefs and practices among group members are often the causes of conflict in learning groups. These conflicts make it difficult for groups to work together and diminish the amount of learning that happens. Thus, teachers must find and apply effective ways to handle and address these conflicts.

Because of the difficulties this poses, teachers must develop skills in both conflict resolution and understanding of different cultures.

Language can add to the difficulties teachers have when communicating in a culturally diverse classroom. According to Franklin (2001), language can be a barrier to learning, since learners from diverse cultures might speak different languages which complicates the process of interacting in the classroom. Even when learners speak the same language, different dialects may cause problems in understanding. Besides, when English is the main language used in African schools, learners may feel excluded and unable to take part in class discussions fully (Dimo, Marokoh, & Waswa, 2021). The authors assert that having to learn in a new language puts a strong emphasis on using materials and approaches that support communication and understanding among learners. Along with language, teachers must deal with the different backgrounds of their learners which also complicates teaching.

Dimo et al. (2021) add that for teachers to include culturally important examples in their teaching, they need to devote a lot of effort and time to learn about and value the different cultural backgrounds of every learner. This way of teaching calls for teachers to appreciate the cultural background of their learners and make sure their own cultures are incorporated. Dimo et al. (2021) state that this task might be tough, especially when there are many cultural backgrounds present in the classroom. Teachers are further challenged by the resources that cater for the different cultural perspectives which keeps them and their learners from practising culturally responsive education.

2.7 Research gap

Several ways to teach probability are already well-researched, though the use of argumentation has received little attention. During argumentation, learners develop critical thinking and can share and uphold their ideas. It has been shown to improve understanding in different subject areas. Even so, research on using argumentation in probability teaching is not widespread. Most studies focus either on standard ways of teaching or on other modern strategies, hence the special advantages of argumentation for culturally diverse learners in learning probability are often overlooked.

There is a clear lack of research on teaching probability using argumentation in Zimbabwe multicultural schools, while similar studies have been conducted in other parts of the world. The particular cultural, social and educational context of Bulawayo could play a major role in determining how effective argumentation will be in teaching probability. This study assesses the effect of the argumentation instructional approach in teaching of probability in Bulawayo Central. In this way, the study expands the current knowledge base and supports teaching practices that suit local contexts. This research could provide insights to effecting teaching in culturally diverse contexts and can clarify the role of argumentation in mathematics education throughout the educational system.

2.8 Chapter summary

This chapter gave an in-depth review of the literature dealing with probability, argumentation and culturally responsive pedagogy. The first emphasis was given to understanding the main theories that influence argumentation, concentrating on how they help enhance critical thinking and support knowledge of mathematics. The discussion also investigated earlier studies on teaching probability which revealed that various learners struggle and require strategies adapted to their circumstances.

Then, there was a shift towards highlighting how cultural diversity in schools affects learners' involvement and comprehension of mathematics. Also, the chapter covered challenges related to incorporating argumentation in teaching probability to learners from various cultures. Through the synthesis of these interconnected themes, the chapter established a robust foundation for the present study, highlighting the imperative of incorporating argumentation into probability instruction to better accommodate the educational requirements of learners in Bulawayo Central schools.

This chapter serves as a critical precursor to Chapter 3, which delineates the research design and methodologies employed in this investigation. By identifying existing challenges and gaps within the literature, the methodology chapter introduces a strategic approach aimed at effectively assessing the influence of argumentation on learner engagement and understanding, thereby reinforcing the vital role of culturally responsive pedagogy in the domain of mathematics education.

While the basic principles of argumentation and probability are established in literature, this literature review includes contemporary research to ensure that the study is situated within current global and regional pedagogical shifts. Through this synthesis, this chapter bridges the gap between traditional ethnomathematics and school probability providing the necessary academic currency to support the proposed E-API framework.

CHAPTER 3: RESEARCH METHODOLOGY

3.0 Introduction

This methodology chapter addresses the following research questions that guide the study:

a) What are the academic performance abilities of culturally diverse secondary school learners prior to the intervention?

b) How does the integration of argumentation in teaching probability affect the academic performance of culturally diverse secondary school learners?

c) How does the argumentation teaching approach influence learner's attitudes and motivation towards learning probability?

d) How does the argumentation intervention influence collaboration, and active participation among culturally diverse secondary school learners during probability learning?

The mixed-methods approach employed in this study was designed to address these questions through both quantitative measurement of learning outcomes and qualitative exploration of learners' experiences with the intervention.

3.1 The methodological framework of the study

Figure 3.1 below summarises the methodological framework used in this study. It outlines the important aspects of the approach.

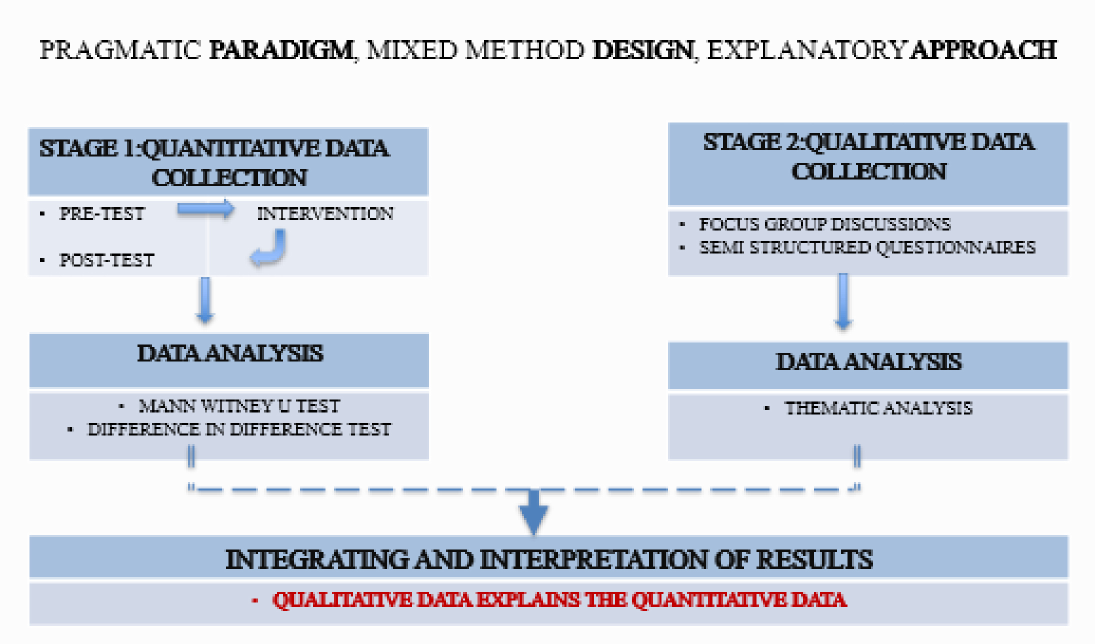


Figure 3.1: Methodological Framework

The flowchart depicted in this figure delineates the overarching methodological framework employed in the study. At the apex of the chart is the pragmatic paradigm, which subsequently transitions downwards through a mixed-methods approach, encompassing a sequential explanatory design, sampling procedures, intervention implementation, data collection methodologies, and analytical techniques. Interconnecting arrows illustrate the relationships that exist among these various components. This visual representation effectively synthesises the primary methodological elements addressed in this chapter, thereby affording readers a comprehensive overview of the interplay between the components as they collectively contribute to the answering of the research questions. A detailed examination of each methodological element will be provided in the subsequent sections.

3.2 Research paradigm

In this study, the research is supported by four interlocking layers that move from the philosophical to the practical. At the foundation is the pragmatic research paradigm, which justifies the use of mixed data to solve the practical problem of probability understanding. To support the paradigm, the sociocultural theoretical frameworks and argumentation theories provide the lens for interpreting the learners' social interactions. The conceptual framework provides the internal roadmap, specifically the E-API model, while the Analytical Framework provides the precise procedural tools, such as the argumentation assessment rubric, Mann Whitney U test and thematic analysis used to evaluate the impact of the intervention and quality of learner discourse.

Creswell and Plano Clark (2018) present a paradigm as a set of beliefs that permits a researcher to examine a phenomenon by acceptable methods. This permits the application of findings in ways that can produce credible results. The authors posit that the paradigm employs practical outcomes as a measure for assessing the worth and accuracy for the research. Pragmatism permits employing both quantitative and qualitative data in one study (Creswell & Hirose, 2024). The blending of methods achieves completeness and sufficiency to the research results (Creswell & Hirose, 2024). When a paradigm is used the researcher considers both the objective and/or subjective perspectives of participants. The use of quantitative and qualitative data permits a deep understanding of the research problem.

The current study considers a pragmatic paradigm as a suitable option as it supports theory with implications for reforming practice (Creswell & Hirose, 2024). This study's main purpose was to evaluate the effect of incorporating argumentation to the teaching of probability to culturally diverse secondary school learners in Bulawayo. The researcher can critically evaluate current practices with the goal of not only describing the nature of the intervention but also determining how to improve it. As a practical, result-focused philosophy of inquiry that identifies what works, the results helped the researcher explore the practice regarding how, in their context, learners in the experimental group responded to the intervention. The choice of a pragmatic paradigm is supported by the fact that it permits flexibility in choosing methods that enable the researcher to effectively respond to the research questions on the effect of teaching probability through argumentation.

The study also uses the pragmatic research paradigm because it is suitable for educational settings that seek to develop practical solutions to problems (Creswell & Hirose, 2024). Pragmatism focuses on using interventions that matter in real life and evaluating their results, rather than sticking to strict theoretical rules (Kaushik & Walsh, 2019; Morgan, 2014). The use of different approaches assists in combining qualitative and quantitative techniques as well as making it easier to explore the effect of argumentation when teaching probability to culturally diverse, secondary school learners in Bulawayo.

3.3 Research approach

Zhang and Creswell (2013) assert that a research approach provides steps, specific methods of data collection, analysis and interpretation. The definition clarifies that the researcher should specify the decisions made regarding the choice of the research method, data collection and the data analysis techniques employed.

*3.3.1 **Mixed methods approach***

Guided by the research paradigm this study uses a mixed method research approach. The mixed methods approach collects quantitative data in one phase of the study and qualitative data for another to understand and address the research questions more fully (Zhang & Creswell, 2013). Using qualitative and quantitative approaches, this approach is flexible in the choice of multiple methods than using each approach separately (Santos et al., 2020). Research supporting this approach assert that the mix of research approaches leads to strong research conclusions as each data set will support the other (Zhang & Creswell, 2013; Green, et al., 2019). The author suggests that mixing qualitative and quantitative results improves the research's validity while also making the research better.

The mixed methods approach has a strength in that it addresses the weaknesses in one method by the strengths of the other (Taherdoost, 2016; Fetters et al., 2013). For this study the mixed methods approach was used because it enabled the collection of a dual set of data on the effects of integrating argumentation in teaching probability to culturally diverse secondary school learners in Bulawayo Central in a way that added value to the study results. For the current study, the need for data triangulation is the main reason for the use of mixed methods, as it makes the findings more accurate and consistent. Zhang and Creswell (2013) believe combining qualitative and quantitative methods leads to findings that are more complete and more precise.

The use of a sequential explanatory mixed-methods design in this study allowed for the resolution of the contradictions between cultural and probability perspectives of the learners. While the quantitative phase provided the breadth by identifying the statistically significant shifts in learner performances, the qualitative phase provided the depth required to understand the why and how of those statistical values. For complementarity purposes, triangulation was used to ensure that the limitations of one data strand were offset by the strengths of the other, to produce a more holistic instructional model.

3.4 Research design

Scholars offer different definitions for the research design. Variation in the definitions shows the multidimensionality of the concept. Zhang and Creswell (2013) assert that it is a logical framework and procedural roadmap that provide the practicalities of data collection and analysis. Taherdoost (2016) and Tobi (2022) provide another view of the definition and assert it is a blueprint that outlines methods and procedures to be used in the collection and analysis of the research data.

Informed by the paradigm and the research approach, this study uses the sequential explanatory design. The design is carried out in two distinct interactive phases as shown in Figure 3.2. Quantitative data has a dominant position and were collected first and it benefits from the qualitative data collection (Johnson, Onwuegbuzie, and

Turner, 2007). In the first phase, quantitative data were collected and analysed to address the research questions meant for quantitative data. The second phase collects and analyses qualitative data that refines and explains the attained quantitative results (Taherdoost, 2016). The analysed data were then combined during interpretation (Ivankova et al., (2006). According to Zhang and Creswell (2013) this approach is characterised by its use of the qualitative result to explain the quantitative results. This explanatory design seeks to explain quantitative results using follow-up qualitative results (Zhang & Creswell, 2013). The author adds that the approach's strength is its straightforwardness, clear and separate stages though the implementation of the design takes. The following figure illustrates the sequential explanatory design as employed in this research.

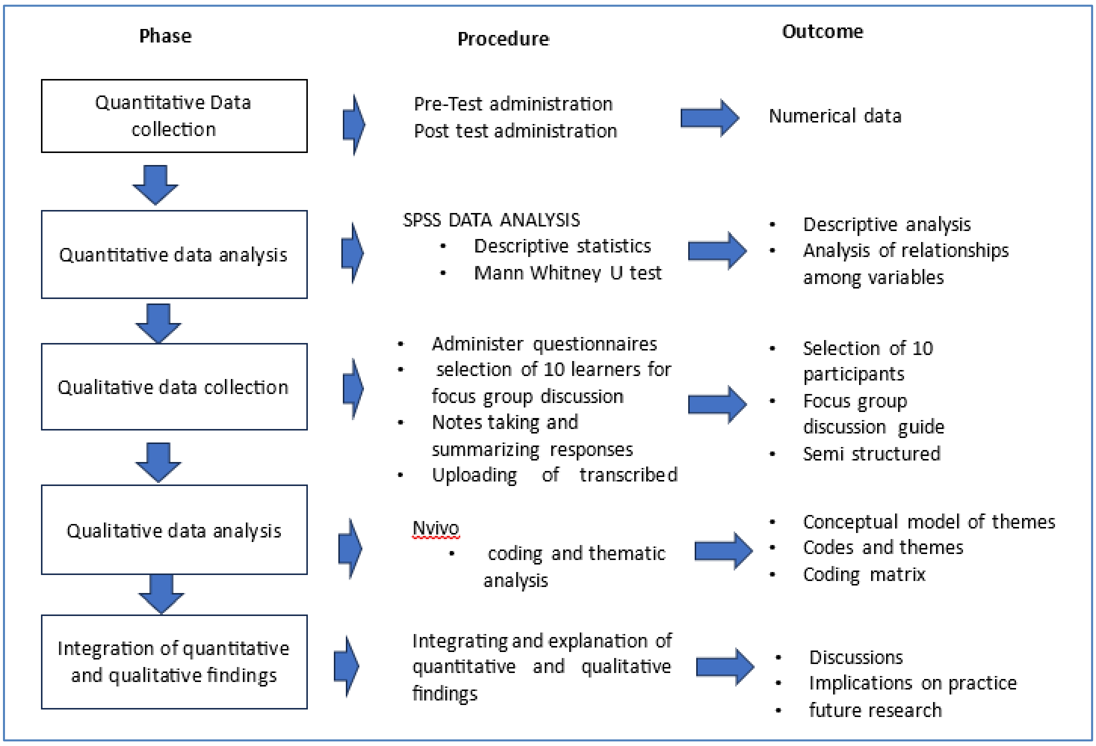


Figure 3.2: Sequential explanatory data collection and analysis: Researcher's own illustration: *Researcher illustration.*

Figure 3.2 shows the intervention process: pre-test → intervention → post-test which was used in the collection of quantitative data followed by the collection of qualitative data through semi-structured questionnaires and focus group discussions. Data from both types were analysed separately and the integration of the data occurred during the interpretation process. The integration made it possible to compare data from different sources which strengthened how reliable the findings were, as mentioned by Zhang and Creswell (2013). After establishing the research design, it was important to find out which population and sample would be needed for data collection. The next sections explain the steps and decisions taken when sampling.

3.5 Population, Sample and Sampling procedure

3.5.1 ***Population***

The target population comprised 40 secondary school mathematics learners in two classes at the selected school in the Bulawayo Central district. The cultural composition of the population included learners from Ndebele, Shona, Kalanga, Tonga, and mixed descent communities, consistent with the multicultural profile of Bulawayo urban (Mapuranga & Bukaliya, 2014). Since the population was small and accessible, all 40 learners were included in the study. Because of the variety of cultures in the region, the researcher managed to examine questions related to argumentation in this multicultural mathematics context.

A range of considerations led to the choice of the specific school in Bulawayo Central out of the ten high schools that offer secondary school studies. Being a teacher at the school allowed the researcher to easily access, communicate with participants, access needed resources and collection of data which made the research method more convenient. Concentrating on a single school meant a more unified and orderly environment which was very convenient for carrying out the intervention and checking its outcomes. This design greatly decreased the chances of encountering problems that could result from using multiple schools with different characteristics.

Because the study was limited to one location, the researcher managed to study the local context in-depth. A thorough understanding of the cultural context likely produces rich qualitative data which helps in understanding the outcomes of the research (Morgan, 2014). The close connection with the school facilitated collaboration with the administrators and that enabled the successful implementation of the argumentation strategy.

3.5.2 ***Sample and sampling procedure***

The population consisted of two classes each with 20 learners. Hence, a census approach was adopted ensuring that every member of the group participated in the study, providing complete representation and eliminating choice bias. The sample of 40 mathematics learners comprised intact groups whose cultural and linguistic backgrounds reflected the diversity of the Bulawayo Central district. Table 4.1 in Chapter 4 provides the detailed breakdown of participants by cultural group for both the experimental and control groups. The cultural groups represented include 18-Ndebele, 8-Shona, 2-Sotho, 1-Venda, 4-Kalanga, 5-Tonga, and 2 learners of mixed cultural heritage. This diversity was a deliberate feature of the study design, allowing the investigation to examine argumentation in an authentically multicultural classroom context. The focus on Mathematics learners was consistent with the purpose of the study, which sought to examine learning and reasoning within the mathematics context. However, to ensure adequacy of the 40 learners, a G *Power 3.1 was used to do a prior power analysis to check on adequacy of the sample size (Cohen, 1988; Palinkas (2014). The parameters chosen were an alpha level (α) of 0.05, a power (1-β) of 0.80 and a medium effect size (d = 0.50) for the Mann Whitney U test. The calculation yielded that at least 34 participants, split equally into 17 participants per group, were necessary. However, a sample of 40 secondary school learners was used, since the participants were studying statistics and Pure mathematics which had probability as a topic.

As a result of the attained power of 0.86, the outcomes are more statistically reliable (Taherdoost, 2016). Also, the sample size of 40 participants is in line with the recommendations by Staffa and Zurakowski (2019) which suggest using a minimum of 40 participants when interventions are designed to measure variations in learning. Also, the sample size is consistent with other studies on argumentative interventions in mathematics education which found similar medium effect sizes when using comparable sample characteristics (Gamarra-Moncayo & Prada-Chapoñán, 2025).

As an insider researcher employed as a mathematics teacher at the study site, it is important to note that the researcher taught the two classes during the intervention period. This is acknowledged as a potential source for researcher bias and was mitigated through specific measures which included standardised lesson scripts, independent test marking and member checking.

3.5.3 Assignment to experimental or control group

Cluster randomisation was applied to assign the either group to the experimental or control group through a computer-generated sequence. Several organised steps were implemented and involved:

1. Allocating each class, a number (1 or 2) as its identifier.
2. Using the Microsoft Excel RAND function to get a random number between 0 and 1.
3. The group assigned as experimental was number 1, only if the random number generated was less than 0.5. If the random number generated was 0.5 or higher, the class was labelled as the control group (group 2).

A teacher who was not involved in the study was requested to perform the randomisation process to avoid assignment bias (Hennink et al., 2020). The Head of the Mathematics Department was present during this randomisation process for verification. This was done to ensure the process was carried out fairly and properly. The process is based on the most effective practices used in educational research (Johnson et al., 2007). The authors argue that the strict use of randomisation made it unlikely that researchers' expectations or past differences among the learners could have affected who was assigned to the conditions.

3.5.4 Control and Experimental Groups' instructional approaches

In order to safeguard the internal validity of this quasi-experimental design and to account for the Hawthorne effect (Stratton, 2021), some control measures were implemented. As an insider researcher, the treatment was delivered through a standardised instructional script that minimised bias that could be introduced by the researcher's teaching style. For the control group, a placebo condition was established where the learners were taught through a structured traditional teacher-centred series of problems. This ensured that the observed experimental group gains were a true reflection of the argumentation-based pedagogy. To mitigate the Hawthorne effect and ensure the control group did not feel disadvantaged or deceived, participants were informed at the outset that the structured traditional instruction they received formed an essential and equally valued component of the study design. At the conclusion of the study, learners in the control group were provided access to the argumentation-based instructional materials and a debrief session, addressing any perceived inequity and fulfilling the researcher's ethical obligation to all participants (Stratton, 2021).

3.5.4.1 Experimental Group Instructional Approach

The argumentation instruction approach in the experimental group focused on integrating argumentation into probability lessons. The structure of the activities supported teamwork and included learners from culturally diverse backgrounds to allow different viewpoints to interact. Each activity began with a scenario that connected to the learners' personal perspectives and cultures. During discussions, facilitators encouraged learners to share their thoughts and to consider ideas from others. Learners participated in simple experiments, such as the rolling of dice or drawing cards, to gather data for later discussion. Following each activity, learners concluded with reflective discussions to consider their discoveries and the arguments presented. Through group reasoning and cooperation, the objective was to promote critical thought and to assist learners in understanding probability concepts more fully.

3.5.4.2 Control Group Instructional Approach

The way the mathematics curriculum on probability was taught to the control group matched the established framework for secondary school mathematics. It involved several well-defined components.

1. Probability was taught mainly by lectures, illustrating concepts with examples from the textbook and often a chalkboard.
2. Learners observed probability problems being systematically solved by the teacher and then practised similar problems themselves.
3. Learners were given homework from the textbook and afterwards, the teacher gave correct responses and detailed explanations.
4. Regular exercises and homework ensured learners practised using probability formulas and building procedural fluency. While a few informal discussions occurred in the classrooms, the control group's teaching method did not involve planning activities around argumentation, critiquing others' ideas or teaching how to build mathematical arguments. Both groups had the same number of lessons, each lasting forty minutes and they covered the same material on probability. What separated the groups was the type of instruction used to teach them probability.

3.5.5 Pre-Intervention procedures

Before the intervention, some actions were carried out to help participants fully understand the study and be well prepared for involvement. The research activities were centred on making the research objectives clear, emphasising that argumentation should be a part of teaching probability. It pointed out that being involved in the process can support improved learning practices.

The first step in the pre-intervention framework was setting up ground rules. It was necessary to encourage a positive and helpful environment during the discussions which would help build an inclusive setting for everyone's opinions. Since Bulawayo Central is very culturally diverse, this approach was necessary to help participants feel welcome and encourage open discussions among the children. The researcher drafted the following guidelines to ensure the intervention was carried out effectively.

3.5.6 Classroom discussions guidelines

Learners were encouraged to take part in discussions and openly speak about what they think. Learners were discouraged from teasing each other to encourage a respectful classroom. In cases where learners disagreed, everyone had to show their reasons with correct facts and interact in a respectful way. They were also encouraged to express the reasons why they agreed or disagreed. The focus was on using evidence to explain and support one's opinion or challenge the opinions of others. Learners were encouraged to talk openly in the discussions and express their own opinions. It was strongly emphasised that no one should make fun of others to maintain a respectful atmosphere during lessons. If the learners disagreed, they needed to back up their arguments with proper evidence and respond with respect towards the opposing side. It was also suggested that learners should spell out the reasoning behind their points of agreement or disagreement. Evidence and facts were central in the discussions to justify one's arguments or to handle arguments from others.

3.5.6.1 Respect for others' contributions

It was determined that all learner input would be respected, no matter who made it. It was explained that no answer is truly right or wrong which allowed everyone to discuss freely. Learners were prompted to offer logical explanations for their opinions and were directed to use evidence to defend their ideas or challenge different opinions. In addition, learners were informed that if they thought their arguments were weak, they could adjust their positions.

3.5.6.2 Making contributions

Encouraging all learners to contribute, even if their ideas seemed insignificant, was important, as those ideas could join forces and become strong arguments. An emphasis was placed on making sure the atmosphere was safe, and everyone was treated with respect, and their opinions were valued.

3.5.6.3 Participation rules

Those involved in the discussions needed to listen carefully to keep the conversations smooth and unbroken. The focus was on the idea that the main aim of the study was to support collaborative learning.

3.6 Experimental group's Argumentation training

To enhance argumentation skills, participants were introduced to a special programme that taught them the necessary argumentation skills. The main goal of this programme was to explain the foundations of argumentation, improve critical thinking skills and stress the need for evidence-based thinking in mathematics. The activities were guided by prompts in Annexure M.

The purpose of the structured activities was to improve the participants' skills in presenting, defending and analysing arguments in probability. In this training, learners were asked to practice using the ThinkCERCA methodology which lists the main elements as Claim, Evidence, Reasoning, Counterargument and Audience. The steps of this approach are shown in Table 3.1 below:

Table 3.1: ThinkCERCA approach for teaching argumentation

Stages	Example
It starts with an essential question	Should robots be used to take care of the elderly?
Development of a Claim **Learners develop a claim in response to the question**	"Robots **should** be used to take care of the elderly"
Evidence **Learners look for credible evidence to support the claim**	"There are robots that can lift a person
Reasoning **Learners connect the evidence to the claim with reasoning**	"Which means robots could help if an elderly person fell down"
Counterargument **Learners consider other points of view and explain why this claim is strong**	Some people say that robots **should not** be used to take care of the elderly because they are expensive
Audiences Learners consider the audiences and use appropriate language while writing their formal arguments	In conclusion: robots should be used to take care of the elderly

Adapted from ThinkCERCA. (n.d.). ThinkCERCA: A Framework for argumentation. ThinkCERCA

The framework presents a systematic methodology for the formulation of well-reasoned arguments, thereby facilitating the clear and persuasive presentation of ideas by learners. As detailed in Table 3.1, the implementation of the CERCA approach served as the instructional foundation for elucidating the process of argumentation.

In conjunction with the ThinkCERCA approach, Toulmin's Argumentation Pattern (TAP) was employed to further delineate the structures underlying the argumentation process. TAP delineates key components of argumentation, which encompass the claim, grounds (evidence), warrant (justification), backing, rebuttal, and qualifier. By engaging learners with these components, the training sought to develop their ability to construct coherent and logical arguments, particularly in the context of probabilistic reasoning. Collaborative activities during training sessions provided learners with opportunities to practice argument formulation grounded in probability problems, engagement in peer review, and exchange constructive feedback. This collaborative framework not only reinforced essential argumentation skills but also fostered a sense of collaboration among participants, an important aspect in culturally diverse classroom settings.

3.6.1 ***Argumentation Training Activities***

To develop the learners' abilities to argue, three 35-minute group exercises were implemented. The argumentation exercises were based on the following topics: "*Is digital technology good for children?*", "*Are learners entitled to money for attending school*?" and "*Is reading better than watching television*?". All the activities were designed to help learners enjoy talking and focus on the lesson. The topics were general such that no cultural group of learners could feel offended discussing the topic. Each learner was assigned to a group of three participants and given a topic to engage in a discussion as group members to encourage focused discussions and teamwork. The facilitator assisted the groups in their discussions to ensure they thought deeply about cultural viewpoints involved through probing prompts. If learners found it difficult to talk about their claims, facilitation guided them to consider their experiences and cultures which helped them form claims based on where they came from. Every group had to form a claim, support it with proof and give reasons for their argument. They were prompted by the facilitator to use proper language to consider the group membership, the facilitator and the class in general. This encouraged learners to consider their audience when making contributions and made them more careful in what they discussed. Each group discussed their assigned topic from both perspectives (supporting and/or opposing) which made sure learners understood the topic well and accepted different perspectives. After presenting, groups were asked to express new opinions and ideas which helped keep the discussion on topics alive and allowed everyone to participate fully.

Having argumentation skills is vital for learners in schools with culturally diverse classrooms. This helps them deal with matters involving cultural ideas and probabilities. Training approaches in this study were based on the ThinkCERCA method (Claim, Evidence, Reasoning, Counterargument and Audience) and Toulmin’s Argumentation Pattern (TAP). The way the ThinkCERCA framework is structured shows a structured way to teach argumentation, making critical thinking and effective communication key aspects. The approach was used in all three topics used for training and the different stages were adjusted to include ideas about probability and culture. Besides using empirical data, the Claim stage also relied on cultural beliefs about handling risk and uncertainty. As an example, when asked "*Is digital technology helping children?*", learners were inspired to think about the common views on technology in their cultures, since these often shape their understanding of what technology brings. Because of this, learners could express ideas that reflected their own lived experiences and culture.

At the Evidence stage, the focus was on finding helpful cultural data sources and combining statistical facts with perspectives from personal experiences. When addressing the question "Should learners get paid for attending to school?", learners were guided to use their local customs and economic conditions to guide their opinions. Having two kinds of evidence made the discussion go deeper and kept the argumentation strong. Researchers often point out how background and culture affect the way learners think about probability and risk. Adding these cultural viewpoints greatly improved the usefulness and relevance of the learners' arguments.

The Toulmin's Argumentation Pattern uses six categories which are Claim, Grounds, Warrant, Backing, Qualifier and Rebuttal these help in the analysis of arguments. While groups were presenting their ideas, TAP elements were used in the analysis of learners' arguments. Once a group finished its presentation, participants broke it down using TAP, identifying what the group claimed, what data (evidence) it provided and the reasons (warrants) that supported the data. This analytical process yielded a clearer understanding of how learners constructed arguments and the role of cultural perspectives therein. In discussions surrounding the prompt "*Is reading better than* *watching television*?" the warrants articulated by learners revealed underlying cultural values relating to literacy and education. The rebuttal stage further allowed learners to critically evaluate counterarguments based on divergent cultural experiences with media consumption, thus enriching the discussion and promoting critical engagement with opposing viewpoints. An annexure containing the prompts employed during the group activities is provided for further examination of the methodologies utilised within this study (Annexure M).

3.6.2 Importance of the argumentation training scenarios

The three intervention scenarios were selected for their propensity to engage learners with common perceptions of probability and their relevance in developing argumentation. The first scenario focuses on digital technology, which relates to the ongoing debates on the effect of technology on young people. It is, therefore, relevant to current discourses as it incorporates the learners' social contexts, allowing them to relate to their daily lives. Additionally, this scenario allows learners to consider a variety of views regarding the benefits and drawbacks of technology, each grounded cultural and individual beliefs. The second question of being paid for attending school involves two issues: economic concerns and equity in education. Each issue represents concerns for most communities. This question invites learners to estimate the probability of events associated with issues of financial motivation in education; hence, encouraging critical thinking about socioeconomic elements surrounding their school experiences while improving their argumentation skills. In the third scenario, learners reflect on the relationship between media consumption, culture and literacy. It should be appropriate when it helps learners compare the chances of several educational outcomes based on the media they experience and their cultural backgrounds.

3.6.3 Facilitation and assessment strategies

How argumentation training was facilitated was designed to encourage participants to engage and discuss the topic actively. Several techniques were used by the facilitator to help the discussions remain focused. The main method was supporting learners to look at their own experiences and cultural situations when making arguments and providing evidence. It made the arguments more engaging and increased everyone's knowledge about probability. As the training went on, formative assessments were used to give ongoing feedback as the discussions took place. As a result of a peer review process, individuals became more critical of their own arguments and practised observing others' arguments. Here, the facilitator made use of different questions, prompts (Annexure M) and a rubric (Annexure N) to help the participants structure their arguments. Critical reflection was encouraged in peer review by asking learners questions such as: What argument is the author making and how clearly is it presented? Are there any facts or proof to back up the claim and are they taken as meaningful and reliable? How does the argument deal with possible counterarguments?

The questions were carefully planned to help learners think deeply and communicate constructive opinions with their classmates. Also, questions that pushed for more careful thought about their arguments asked: What are the assumptions behind your claim? What cultural viewpoints might exist about this argument?

A detailed rubric (Annexure N) was used to judge the effectiveness of the arguments presented by learners. It included several aspects, for example, clarity, evidence, sound reasoning, evaluation of opposing arguments and overall organisation. Every dimension was scored between 1 and 4 and descriptors were given to explain each score.

Simply put, the scenarios chosen for the study were based on what the learners experience and believe which fits with the main research questions. The intervention used certain ways of facilitating and assessing activities to encourage participants to think critically and form well-supported arguments.

3.7 Experimental group's argumentation intervention

The purpose of the intervention was to improve how learners in Bulawayo Central comprehend probability by using argumentation techniques. A complete explanation of argumentation and its uses was given to the learners. Emphasising argumentation, the facilitator guided learners to defend their ideas by citing evidence and using reason.

A conducive environment for learning was created and learners were encouraged to feel included and to share their thoughts. Learners were advised to use respectful language and really listen to and value everyone else's opinions. In addition, the argumentation tasks designed by the researcher gave learners an opportunity to analyse, assess and structure arguments about probability concepts that were important and meaningful to them. A complete explanation of argumentation and its uses was given to the learners. Emphasising argumentation, the facilitator guided learners to defend their ideas by citing evidence and using reason.

In order to facilitate collaborative argumentation, learners were placed into 5 groups of 4 members each. The members of each group were deliberately placed into each of the groups ensuring that each cultural group is represented in that group. Each group had at least 2 minority cultures, minority learners are spread and no group was dominated by one cultural group. These group formations served the purpose of encouraging discussions and debates that explored a variety of perspectives, scrutinised underlying assumptions, and contributed to the formulation of coherent arguments. The argumentation tasks were adapted from Sharma (2014) and were aligned with the specifications outlined in the respective lesson plans (Annexure HA, HB, HC). The structured approach enabled a deeper engagement with the content while simultaneously fostering critical thinking skills among the learners.

*3.7.1 **Intervention activities***

The following activities were implemented in the proposed intervention. Possible explanations of each scenario by learners, sources of perception, expected outcomes, and relevance of the scenarios are outlined.

Scenario 1: Dice Game

In the scenario provided, three individuals - Johab, Noma, and Thabo-exhibit distinct belief systems that influence their perceptions of luck and success. Johab adheres to a Christian faith, which leads him to assert that divine intervention governs his actions. Conversely, Noma’s conviction lies in cultural practices, wherein she possesses a charm that symbolises her faith in its protective power. Thabo, on the other hand, subscribes to the notion that number six constitutes his personal emblem of fortune (his lucky number).

The three meet up at a carnival, where they take part in a game centred on rolling a standard six-sided die. Players must get a score of six to win or any other result causes them to lose. Before they went to the carnival, Johab prayed for help, whereas Noma pulled out her charm hoping it would give her an advantage.

By including all the different thoughts on luck, this scenario demonstrates how they can affect how a probabilistic game ends. The assignment for learners is to determine the most likely winner among the three - this judgement is affected by Johab's faith, Noma's trust in charms and Thabo's lucky number. The discussion encourages the learners to investigate how probability can be affected by cultural perspectives.

The expected responses from the three individuals about chance are shown in Table 3.2 below:

Table 3.2: Anticipated explanations for lesson 1 scenario

Learners' explanation	**Source(s) of the explanation**	**Outcome**	**Relevancy to the study**
That 6 being Thabo's lucky number, he is likely to win. That God controls everything hence, Johab is likely to win That charms lead one to success hence Noma is likely to win	*Beliefs systems:* Religious background Cultural beliefs	Since there are 6 possible outcomes and the probability of a six showing is one out of six. Hence, the probability of a win is $\frac{1}{6}$	Cultural Beliefs in luck and religion reveal common misconceptions about randomness.

The scenario was designed to elicit potential perceptions about randomness and explore whether cultural beliefs about luck and religion affected probability reasoning.

Scenario 2: Cultural Belief vs. Probability

In the second situation, Teddy and Laro were walking during a thunderstorm. Teddy wore a dark shirt while Laro wore a red T-shirt. Participants were instructed to determine who was more likely to be struck by lightning. Here, the scenario links beliefs about lightning with the idea of probability which means learners needed to review the risk of lightning strikes. The discussion resulting from this scenario allowed learners to discuss how probability and culture are connected and different. The expected reactions of participants are shown in Table 3.3.

Table 3.3: Anticipated explanations for lesson 2 scenario

Learner's explanation	**Source(s) of the explanation**	**Outcome**	**Relevancy to the study**
i) Laro is likely to be struck because of wearing a red T-shirt when it's raining. **ii) They were walking in the open during a storm**	Cultural beliefs that lightning strikes people wearing red clothing. That Laro was bewitched.	The probability of being struck will largely depends on the historical background of how many people are struck by lightning in their area given the population of that particular locality during the same period. Hence, additional information is required.	Beliefs in lightning strike influence probability perceptions, hindering learner understanding.

The scenario was designed to elicit potential perceptions about randomness and to further explore whether cultural beliefs about lightning influenced probability reasoning.

Scenario 3: The Fairness Game

The third question was formulated based on a dice rolling game and reads: Two girls, Eisha (8 years old) and Sarah (12 years old), decided to participate in a dice-rolling game. They alternated rolling two fair dice and computed the difference (larger number minus smaller number) between the shown numbers. If the difference of the scores was 0, 1, or 2, Eisha won. If it was 3, 4, or 5, Sarah won. The learners were then prompted to elucidate whether the game was fair based on their individual perspectives and understanding. The anticipated responses are illustrated in Table 3.4.

Table 3.4: Anticipated explanations for lesson 3 scenario

Learner's explanation	Source(s) of the explanation(s)	Outcome	Relevancy to study
Some may not see anything wrong with the game. **Others will hold onto the belief that one number is just luckier than the others.** **Some may base their arguments on age, Eisha being 8 years old and Sarah being 12 years old may be considered wiser than Sarah.** **Some may think that there are six possible scores and three have been assigned to each player hence the game is fair.**	Beliefs on luck Cultural beliefs that when one is older, he or she may be wiser They may be informed by the notion of equi-probability	There are 36 possible combinations, 24 for Eisha and 12 for Sarah. Some learners will realise that the game is not fair since one of the two players (Eisha) wins more times. In this context, "fair" means that each player has the same theoretical chance of winning the game. (It is assumed that the dice being used to play the game are themselves fair.) Some learners may have in-school beliefs about the notion of fairness while at the same time holding contradictory beliefs about this notion in out-of-school situations	Cultural perceptions about such as luck, respect for elders and equiprobability reveal common notions about randomness in learners which influences probability thinking

The aim of this scenario was to prompt the learners' understanding of probability language and links to their pre-existing knowledge of specific probability terms, notably the concept of 'fairness,' which holds unique meanings in everyday language as opposed to probability.

3.8 Example lesson structure

Three intervention lessons were implemented, with only one presented here to exemplify the implementation of all three, as they followed the same design outlined below. This lesson structure (Annexure HB) as detailed here resulted in the development of a refined **Enhanced-Argumentation based Probability Instruction Model (E-API);**

Lesson Plan 2: Probability and Cultural Perspectives

Objective: Learners will understand the concept of probability and apply argumentation skills to analyse the scenario of Teddy and Laro walking during a thunderstorm, considering cultural perspectives.

Introduction (10 minutes)

The lesson began with a discussion on the concept of probability and its significance in daily life. Learners will be prompted to identify scenarios where probability is involved, such as weather predictions, sports outcomes, or games.

- The teacher will then introduce the scenario of Teddy and Laro walking during a thunderstorm, with Teddy wearing a dark shirt and Laro wearing a red t-shirt.

- Learners will be prompted to consider which of the two, Teddy or Laro, has a higher likelihood of being struck by lightning and to provide a brief rationale for their initial reflections.

Group discussions and argument development (15 minutes)

- Learners will be divided into four groups and tasked with deliberating on the scenario from a single perspective chosen by the group members. They will construct arguments to justify their opinions on who is most likely to be struck by lightning.
- Learners will be encouraged to consider cultural beliefs, superstitions, folklore, and scientific reasoning in their arguments.
- They will be reminded to consider the elements of TAP (claim, grounds, warrant, backing, rebuttal) while formulating their arguments.

Group presentations and peer feedback (20 minutes)

- Each group will present their arguments and cultural perspectives to the class, followed by peer feedback sessions to encourage constructive discussions.
- Learners are encouraged to actively listen and respect questions from other groups during the presentations.
- After each presentation, the teacher will facilitate a constructive feedback session where other groups provide feedback on the strength of the arguments, evidence used, and cultural perspectives shared.
- Critical engagement, open-mindedness, and understanding of different viewpoints during the feedback session will be encouraged.

Whole-class discussion (10 minutes)

- Lead a whole-class discussion to synthesise the arguments and cultural perspectives presented. Encourage learners to reflect on the various viewpoints shared and to consider how different cultural beliefs and scientific reasoning intersect in the context of the scenario. Emphasise the importance of critical engagement and open-mindedness in understanding and appreciating diverse perspectives.
- Discuss the elements of TAP in the arguments, highlighting how each group supported their claims with grounds, warrants, and, when applicable, backing and rebuttals.
- Explore the similarities and differences in cultural perspectives and analyse how they influenced the learners' arguments, emphasising the role of TAP elements in supporting their claims.
- Encourage learners to reflect on the role of cultural beliefs and scientific reasoning in probability assessments.

Reflection and individual writing Task (10 minutes)

- Learners should work on an individual assignment that invites them to consider the lesson material as well as the way they relate to their own culture. Those involved are told to assess the influence of their cultural background on their thinking and reasoning.
- Learners are encouraged to examine how cultural beliefs and science interact, mainly in the context of figuring out probabilities.

- The overall aim of this task is to help learners describe their thoughts about the situation, focusing on the need to consider different cultural opinions when considering the chances of an outcome.

Conclusion (5 minutes)

It pointed out the main features of arguing, the cultural viewpoints and how TAP elements can be used to predict probabilities. A careful study of these aspects was done to see how they affect the evaluation process. It was also stressed that decision-making relied on careful thinking, evidence-based reasoning and an open-minded attitude when dealing with probabilities in several situations.

Learners were invited to use their personal experiences to understand the connections between culture and probability. Using argumentation, learning about various cultures and considering TAP helped both critical thinking and the study of different opinions. As a result, learners gained a clearer view of how culture impacts assessments of probability which also helped them become more aware of cultures and better at arguing.

The results of the arguments in the lesson differed greatly because of the strength of the arguments, how well the information was communicated and the learners' willingness to accept different opinions. The following three outcomes were systematically identified and analysed.

Table 3.5: Results on who is likely to be struck by lightning

Facilitator Prompts	Group Two presentation
Question:	**Claim**: Laro had the highest probability of being struck by lightning.
What evidence can you site to support your claim?	**Grounds**: wearing a red shirt attracts lightning due to the colour symbolism in our cultural beliefs or folklore.
What logical connection is there between the evidence and claim?	**Warrant**: The logical connection between the evidence and the claim is that the cultural beliefs surrounding the colour red and lightning makes it more likely for Laro, wearing a red shirt, to be struck.
What additional evidence or examples can you give to reinforce the claim?	**Backing**: from cultural contexts there are traditions that associate the colour red with lightning strikes. This was supported by a learner in group One who said, "*when raining their parents always hide red items, indicating the danger it posed during rains*".
Is there any evidence we can use to challenge the claim?	**Rebuttal**: lightning strikes often occur if learners walk or stand in the open when it's raining or are standing under tall objects such as trees. Hence, this was the case with the two who were walking in the open while it was raining.

Table 3.5 shows the results of the discussion that raised three important outcomes as explained below.

Considering whether someone is likely to be struck by lightning shows how important it is to look at cultural and scientific beliefs together to understand probability. Learners presented their arguments based on either cultural beliefs or scientific research. From Group Two's presentations it is shown that it is important to recognise and involve various perspectives in discussion about probability.

The use of counterarguments in group settings and feedback sessions was helpful for analysing the assertions and the evidence offered. It led to learners becoming better thinkers and able to form strong objections to opposing ideas. The in-depth arguments allowed learners to adjust their arguments and consider other viewpoints which helped them improve their reasoning skills. One of the clear observations from the discussion was that opinions differed about which learner was most likely to be struck by lightning, Group Two suggested, "*maybe they were bewitched*..." This commentary highlighted the fact that there are many aspects to understanding probability and learners from different cultures may offer different explanations to probability situations.

The results emphasise the role of open and respectful conversations about differing opinions, even when reaching agreement is difficult. Though no unique agreement was reached, the process of arguing about probability helped learners understand how probability is connected to their cultural background. This process assisted in realising the importance of culture in understanding probabilities, while emphasising the need for critical thinking when evaluating arguments. While it is important to pay attention to cultural perspectives, it is also important to consider the scientific and mathematical methods to confirm claims. In Zimbabwe, this balance matters a lot since examinations usually consider the scientific viewpoints when assessing learners.

3.8.1 ***Experimental group's argumentation practice outcomes***

The pre-test showed that both groups lacked skills in arguing effectively. For this reason, the experimental group learners were involved in argumentation using the CERCA method, particularly by discussing the question, "*Should we trust weather forecasts when we organise outdoor activities?*" This question prompted learners to come up with claims and reasons, using old weather records and judging the reliability of forecasting methods which involved using probability.

During discussions learners were placed into groups and then shared their ideas as a group. They were guided in creating arguments, using claim, evidence and reasoning which they practised on probability-related questions. As the weather forecasting question was discussed, the development of learners' argumentation skills was evaluated. Besides, this assignment allowed for the exploration of cultural ideas about predictive practices related to formal probability theory.

Using probabilistic activities, learners gained experience in both probability thinking and basic arguing which would be key for future applications. Because the scenario mentioned the local weather in Bulawayo, it was easier for the learners to connect with the material. Starting with an introduction to weather forecasting and what it means in real life, the dataset was introduced, followed by a discussion of the task which was to assess how accurate the weather forecasts were for Bulawayo during the past week.

During the activity, critical thinking, reasoning with evidence and constructive argumentation were all strongly emphasised. Learners were required to assess the accuracy of the weather forecasts for Bulawayo from the data provided in Table 3.6. The scenario linked learners' everyday observations about weather variation with formal probability ideas and gave them a chance to discuss how traditional beliefs affect forecasting.

Table 3.6: Weather dataset

Day	Weather condition
Day 1:	• Forecast: Sunny • Actual Weather: Mostly sunny with brief showers in the afternoon
Day 2:	• Forecast: Cloudy with a chance of rain • Actual Weather: Overcast with heavy rain throughout the day
Day 3:	• Forecast: Partly cloudy • Actual Weather: Clear skies with no clouds
Day 4:	• Forecast: Thunderstorms • Actual Weather: Thunderstorms in the evening, otherwise mostly sunny
Day 5:	• Forecast: Rain showers • Actual Weather: Intermittent rain showers throughout the day
Day 6:	• Forecast: Sunny and hot • Actual Weather: Sunny and hot
Day 7:	• Forecast: Cloudy • Actual Weather: Cloudy with occasional drizzles

The learners were told to make claims about how accurate weather forecasts are, with all claims requiring justification from information in the dataset. The exercise involved analysing the weather and prediction systems. Hence, the initiative led to detailed discussions about the uncertainty in forecasts, causing learners to reconsider whether predictions can be seen as definite outcomes.

Working together, group members realised that, despite their inaccuracies, forecasts give useful probabilities for making decisions. Analysing and comparing the data probably revealed that the predicted outcomes and the actual weather were not always the same, which helped scientists better understand the unpredictability of weather forecasts.

By asking thoughtful questions, the teacher encouraged learners to think critically. Some of the questions asked were, "*How can cultural beliefs about weather change* *the way learners interpret these reports?", "how can we utilise probability calculations to support or refute claims regarding forecast reliability?" and "How much weight should we give to weather forecasts when organising outdoor activities*?" Such engagements served to deepen the learners' critical evaluation of the reliability and implications of weather forecasting.

3.8.1.1 The argumentation outcome on reliability of weather forecasts

Using the dataset presented in Table 3.6 above, learners engaged in argumentation discussions to evaluate the accuracy of weather forecasts and presented evidence-based arguments. They deliberated factors such as historical weather patterns, the reliability of forecasting methods, and personal experiences to support their claims. The following are the arguments articulated by the learners for the weather forecasting assignment on whether to "*trust weather forecasts when we organise outdoor activities?*"

Table 3.7: Arguments and counter arguments for the weather forecast scenario

Group	Argument	Counter argument
1	Weather forecasts are generally accurate because on Day 1, the forecast was "sunny," and although there were brief showers, the majority of the day was indeed sunny.	
2		Weather forecasts are unreliable because on Day 2, the forecast was "cloudy with a chance of rain," but it rained heavily throughout the day. This indicates that the forecast was inaccurate and misleading.
3	Weather forecasts are influenced by various factors, such as local microclimates. On Day 3, the forecast was "partly cloudy," but the actual weather was clear. This could be attributed to a localised weather pattern specific to that area.	
4		Weather forecasts are generally accurate as on Day 6, the forecast was "sunny and hot," and the actual weather matched the forecast. This demonstrates the reliability of the forecasting methods used.
5	Weather forecasts provide useful information, but they are not always precise. On Day 7, the forecast was "cloudy," and although there were occasional drizzles, the overall cloudiness was consistent with the forecast.	

Table 3.7 presents the analysis of what transpired on that day and each group made conclusions guided by the data. This facilitated significant engagement and fostered critical reflection about the reliability and limitations inherent in meteorological predictions. Learners demonstrated active involvement in constructive exchanges, articulating evidence and reasoning to validate their respective positions while also acknowledging the cultural contexts and personal experiences relevant to weather forecasting practices in Bulawayo.

3.8.1.2 Discussion of the results in Table 3.7

At the beginning, the groups were presented with the weather forecasting data which they then studied, discussed and arranged their ideas, made presentations and received feedback. It was important for formative assessment to show learners how to use words suggesting probability, make logical links between evidence and claims and consider different ways learners interpret the same evidence. Besides receiving advice from teachers, learners evaluated each other's arguments using a rubric that looked at how clear the claims were, how evidence was included and how learners valued other opinions. This way of learning assisted learners to develop arguments argumentation skills. The strategy adopted for this discussion involved small group discussions before involving the whole class, to motivate shy learners to join in.

3.8.1.3 Arguments about Day1 prediction

Group One learners contended that, notwithstanding the forecast predicting clear skies on Day 1, the actual conditions were characterised by rainfall. The Group One posited that the precipitation could be attributed to a rainmaking ceremony conducted at Njelele in Matobo, wherein elders sought invocation from ancestral spirits for rain. Conversely, Group Three dismissed the correlation with Njelele, instead asserting that the observed rainfall was the result of cloud seeding operations. Observations of a small aircraft traversing the skies above Bulawayo were interpreted as indicative of such cloud seeding efforts, thereby explaining the precipitation.

Because of these discussions, Group One concluded that the forecast was valid and outdoor activities planning could rely on weather forecasting regardless of whether the rain started by itself or was artificially induced.

3.8.1.4 Assessing the argumentation quality of the prediction

Analysis of learners' arguments using TAP showed that several learners needed guidance in finding scientific evidence for forecasting, since many started by referencing personal experiences or cultural beliefs. Frequently, learners tried to explain weather by using cultural ideas and beliefs about seasonal changes. For instance, mention was made of traditional rainmaking ceremonies that show the connection between cultural understanding and science. It points out that causal reasoning includes both formal and informal methods. Following the TAP analysis, learners deliberated on the merits and drawbacks of each argument, discussing the relative significance of scientific evidence versus personal observations. While most arguments included grounds and warrants, backing was frequently lacking or based on anecdotal evidence. This indicates a tendency to prioritise immediate results over the broader probabilistic principles underpinning forecasts. Argument excerpts were extracted from learners' written reflections during the practice activity. For instance, argument analysis using the TAP framework revealed a common need for reinforcing backings. Although the TAP model offered a valuable framework, some learners employed forms of reasoning rooted in personal experience or observation, which do not align neatly with this structure. This suggests a potential area for future exploration. This analysis paves the way for the intervention, with the post-test set to assess whether learners incorporate more sophisticated warrants and backings in their probability arguments.

3.8.1.5 Assessment and Feedback

Using assessment and feedback strategies greatly helped learners develop their skills in argumentation. While learners were doing the meteorological forecasting assignment, formative assessment was used to assess their understanding and skill level in argumentation. The assessment process emphasised the development of strong arguments backed by evidence and the involvement in group discussions and the analysis of multiple viewpoints.

Feedback from the teacher was provided promptly, informing learners what they were good at and what they needed to work on. Learners were also urged to assess their argumentation abilities and find ways to improve themselves. By including argumentation in probability lessons, learners were given an environment that encouraged everyone to think critically and use reasoning. Besides improving understanding of probabilities, these tasks helped hone the learners' critical thinking skills for the next part of the intervention.

3.9 Data collection methods

During the study, the researcher used a variety of ways to collect data to help understand the goals of the research. The key parts of the methodological framework included pre-tests, post-tests, semi-structured questionnaires and focus groups.

Because each of these components had specific functions, the study design was richly detailed, and the data collected was informative.

3.9.1 ***Quantitative data collection***

The quantitative component of the data were collected through Pre-test and Post-test Designs. Creswell and Creswell (2023) define pre-test and post-test design as quasi-experimental or experimental frameworks where measurements are taken both before and after an intervention to determine its effectiveness. This design allows for the calculation of gain scores, providing a direct measure of change within the same group subjects, which enhances internal validity by controlling for individual differences (Bloomfield & Fisher, 2022). However, it is susceptible to testing effects, where participants become sensitised to the research objectives during the first measure, potentially biasing the post-test result (Stratton, 2021). In this study the use of this design was indispensable the research objective was to evaluate the efficacy of the argumentation intervention (Tobi & Kampen, 2022).

3.9.1.1 Pre-test

A pre-test was carried out before the intervention to assess how well participants knew probability and argumentation. By using quantitative data, the researcher set out to assess how the intervention impacted academic outcomes. Both the experimental group and the control group were given a pre-test, as described in Annexure E, one week before the intervention.

The design of the pre-test included four items aimed at evaluating distinct dimensions of probability concepts. It comprised three structured questions that necessitated calculations, alongside one extended response question. The intent was to profile learners by assessing their theoretical understanding of probability, their problem-solving capabilities related to probability scenarios, as well as their proficiency in articulating arguments effectively.

3.9.1.2 Post-tests

Following the completion of the intervention, a post-test (Annexure I) was administered to both participant groups. This assessment was designed to mirror the pre-test in structure while fundamentally differing in the numerical values assigned to each question; however, the overall level of difficulty was intentionally maintained to ensure consistency. The purpose of the post-test was to assess variations in learners' understanding of probability concepts, evaluating several dimensions of their educational outcomes, including theoretical understanding, problem-solving abilities related to probability, and competencies in argumentation following the intervention.

To mitigate influences that could arise from repeated testing, the items within the post-test were randomised. After that, a comparison of the pre-test and post-test results made it possible to develop a conclusion about the learners' understanding of probability and their ability to argue.

Using both pre-tests and post-tests was needed to measure the success of the lessons and to assess the effectiveness of the argumentation-based instruction. According to Johnson et al. (2007), using these tests assists in judging the success of educational interventions, since they allow the measurement of what learners have learned during a given period.

The inclusion of lower-order Bloom's Taxonomy items in both pre-test and post-test (Annexure E and Annexure I) was a deliberate pedagogical strategy. These items were designed to establish foundational probabilistic fluency among learners before transitioning to high-order argumentation. By ensuring that all participants possessed basic conceptual knowledge, the model empowered them to construct the warrants and backings required for the more cognitively demanding phases of the intervention.

3.9.2 ***Qualitative data collection***

3.9.2.1 Semi-structured questionnaires

Both before and after the intervention, semi-structured questionnaires were administered to the experimental group (Annexure G and K). According to Hennink et al. (2020) a semi-structured questionnaire is an instrument that combines closed-ended with open-ended questions, allowing for both quantitative (Section A) and qualitative data (Section B). It offers a balance between replicability and flexibility. Researchers can use it to collect comparable data across a large sample while capturing unexpected insights through the open-ended sections (Varpio et al., 2020). However, analysing qualitative components requires rigorous thematic coding, which can lead to researcher bias if not properly triangulated (Noble & Heale, 2019). The instrument was suitable as the research used a mixed-methods design where the study sought to quantify the learner performances while also exploring the "how" and "why" behind the quantitative data (Creswell & Hirose, 2024).

Section B collected qualitative feedback about learners' perceptions, motivation and views about probability and to assess how their academic performance changed following the intervention. Because the questionnaire items were semi-structured, they allowed the learners to communicate their understanding and elucidate their views about probability (Varpio et. al., 2020). Besides the right answers, the learners were required to explain their thoughts, aiming to assess if they could argue well.

Measuring the outcomes both before and after the intervention allowed the assessment of how well learners constructed arguments and the overall impact of the intervention. In addition, the purpose of the research was to understand what learners go through, think about and struggle within probability, while evaluating argumentation as a teaching approach.

3.9.2.2 Focus group discussions (FGD) guide

An FGD guide consist of predetermined set of open-ended questions designed to facilitate a moderated, interactive discussion among a small group of participants (Sim & Waterfield, 2019). It leverages group dynamics to generate data that might not emerge in individual settings. The interaction between participants can trigger ideas or clarify collective norms and social values (Hennink et al., 2020). However, dominant voices overshadow quieter participants, potentially masking dissenting views (Sim & Waterfield, 2019). It is advantageous to use when social constructs, shared cultural meanings, or community-level attitudes are sought rather than individual experiences (Shabina et al., 2024).

To achieve cultural diversity, purposive sampling was employed to select 10 FGD participants ensuring the unique cultural lenses of each group. The study group constituted six mutually exclusive strata based on cultural identity (Ndebele, Shona, Tonga, Venda, Kalanga and Mixed decent). The number of participants per stratum was purposively fixed to the selection of 4 Ndebele, 2 Shona, 1 Tonga, 1 Venda, 1 Kalanga and 1 of Mixed decent. Within each stratum each learner was assigned a code which was used to carry out a simple random selection within the stratum. The codes were placed into a box and an independent member randomly chose the participants without replacement. This ensured each learner in each stratum had an equal chance of being selected to participate in the FGD.

Structured discussions were carried out using a discussion guide (Annexure L) which allowed participants to talk about argumentation, cultural influences and their personal views on the usefulness of argumentation methods in probability. Using open-ended questions, the discussions encouraged learners to share and discuss their own experiences with the argumentation training, as well as its connection to the study of probability. Because the approach supported interactivity and exchange, it allowed participants to expand on each other’s ideas and contributed to an understanding of what learners from different cultures have in common.

The reason for using focus group discussions was the assumption that they could reveal social and cultural factors that are not always visible in tests and questionnaires. According to Johnson et al. (2007), focus group discussions help in exploring complex subjects, as members can explain their views in a friendly environment. Focus group discussions were considered because their design allows modifications to suit what the participants say. Being adaptable helps to understand new aspects of the data more deeply as data collection proceeds. This approach follows the view of Santos et al. (2020), who encourage using qualitative studies to support educational interventions.

*3.9.3 **Rationale for focus group discussion with experimental group***

The discussion in the experimental group was meant to gather detailed feedback and emotional responses about the argumentation approach which provided a clear picture of learners' experiences. By using this qualitative method after the quantitative data, the researcher could enrich their understanding of the learning process.

The test scores provided numerical evidence, but the group discussion helped to explore the reasons behind what was attained from the test. For this reason, learners' views on probability, different ways of facilitating learning and cultural awareness, assisted in the interpretation of the test results more completely.

3.10 Validity and reliability of instruments

To ensure the scientific rigour of this study, several measures were taken to establish the validity and reliability of the data collection instruments.

*3.10.1 **Quantitative Data Instruments***

3.10.1.1 Content Validity

Both the pre-test and the post-test assessments were designed based on the curriculum for secondary school probability. To make sure the content and effectiveness of these instruments were valid, two experienced mathematics teachers and two education experts reviewed them. Insight from the review feedback were incorporated into the instruments to make certain they covered the important concepts and abilities in argumentation.

3.10.1.2 Construct Validity

Fifteen secondary school learners were assessed in a pilot study but were not part of the study context but another school, to assess if the instruments measured the specific constructs appropriately. The pilot data analysis by factor analysis showed that the test items clustered accordingly to the defined areas of probability knowledge and argumentation abilities. The factor analysis results presented in Annexure U confirmed that the test items loaded onto two primary factors, probability knowledge and argumentation quality, with all item loadings exceeding the 0.40 threshold recommended for construct validity in educational measurement (Hair et al., 2019), supporting the construct validity of the instruments

3.10.1.3 Reliability

Using Cronbach's alpha showed that the pre-test and post-test assessments were internally consistent. The coefficients turned out to be 0.82 for the pre-test and 0.85 for the post-test which are both above the commonly accepted threshold of 0.70 in education, (Johnson et al., 2007; Mohajan, 2017).

3.10.2 ***Qualitative Data Instruments***

3.10.2.1 Credibility

The creation of semi-structured questionnaires and the focus group discussion guide were designed based on the set research questions and theoretical framework. To yield correct data, a member checking process was set up which permitted participants to verify their summaries and confirm that what was written accurately reflected their views (Golafshani, 2015).

3.10.2.2 Dependability

An organised record of the steps taken in data collection and analysis was maintained which also includes a record of all the decisions made during the qualitative coding process. Keeping records in this way guarantees that others can follow the research methodology used.

3.10.2.3 Transferability

There were clear and detailed descriptions of the participants and their context, so that others can assess how the results relate to situations apart from those studied.

3.10.2.4 Confirmability

Reflexivity by the researcher was an important part of the study throughout its progress. Proper handling and awareness of possible biases was helped by talking with colleagues who did not contribute to the research. Using this technique helped minimise how personal opinions affected the research outcomes.

3.11 Data analysis

A dual method of analysing quantitative and qualitative data helps make the findings more reliable and to effectively respond to the research questions (Bloomfield & Fisher, 2022).

3.11.1 Quantitative data analysis

The effectiveness of the argumentation-based intervention was checked by analysing the results of pre-tests and post-tests using SPSS version 16. Initially, descriptive statistics, specifically medians and interquartile ranges, were computed to provide a summary of the pre-test and post-test scores across both groups. Additionally, means and standard deviations were reported to offer further insight into the data characteristics. The normality of the dataset was evaluated through visual inspection of histograms, which served as a preliminary diagnostic tool. The generated histograms indicated that the data followed a non-normal distribution pattern. As a result, the Mann-Whitney U test was applied, which is a non-parametric method recognised for its suitability in comparing independent groups when the assumptions of normality are not satisfied.

Effect sizes were calculated to determine the practical significance of the intervention using the rank-biserial correlation, given by:

$$r_{rb} = 1 - \frac{2U}{(n_1\ x\ n_{2,)}},$$

$$where\ U\ is\ the\ Mann\ Whitney\ test\ value, n1\ and\ n2\ are\ the\ sample\ sizes,$$

To compliment the statistics above, DID (Difference-in-Differences) analysis was applied to evaluate the changes in test scores over time between the two groups. By comparing the pre-test and post-test scores, DID analysis assisted in understanding the causal effect of the argumentation intervention on learners' performance in probability. This approach was beneficial in establishing a causal relationship between the intervention and the observed outcomes. Figure 3.2 below shows the DID analysis illustration used.

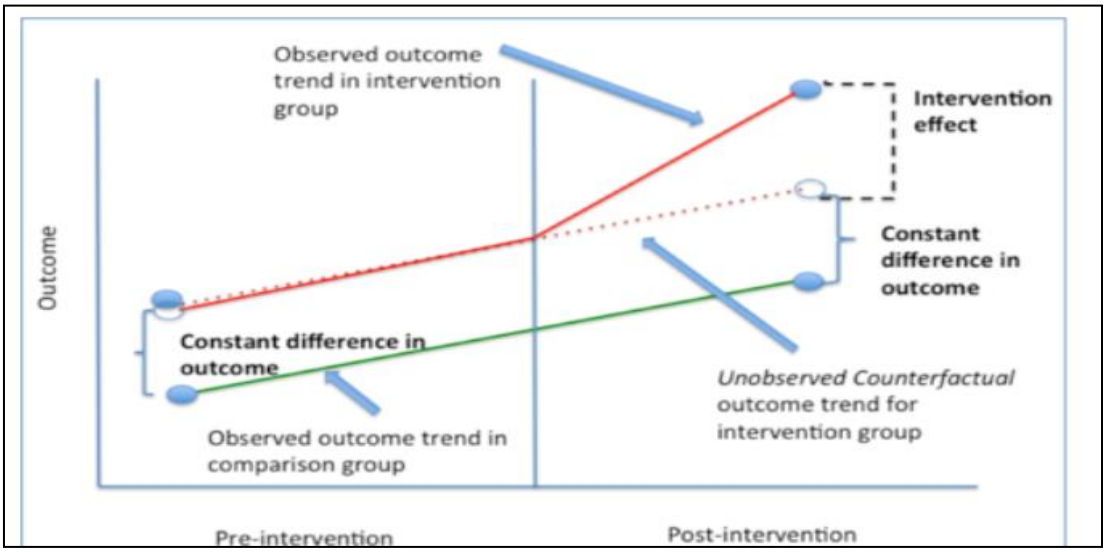


Figure 3.3: Difference-In-Difference. Adapted from *Population health methods*. (2023, March 28). Columbia University Mailman School of Public Health.

These statistical methods were collectively designed to address the research questions by providing empirical evidence of the intervention's effectiveness and contributing to the understanding of the effect of the intervention on learner performance.

*3.11.2 **Qualitative analysis***

An analysis of the qualitative data from the open ended semi-structured questionnaires (Section B) and focus group discussions was conducted through a systematic method

(Braun & Clarke, 2006). This facilitated an in-depth exploration of learners' experiences with argumentation and its influence on their understanding of probability. There were various distinct steps included in this analytical process using NVivo 10.

1. At the beginning, the familiarisation stage meant the researcher spent much time reading and re-reading the transcripts. The purpose of the immersion was to grasp the participants' views deeply.
2. After that, the first codes were made by identifying patterns or similar themes that appeared in the data. By doing this phase, the researcher was able to organise the data into groups that analysed in detail. At the beginning, the questionnaires were studied closely and key statements, phrases and ideas that explained participants' experiences and views were marked. Hence, by producing these first codes, the researcher was able to divide the data into smaller groups which made analysis simpler. This way of working supports Saldaña's (2016) view that coding in qualitative research is the way to organise and summarise data. Following this, the codes were used to come up with themes and ideas that appeared in the dataset.
3. Once all the codes were created, these were grouped into themes to capture the main points shared by the participants. By analysing the themes, the researcher could clearly identify what learners learned about probability while arguing. Every theme was carefully established and modified to show true reflections of the participants. Building themes from the data were important for organising the research and finding significant results. Braun and Clarke (2006) highlight that thematic analysis is a flexible way to analyse qualitative data, making it possible to understand the richness and complexity in what

participants say. Sorting the data into sensible themes helped to better observe the connection between argumentation and learners' grasp of probability.

The thematic analysis yielded the following primary themes that structure the qualitative findings reported in Chapter 4: (i) Inadequate pre-conceptual mastery (ii) Socio-cultural inclusivity (iii) Affective transformations (iv) Cognitive and social dimensions development (v) Potential transfer of learning (vi) Argumentation quality aptitudes. These themes are elaborated in Section 4.7.

3.12 Data integration

The interpretation step in sequential explanatory research uses both quantitative and qualitative data to give a clear picture of what happened in the study (Mohammad, 2021). At first, the research relied on gathering quantitative data from pre-tests and post-tests to identify and describe the trends in the data. Next, qualitative data were gathered by discussing with learners through semi-structured questionnaires and focus group discussions which helped explain the reasons for the quantitative results.

To explain the numerical outcomes, qualitative data were used during interpretation, adding a framework of personal perspectives and experiences that may not be captured by the quantitative data alone. At this stage, the researcher compared the outcomes from both datasets which strengthened the conclusions by using triangulation. Synthesising statistics and individual perspectives allowed the researcher to draw conclusions.

3.13 Ethical considerations

All the needed ethical standards for research with human participants were respected while conducting this study, especially considering the cultural diversity in classrooms. The following are the ethical issues that were considered:

3.13.1 ***Institutional Approval***

Ethical permission to conduct the research was obtained from Bindura University of Science Education’s Research and Postgraduate Studies Committee (Annexure A). The research was approved by the Permanent Secretary of the Ministry of Primary and secondary Education (Annexure B) and the Provincial Education Director for Bulawayo Metropolitan Province (Annexure C). Authorisations like these were needed to make certain that all studies in education followed the proper rules.

3.13.2 ***Informed Consent***

Learners gave their consent (Annexure D) as they were all above 18 and there was no need for their parents or guardians to give additional consent. They were reminded that being in the study was their choice and that they could drop out at any time without being penalised.

3.13.3 ***Confidentiality and Anonymity***

Researchers did not use the participants’ names in their records, instead used codes that did not identify anyone. Every recording and its transcript from the focus groups were kept securely and only the researcher could see them. Care was taken to prevent the identification of any of the individuals in the group during the reporting of results or usage of quotations.

*3.13.4 **Minimising Harm and Maximising Benefits***

The research team made sure that the participants would not be harmed or made uncomfortable. For participants not to miss out on their regular lessons, argumentation training was incorporated into their existing lessons.

*3.13.5 **Cultural Sensitivity***

Because of the range of cultures in the participants' group, the researcher was extra sensitive to cultural issues during the research process. The intervention scenarios were carefully created to include all participants' cultural backgrounds. Moreover, the person leading the sessions was trained to handle the different cultural groups. Classroom ground rules made it clear that cultural diversity should be respected and no forms of cultural stereotyping or discrimination were allowed.

The importance of ethics helped guarantee that the study was done right and the results were reliable. This approach made sure to respect the rights, dignity and cultural backgrounds of all those involved in the research.

*3.13.6 **Managing contamination between the two groups***

Given that the experimental and control groups were situated in the same school, specific protocols were implemented to prevent knowledge contamination between groups. These included scheduling the two groups' mathematics lessons at different times of day, ensuring no shared break-time discussions about lesson content took place through participant briefing and physically separating the groups during data collection events. As an insider researcher who taught mathematics to the two groups, the researcher ensured tests were marked by other teachers and member checking was used for learners to confirm their responses.

*3.13.7 **The research team***

The research team comprised: (1) The principal researcher who was responsible for delivering both the experimental and control group instruction and coordinating data collection; (2) Two independent markers (experienced mathematics teachers who were not affiliated with the study) who were responsible for scoring pre-test and post-test responses blind to group allocation, minimising marker bias; (4) The supervisory team (Dr D. Dziva and Dr Z. Ndemo) who were responsible for oversight of research design, data analysis, and report writing. Roles were clarified in a researcher briefing conducted prior to the study commencement.

3.14 Chapter summary

The chapter spells out the strategy used to examine the effects of argumentation on probability teaching to secondary school learners from diverse cultures. The research was conducted using a pragmatic approach and a mixed-methods approach, with a sequential explanatory design designed to answer the research questions.

A group of 40 secondary school learners was purposively chosen and randomly assigned into experimental and/or control groups. The experimental group was taught probability using methods that involved argumentation. The lessons were similar in content so that none of the groups would be disadvantaged. However, the control group used the traditional lecture approach. The intervention was designed to support the development of argumentation skills by applying the ThinkCERCA approach and Toulmin's Argumentation Pattern, while considering both cultural beliefs and probability concepts in various scenarios.

Both quantitative methods (pre-tests and post-tests) and qualitative approaches (semi-structured questionnaires and focus group discussions) were used to collect data. The use of more than one way of data collection made it possible to understand both the quantitative and the feelings of the learners involved. Both the quantitative data and the qualitative data were analysed using appropriate statistical methods and thematic analysis to find patterns in learners' opinions.

While carrying out the research, it was important to follow ethical guidelines, acquire the required approvals and seek informed consent, confidentiality and cultural sensitivity. Applying various methods to ensure the instruments' validity and reliability increased the confidence in the findings.

The chapter presents the approach that guided the study on integrating argumentation in teaching probability to culturally diverse secondary school learners in Bulawayo. The results obtained using this approach will be presented, analysed and discussed in Chapter 4.

CHAPTER 4: DATA PRESENTATION, ANALYSIS AND DISCUSSION

4.0 Introduction

This chapter summarises the findings of this study. The focus is on the effect of argumentation on the developmental process of learners in probability learning and their conflict-resolving strategies between formal and informal reasoning. The learners' arguments showed changes in their thinking process. Quantitative test score analysis will show the progress made in the acquisition of probability knowledge while qualitative data will show the processes through which learners produced their arguments and the impact of the discourse on their knowledge of probability. The combination of quantitative and qualitative findings will show the success of the argumentation instructional approach in the teaching and learning of secondary school learners from culturally diverse backgrounds.

4.1 Learner demographic data

To evaluate the argumentation skills of the groups preceding the intervention, a pre-intervention questionnaire (Annexure G), consisting of both open-ended and closed-ended questions, was administered (Section A and Section B respectively). Questions from Section A were intended to collect demographic data about the experimental group to understand the cultural diversity of the group. The initial pair of questions sought to identify the ethnic backgrounds and religious beliefs of the learners, given that these elements may significantly influence their responses. The outcomes of the questions are presented in Table 4.1.

Table 4.1: Learner Demographics (Section A: Pre-test Questionnaire)

		Experimental Group		**Control Group**		**Total**	
No of learners		20		20		40	
Ethnicity	Ndebele	10	50%	8	40%	18	45%
	Shona	4	20%	4	20%	8	20%
	Tonga	2	10%	3	15%	5	12.5%
	Venda	1	5%	0	0%	1	2.5%
	Sotho	0	0%	2	10%	2	5%
	Kalanga	1	5%	3	15%	4	10%
	Mixed descent	2	10%	0	0%	2	5%
Religion	Christian	13 (65%)		12(60%)		25 (62.5%)	
	Muslim	2(10%)		4(20%)		6 (15%)	
	Other	5(25%)		4(20%)		9 (22.5%)	
Age distribution of the learners							
Age		**Frequency**				**Percentage**	
18		25				62.5	
19		14				35	
20		1				2.5	

Table 4.1 summarises the data, showing the Shona and Ndebele as the most common ethnic groups, while simultaneously reflecting a greater level of ethnic diversity. It's crucial to note that the learners involved in this study were from a wide range of cultural groups, involved in various ethnic and religious traditions. Because of this diversity, learners experience contradictory mathematical experiences to their traditional cultural experiences, mainly in probability.

The examination of the demographic data reveals that the experimental group consisted of 10 Ndebele learners, 4 Shona learners and 2 Tonga learners, plus some learners from several other groups. The control group also had 8 Ndebele, 4 Shona and 3 Tonga learners. The range of ethnic groups may shape learners' opinions about probability; many times, their cultural beliefs shape their thinking about chance and uncertainty (Savard et al., 2014). As an example, learners from different ethnic backgrounds might have different opinions about luck, fate and the role of chance which affects how they think about mathematics in school.

More than half of the learners in both groups were Christian, with 65% in the experimental group and 60% in the control group. On the other hand, only a small group of learners in both groups identified as Muslims at 10% and 20%, respectively, while the majority in the experimental group (25%) and a similar portion in the control group (20%) were part of other religions. Different religious beliefs may shape what learners believe about probability, since some religions claim that things always happen in a fixed order which contrasts with the methods taught in schools.

Recognising the impact of cultural and religious factors is crucial for investigating research questions about the use of argumentation in teaching probability. Besides the issues mentioned, socioeconomic status also impacts the learning of probability. Many learners came from households with low to middle incomes which can make it hard for them to get the help and resources they need for learning (Taeao, & Averill, 2019). Learners from disadvantaged contexts, for example, can have prior mathematical experiences characterised by such socioeconomic backgrounds, including limited exposure to formal mathematical instruction and resources. Consequently, a lack of foundational knowledge may hinder their engagement with probability concepts and affect their performance on argumentation and problem-solving tasks (Ayalon et al., 2018).

Furthermore, learners' educational experiences varied, ranging from a focus on more traditional pedagogical techniques to institutions emphasising more progressive, inquiry-based approaches. Given this disparity in educational exposure to argumentation and the ability to critically engage with mathematical concepts, there exists a disparity in comfort with argumentation (Brown, 2017). For example, learners from collaborative learning environments will have a better understanding of articulating reasoning and participating in probability discussions while learners from more traditional backgrounds may face difficulties in adjusting to innovative pedagogical strategies.

To summarise, the demographic factors described above are likely to have a very significant influence on the outcomes of the study. Performance differences across learners from different ethnic or religious backgrounds may arise in the concepts of probability and in their ability to argue. On that basis, it is therefore necessary to explore how these factors can present themselves in varying degrees of confidence and competence with mathematical reasoning and how cultural narratives can influence learners' understanding of probability scenarios.

Furthermore, Table 4.1 reveals that the distribution of learners' ages predominantly centred around 18 years, accounting for 62.5% of the learners, while 35% were 19 years old and 2.5% were 20 years old. This age distribution substantiates the adequacy of obtaining consent from the learners, justifying the non-parental need for consent, as all learners were classified as adults, being 18 years or older.

4.2 Pre-test data analysis

A pre-test (Annexure E) with marking guide (Annexure F) was administered to both the control group and the experimental group to assess the learners' critical thinking and performance in probability in response to the first research question, *"How do the culturally diverse secondary school learners demonstrate baseline conceptual understanding of probability and argumentation prior to the pedagogical intervention?"* Quantitative data were analysed using IBM SPSS Statistics 16.0, focusing on statistical tests for two independent means for pre-test comparability, these tests provided distinct insights into the effect of the intervention on learner performance relative to the research questions. Additionally, the calculation of Difference-in-Differences (DID) was conducted to further confirm that the results were attributable to the instruction rather than confounding variables. The null hypothesis for the pre-test posited that there would be no statistically significant difference in probability performance between the experimental and control group learners prior to the intervention. For the post-test, the null hypothesis stated that there would be no difference in probability performance between the experimental and control group learners following the intervention. Both tests comprised structured questions, which the two groups were required to answer to evaluate their critical thinking abilities and performance in probability. The results obtained from the pre-test are presented in Table 4.2.

Table 4.2: Experimental and Control groups' pre-test marks

Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Exp.	30	40	35	50	60	35	55	56	64	34	45	46	54	52	44	34	50	57	56	55
Cont.	55	45	56	56	45	50	35	55	45	45	56	45	34	43	56	47	55	24	56	54

Table 4.2 above presents the marks attained by the experimental (Exp.) and control groups (Cont.) prior to the intervention. To decide on the statistical approach to be used, a normality test was carried out on data using SPSS through the analysis of the histograms associated with each data set as shown in Figure 4.1 and Figure 4.2 below.

4.2.1 Pre-test Normality test

Histograms were used in the quantitative data analysis to assess whether the data followed a normal distribution. Histograms provided a visual that figure alone couldn't convey. While a statistical test provides a rigid p-value, a histogram showed the actual behaviour of the learners' scores, showing whether they clustered around a central value or the values were spread to one side of the central value. The checking for skewness or multiple peaks visually assisted in the choice of the statistical approach to use for the quantitative data from the pre-test and post-test.

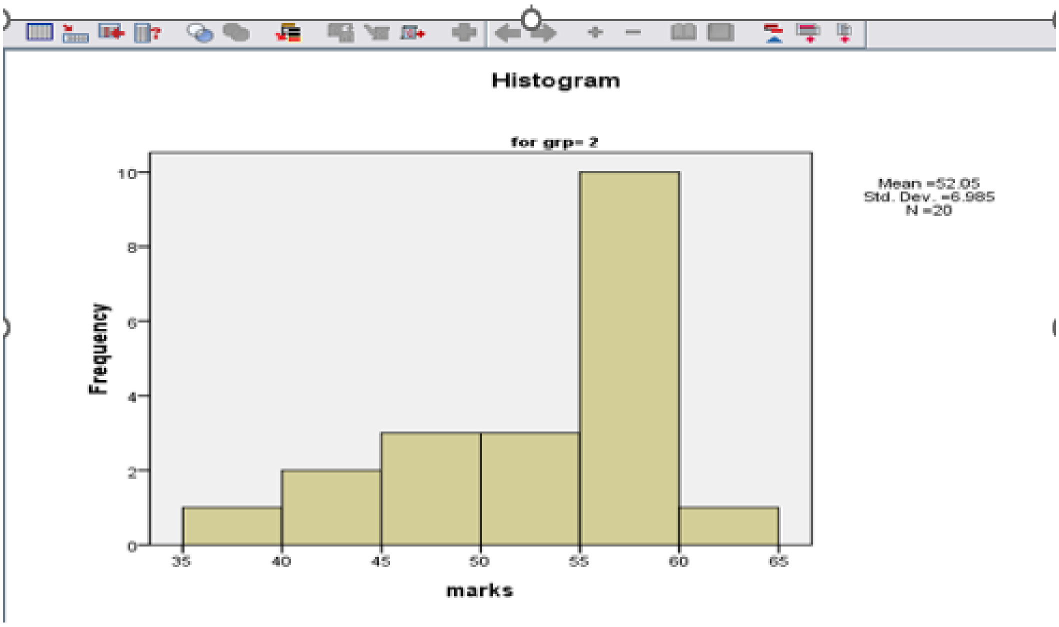


Figure 4.1: Experimental group test for normality (Pre-test)

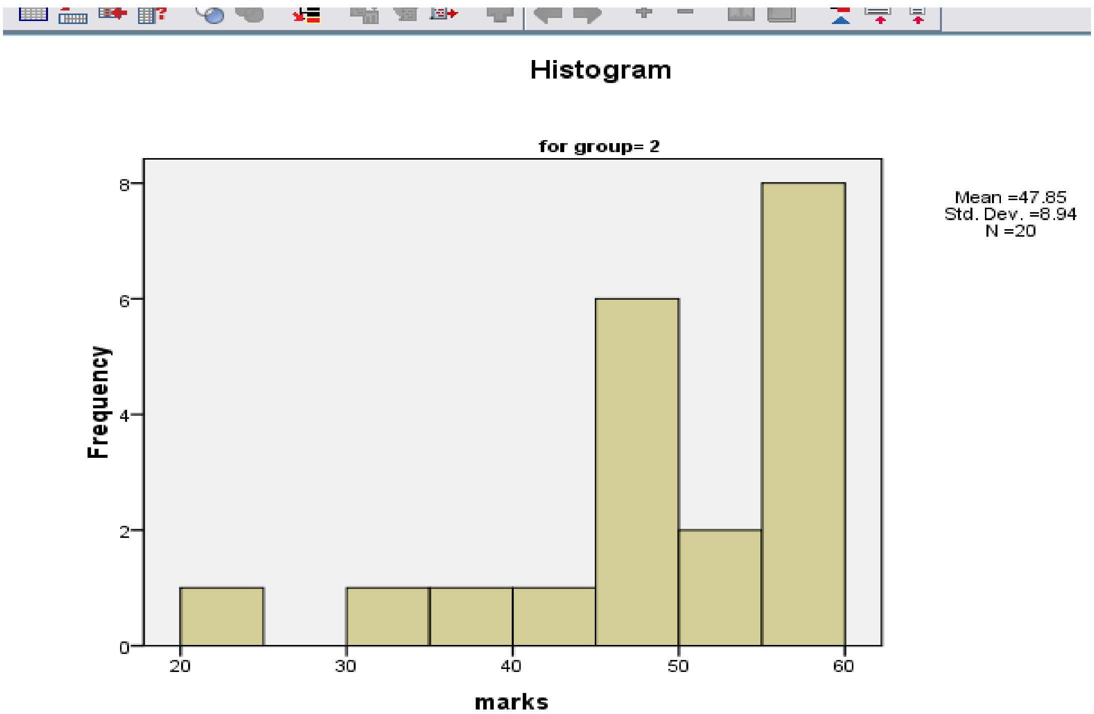


Figure 4.2: Control Group test for Normality (Pre-test)

The histograms in Figure 4.1 and Figure 4.2 show that the marks were not normally distributed. Hence a non-parametric test (Mann Whitney U test) which does not consider normality as a prerequisite was used for analysis. The statistical method is used to determine whether there are significant differences between two independent groups, particularly when the assumptions of normality for parametric tests cannot be met. This test is particularly valuable in situations where the sample sizes are less than 30 or the data are ordinal, as it evaluates the ranks of the data rather than their raw values. The Mann-Whitney U test minimises the impact of outliers and skewed distributions, making it a more reliable choice for analysing non-normally distributed data.

This test was used in this study because the test scores were not normally distributed as shown by the histograms from SPSS outputs and the distributions were largely influenced by outliers. The sample sizes were less than 30 for each group. Due to these conditions the statistics was suitable for analysing the data from the tests. The outputs from the tests are shown below.

4.2.2 Pre-test descriptive statistics

```
[DataSet2] C:\Users\chitera\Documents\post.sav
```

group

Case Processing Summary

		Cases					
		Valid		Missing		Total	
	group	N	Percent	N	Percent	N	Percent
marks	1	20	100.0%	0	.0%	20	100.0%
	2	20	100.0%	0	.0%	20	100.0%

Descriptives

	group			Statistic	Std. Error
marks	1	Mean		47.60	2.242
		95% Confidence Interval for Mean	Lower Bound	42.91	
			Upper Bound	52.29	
		5% Trimmed Mean		47.67	
		Median		50.00	
		Variance		100.568	
		Std. Deviation		10.028	
		Minimum		30	
		Maximum		64	
		Range		34	
		Interquartile Range		20	
		Skewness		-.317	.512
		Kurtosis		-1.109	.992
	2	Mean		48.35	1.875
		95% Confidence Interval for Mean	Lower Bound	44.42	
			Upper Bound	52.28	
		5% Trimmed Mean		49.28	
		Median		48.50	
		Variance		70.345	
		Std. Deviation		8.387	
		Minimum		24	
		Maximum		56	
		Range		32	
		Interquartile Range		11	
		Skewness		-1.381	.512
		Kurtosis		2.434	.992

Figure 4.3: Descriptive statistics for experimental (1) and control group (2)

From Figure 4.3 above, the experimental group (n = 20) had a median score of 50.00 and (Interquartile Range (IQR) = 20) while the control group (n = 20) had a median score of 48.50 (IQR= 11). The median scores are very similar between the two groups

(50.00 versus 48.50). This suggests that, before the intervention, the two groups were equivalent in terms of the outcome variable being measured (performance). To check if the difference in scores was significant the Mann Whitney U test was calculated, and the following results were attained.

4.2.3 Pre-test Mann Whitney U test for two independent means

Insert Format Analyze Graphs Utilities Add-ons Window Help

NPar Tests

```
[DataSet1] C:\Users\chitera\Documents\data.sav
```

Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum	Percentiles		
						25th	50th (Median)	75th
marks	40	47.72	9.378	24	64	43.25	50.00	55.75
grp	40	1.50	.506	1	2	1.00	1.50	2.00

Mann-Whitney

Ranks

	grp	N	Mean Rank	Sum of Ranks
marks	1	20	20.35	407.00
	2	20	20.65	413.00
	Total	40		

Test Statistics[b]

	marks
Mann-Whitney U	197.000
Wilcoxon W	407.000
Z	-.082
Asymp. Sig. (2-tailed)	.935
Exact Sig. [2*(1-tailed Sig.)]	.947[a]

a. Not corrected for ties.

b. Grouping Variable: grp

Figure 4.4: Pre-test Mann Whitney U test for two independent means

Figure 4.4 above shows the results of the Mann Whitney U test that was conducted to determine whether there was a significant difference in probability test scores between the experimental group (1) and control group (2). The results indicate a non-significant difference between the groups [U = 197.00, p = 0.925: p > 0.05]. In conclusion, we fail to reject the null hypothesis and conclude that there is no significant difference in probability test scores between the groups prior to the intervention. The explanation supported by Pineda and Sirota (2018) who analysed the significance of p values in research. To quantify the magnitude of the difference a Cohen's difference (d) was computed using the formula $Rrb = \frac{1-(2U)}{(n1*n2)}$, where n1 and n2 are samples sizes and U is the Mann Whitney Value from SPSS. U = 197, $n1 = n2$ = 20. The attained Cohen's difference was 0.015. This value is far less than Cohen's 0.2 small effect value. The value is essentially zero in terms of practical effect. Hence, there is virtually no meaningful difference between the two groups. Hence, both groups were comparable in performance prior to the intervention. The comparability of the groups was crucial for the validity of the subsequent analysis, as it ensured that the post-intervention differences could be more confidently attributed to the intervention itself rather than to pre-existing group differences. In response to the research question regarding the pre-intervention critical thinking skills of the learners in probability, the descriptive statistics indicate that the means for both groups were below 50%. This suggests that the learners encountered difficulties in solving probability questions before the intervention. Hence, prior to the intervention learners had difficulties solving probability problems.

4.3 Post-test results analysis

Following the intervention, a post-test (Annexure I) with marking guide (Annexure J) was administered to both groups. This analysis sought to answer the second research question: *"How does the implementation of an argumentation-based instructional approach impact the development of probabilistic reasoning and justification skills among secondary school learners?"* The results are shown in Table 4.3 below.

Table 4.3: Experimental and Control groups' post-test marks

Stud	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Exp.	60	76	67	60	70	58	64	58	65	60	65	50	56	65	60	65	62	65	67	68
Cont.	45	56	50	49	57	58	57	56	54	45	44	57	56	58	40	35	60	56	58	50

Table 4.3 presents the marks attained by the experimental and control groups following the intervention. Below is an analysis of the data derived from both the pre-test and post-test scores shown in the preceding table. To decide on the statistical approach to use, an analysis of whether the data followed a normal distribution was carried out using SPSS to come up with Histograms that show the distribution of the data as shown below.

4.3.1 Post-test normality test

The following histograms show the data distribution of the post-test.

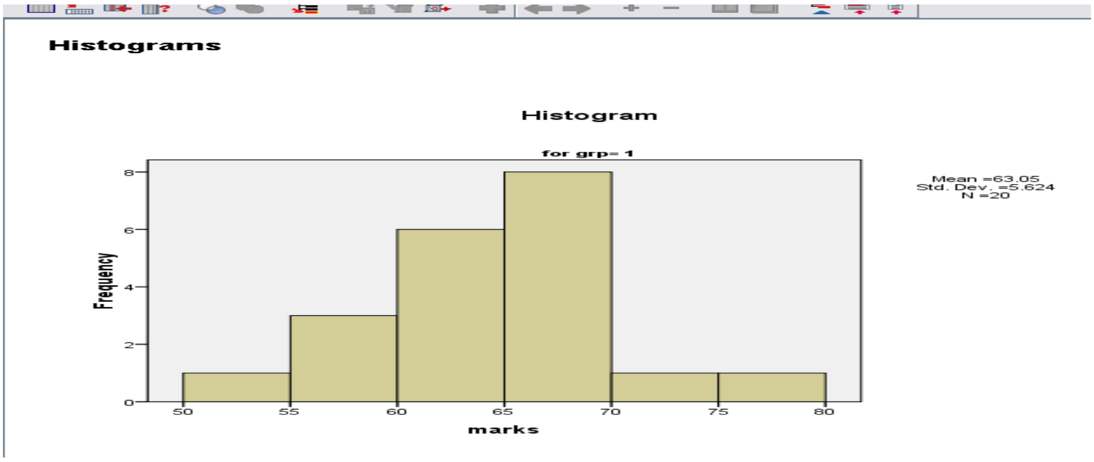


Figure 4.5: Experimental group test for normality (Post-test)

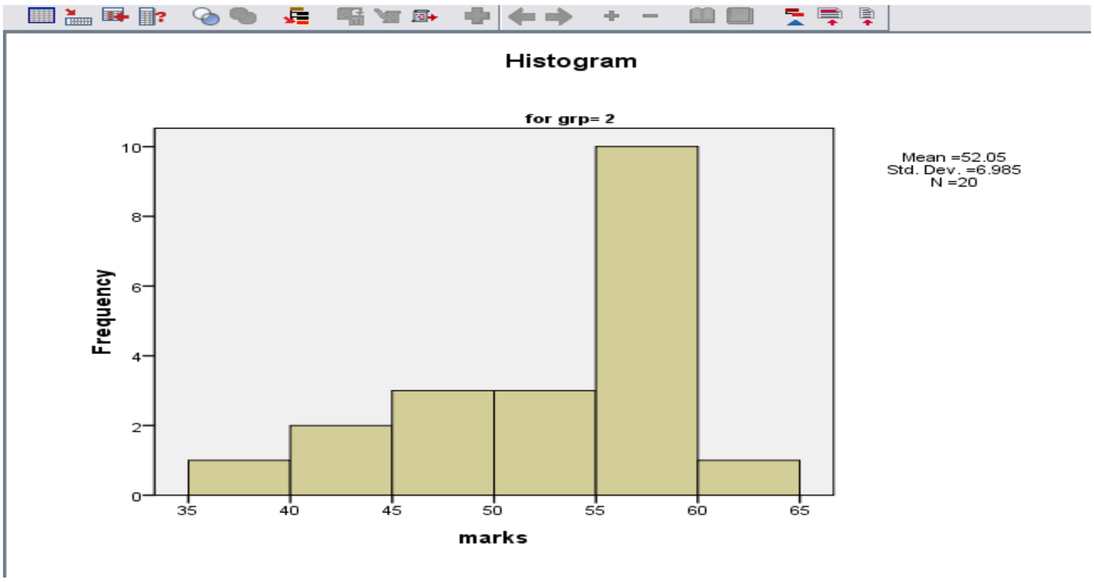


Figure 4.6: Control group test for normality (Post-test)

Figure 4.5 and Figure 4.6 show that the marks were not normally distributed. Hence, the Mann Whitney *U* test was calculated using SPSS, which Stratton (2023) explained was suited for non-parametric data. The results are shown below.

4.3.2 Post-test descriptive statistics

To compare the performance of the two groups after the intervention, descriptive statistics were computed.

```
[DataSet2] C:\Users\chitera\Documents\post.sav
```

grp

Case Processing Summary

	grp	Cases					
		Valid		Missing		Total	
		N	Percent	N	Percent	N	Percent
marks	1	20	100.0%	0	.0%	20	100.0%
	2	20	100.0%	0	.0%	20	100.0%

Descriptives

	grp			Statistic	Std. Error
marks	1	Mean		63.05	1.258
		95% Confidence Interval for Mean	Lower Bound	60.42	
			Upper Bound	65.68	
		5% Trimmed Mean		63.06	
		Median		64.50	
		Variance		31.629	
		Std. Deviation		5.624	
		Minimum		50	
		Maximum		76	
		Range		26	
		Interquartile Range		6	
		Skewness		-.051	.512
		Kurtosis		1.136	.992
	2	Mean		52.05	1.562
		95% Confidence Interval for Mean	Lower Bound	48.78	
			Upper Bound	55.32	
		5% Trimmed Mean		52.56	
		Median		56.00	
		Variance		48.787	
		Std. Deviation		6.985	
		Minimum		35	
		Maximum		60	
		Range		25	
		Interquartile Range		11	
		Skewness		-1.068	.512
		Kurtosis		.267	.992

Figure 4.7: Descriptive statistics for experimental and control group (post-test)

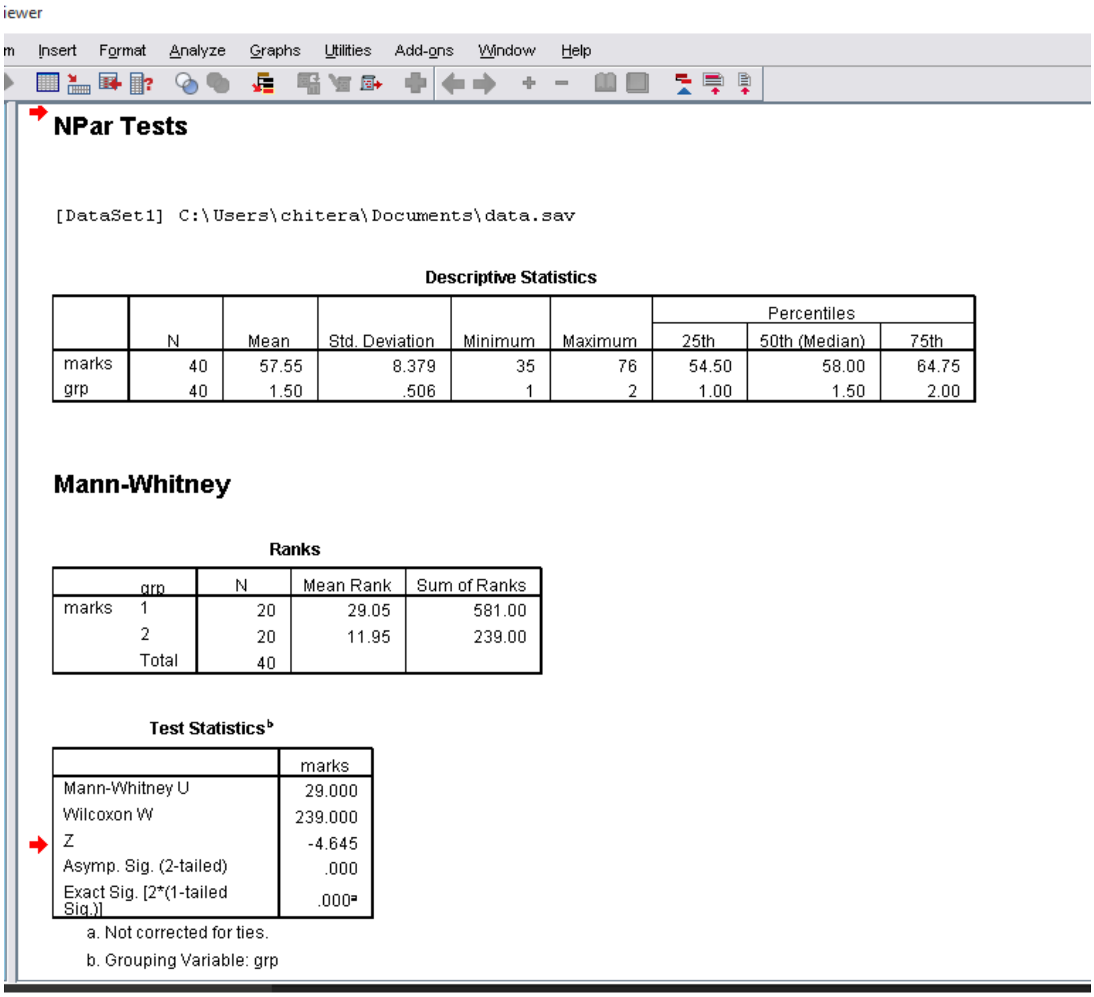
From Figure 4.7 the experimental group [n = 20] had a median score of 64.50 (IQR = 6) while the control group [n = 20] had a median score of 56.00 (IQR = 12). The median score in the experimental group (64.50) is higher than the median score in the control group (56.00). This suggests that, on average, learners in the experimental group tended to score higher than those in the control group. The median is a robust measure, meaning it's not heavily influenced by extreme scores. The IQR for the control group (12) is larger than the IQR for the experimental group (6). This indicates that the middle 50% of the scores in the control group are more spread out than the middle 50% of the scores in the experimental group. In other words, there's more variability or heterogeneity in the scores within the control group compared to the experimental group. The scores in the experimental group are more tightly clustered around the median.

The experimental group not only had a higher median score but also had less variability in their scores compared to the control group. This suggests that the experimental intervention (argumentation) had a consistent effect on the learners, leading to both higher scores and more similar scores among them.

The control group, on the other hand, had lower scores on average and a wider range of scores within the middle 50%. This indicates that the factors influencing their scores were more diverse or that there was a wider range of responses within the control group. This indicates that the cultural aspect could have impacted their responses compared to the experimental group.

In summary, the descriptive statistics suggest that the experimental group performed better and had more consistent scores than the control group. To test the significance of these results the Mann Whitney *U* test was carried out and is described below to test the null hypothesis that there is no significant difference in the probability achievement scores between secondary school learners taught using the Argumentation-based approach and those taught using traditional methods over time.

4.3.3 *Post-test Mann Whitney U test for two independent means*



NPar Tests

[DataSet1] C:\Users\chitera\Documents\data.sav

Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum	Percentiles 25th	Percentiles 50th (Median)	Percentiles 75th
marks	40	57.55	8.379	35	76	54.50	58.00	64.75
grp	40	1.50	.506	1	2	1.00	1.50	2.00

Mann-Whitney

Ranks

	grp	N	Mean Rank	Sum of Ranks
marks	1	20	29.05	581.00
	2	20	11.95	239.00
	Total	40		

Test Statistics[b]

	marks
Mann-Whitney U	29.000
Wilcoxon W	239.000
Z	-4.645
Asymp. Sig. (2-tailed)	.000
Exact Sig. [2*(1-tailed Sig.)]	.000[a]

a. Not corrected for ties.

b. Grouping Variable: grp

Figure 4.8: Post-test Mann Whitney *U* test for two independent means

***H*0:** There is no significant difference in the probability performance scores between secondary school learners taught using the Argumentation-based approach and those taught using traditional methods over time.

The results in Figure 4.8 above indicate a significant difference between the groups [$U = 28.000$, $p = 0.015$: $p < 0.05$]. In conclusion, the null hypothesis is rejected, and it's concluded that there is significant difference in probability test score between the groups following the intervention. The attained $Cohen's\ d = 0.855$. This value is greater than Cohen's 0.8 strong effect value. This means the argumentation-based instruction had a strong positive impact on learners' understanding of probability concepts compared to the control group. Beyond this significance, the intervention is of practical importance. This means the observed gains are likely to be meaningful in real world education settings.

4.4 Difference in difference (DID) analysis of pre and post-test

Difference-in-Differences (DID) analysis is a methodological approach used to evaluate the causal effect of a treatment or intervention by comparing changes in outcomes between a treatment group and a control group before and after the implementation of the treatment. This technique is widely utilised across various fields, notably in assessing the effectiveness of policy interventions and experimental treatments. The core principle of DID analysis involves examining the differences in outcomes over time between the treatment and control groups. By analysing these differential changes, the treatment effect can be isolated from other confounding factors that may influence the outcome. The underlying assumption of this analysis is that, in the absence of treatment, the trends in outcomes for both the treatment and control groups would follow a parallel trajectory. The formula for this calculation is:

$$Difference\ estimator(DID) = (Treatment_{Post} - Treatment_{Pre}) - (Control_{Post} - Control_{Pre})$$

4.4.1.1 Observed mean scores

Table 4.4: below shows the descriptive statistics of the pre-test data.

Table 4.4: Experimental and control groups' Pre and Post means

Experimental Group		**Control group**	
Pre mean	**Post Mean (%)**	**Pre-Mean (%)**	**Post mean**
47.55	62.55	48.10	52.05

Substituting these mean values into the formula:

Difference in difference $= (62.55 - 47.55) - (52.05 - 48.10)$
$= 15.00 - 3.95$
$= 11.05$

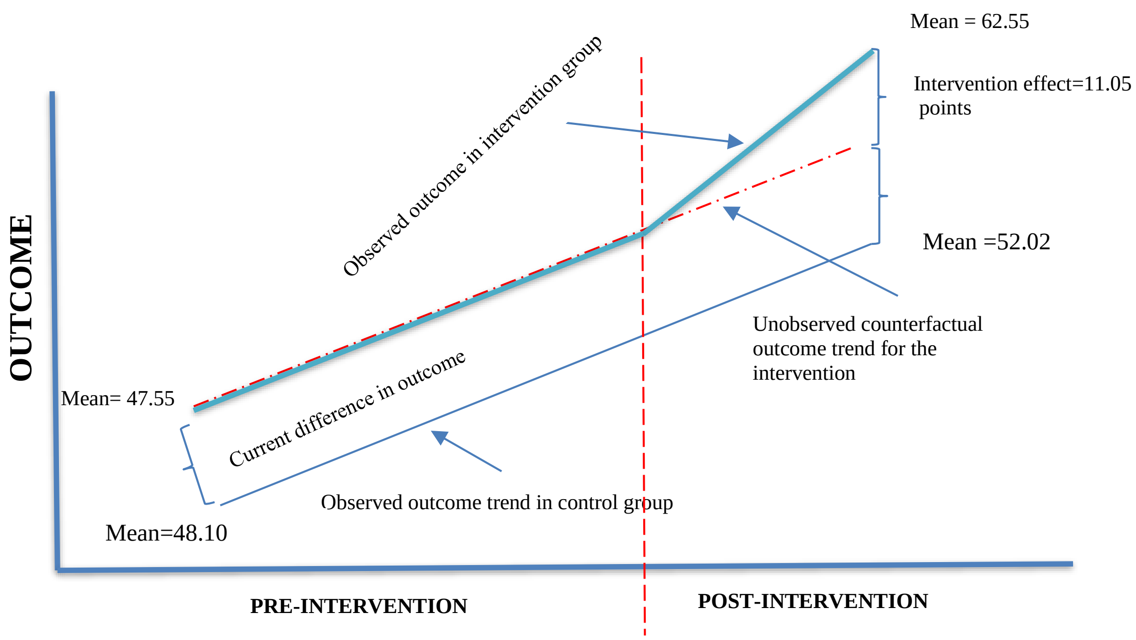


Figure 4.9: Difference in Difference Analysis: *Researcher's illustration*

Figure 4.9 illustrates that, in the pre-test, the mean score for the experimental group was 47.55, while the control group had a mean score of 48.10. The dotted red line represents the unobserved counterfactual outcome trend for the intervention, reflecting the potential outcome for the experimental group had the intervention not been implemented. The vertical dotted line delineates the two intervention phases (pre and post intervention). The pre-intervention period reveals a consistent difference in performance between the two groups. In contrast, the post-intervention period indicates an improvement in the performance of the experimental group. The post-test means for the experimental and control groups were 62.55 and 52.05, respectively. Utilising the DID formula, the effect size was calculated to be 11.05, with a mean difference of 10.53, thereby confirming that the intervention positively affected the experimental group. Given that the DID statistic accounts for other confounding variables, the results affirm that the observed outcomes were attributable to the intervention. These results signify a substantial enhancement in the performance of the experimental group, illustrating improved critical thinking skills in comparison to the control group.

4.5 Quantitative findings from semi-structured questionnaire

To answer Research Question 4, data were drawn from section A of the post intervention semi-structured questionnaires. The quantitative analysis of culturally diverse secondary school learners' perceptions and experiences regarding the use of argumentation in probability learning yielded significant insights pertinent to the research questions. The analysis revealed several key themes that consistently emerged across the focus semi-structured questionnaires and group discussions, further corroborated by learner excerpts. The identified themes include enhanced engagement, improved conceptual understanding, strengthened critical thinking skills, improved communication and collaboration, increased confidence and self-efficacy, and heightened cultural relevance. Below are the Likert responses to the first question in the questionnaire.

Table 4.5: Post intervention attitudes towards argumentation

Key: 1. Strongly disagree 2: Disagree 3: Neutral 4: Agree 5: Strongly agree	**5**	**4**	**3**	**2**	**1**
Q1: I found the use of argumentation in the classroom engaging and interesting.	6	9	4	1	0
Q2: Argumentation helped me to better understand different perspectives and opinions.	9	8	2	1	0
Q3: Through argumentation I improved my critical thinking skills.	8	8	3	1	0
Q4: Argumentation helped me to develop better communication and presentation skills.	6	8	4	1	1
Q5: I felt comfortable expressing my ideas during argumentation activities.	5	7	5	1	2
Q6: Argumentation helped me appreciate and respect different cultures and backgrounds.	6	8	4	2	0
Q7: Argumentation improved my viewpoints with evidence and reasoning.	8	8	3	1	0

The dataset in Table 4.5 consists of responses to seven questions (Q1 to Q7) obtained from a sample of 20 learners. Each question was rated on a scale from 1 to 5, with the objective of assessing various dimensions (e.g., satisfaction, performance, etc.).

Table 4.6 below presents key descriptive statistics for each question.

Table 4.6: Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation
Q1	20	2	5	4.00	.858
Q2	20	2	5	4.25	.851
Q3	20	2	5	4.15	.875
Q4	20	0	5	3.80	1.240
Q5	20	1	5	3.60	1.231
Q6	20	2	5	3.90	.968
Q7	20	2	5	4.15	.875
Valid N	20				

Table 4.7 below shows the analysis of the Means obtained from the Table 4.6.

Table 4.7: Analysis of Mean Values

Question	Mean	Analysis
Q1	4.00	The responses indicate a generally positive evaluation.
Q2	4.25	This question received the highest average rating, suggesting strong agreement or satisfaction among respondents.
Q3	4.15	Similar to Q2, this indicates positive feedback, although slightly lower than Q2.
Q4	3.80	While still positive, this question shows a decrease in average satisfaction, warranting further investigation.
Q5	3.60	This question has the lowest mean, suggesting greater variability in responses and potentially less agreement or satisfaction among learners.
Q6	3.90	This score reflects a favourable response; however, it is still lower than the means of Q1, Q2, and Q3.
Q7	4.15	Similar to Q3, this indicates overall positive feedback

From Table 4.7 the standard deviations for Q1 to Q3 are relatively low (approximately 0.85-0.88), indicating that the responses were closely clustered around the mean, reflecting consistent levels of agreement among learners. The higher standard deviations for Q4 and Q5 (1.240 and 1.231, respectively) suggest increased variability in responses. This indicates that learners held differing opinions, which were further explored through qualitative data or open-ended questions. The standard deviations for

Q6 and Q7 are moderate (0.968 and 0.875), indicating a mixture of agreement levels while exhibiting more consistency than those observed in Q4 and Q5.

This descriptive statistical analysis indicates satisfaction with the argumentation instructional approach among learners, particularly regarding questions Q2, Q3, and Q1. Conversely, questions Q4 and Q5 reveal opportunities for further investigation, as evidenced by their lower mean scores and increased variability.

4.5.1.1 Internal consistency of the Item Q1 to Q2

Internal consistency was achieved through Cronbach's Alpha analysis of the series of items in Annexure K. In this instance, the reliability of the seven questions (Q1 to Q7) was assessed and the results are shown in Table 4.8.

Table 4.8: Reliability Statistics Summary

Cronbach's Alpha	**N of Items**
.982	7

A Cronbach's Alpha value exceeding 0.9 signifies excellent internal consistency (Tavakol & Dennick, 2011). The value of 0.982 obtained in Table 4.8 implies that the items exhibited a high degree of correlation, suggesting that they effectively measured the same underlying construct. This alpha value indicates that the questions were well-aligned, facilitating reliable interpretations of participant responses.

4.6 Post-test results interpretation

The results provide compelling evidence of the intervention's effectiveness. Both groups exhibited comparable performance levels at the beginning of the study, establishing a valid foundation for comparison. While both groups demonstrated statistically significant improvement from pre-test to post-test, the experimental group showed a substantially larger improvement ($Cohen's\ d = 0.855\ versus\ d = 0.015$). The experimental group significantly outperformed the control group on the post-test, with a very large effect size. The DID mean difference of 10.53 points between the groups indicates a considerable improvement attributable to the intervention. Collectively, these results suggest that the intervention was highly effective in enhancing academic performance among learners. In the post questionnaire learners agreed that the intervention improved their understanding of probability and their ability to justify claims through evidence. The consistency of results across various statistical analysis and the large effect sizes observed further reinforce the conclusions regarding the research question.

4.6.1 ***Relating the results to literature***

The study examined the effectiveness of the argumentation intervention aimed at improving learner performance. A significant and substantial positive effect of the intervention on learners' critical thinking skills was evidenced by the high post-test performance of the experimental group compared to the control group. This study's results strongly support the emerging body of literature emphasising the efficacy of targeted educational interventions. Improvement in the experimental group, with an adjusted mean difference of 10.53 points, is consistent with meta-analyses of Hattie's (2009) influential work which shows that targeted interventions can substantially affect learner performance. The effect size of this study was 0.855, falling within the range that Cohen (1988) identifies as "large," suggesting a significant influence of the argumentation instructional approach on learner performance and critical thinking skills. Research by Dignath and Büttner (2008) has demonstrated that appropriately designed interventions can substantially enhance learner outcomes. In their meta-analysis of 74 studies related to self-regulated learning interventions, they reported effect sizes ranging from 0.2 to 0.8. This study contributes to the findings by Dignath & Büttner (2008), demonstrating the effectiveness of the intervention within the Bulawayo educational context, thereby enhancing the generalisability of such approaches. The strong relationship between pre-test and post-test scores, as indicated by the Mann Whitney *U* test results, elucidates the role of prior knowledge in learning outcomes. The equivalence of pre-test scores between groups (U = 197.00, p = 0.925) confirms the success of the random assignment process. Since the learners in both groups performed almost equally in the pre-test, any difference observed in the post-test can be attributed to the intervention rather than preexisting ability differences (Sweller & Paas,1998). While this study identified an overall significant effect of the argumentation intervention, the relationship between pre-test and post-test scores suggests that the benefits of the intervention may have varied based on the pre-test - level knowledge that learners brought to the activity. This phenomenon aligns with the aptitude-treatment interaction hypothesis by Cronbach and Snow (1977) which posits that educational treatments will have varying effects based on learner characteristics. Future studies could explicitly investigate such differential effects. This approach aligns with recent findings by Planas and Civil (2013), who demonstrated that the relative effectiveness of different instructional design elements depends on learners' levels of prior knowledge. The overall effect size of the argumentation intervention (d=0.855 indicates its potential for significant practical importance in teaching probability. Responding to a call from the literature, the argumentation intervention serves as a pertinent example of evidence-based practices in education. The study results suggest the implementation of similar interventions in analogous contexts. Meanwhile, it is essential to consider Bryk (2015) perspective, which argues that understanding how specific interventions function in particular contexts rather than assuming universal effectiveness is crucial. Although the results are promising at this level, further research is needed to identify the specific mechanisms through which the intervention exerted its effect and how these may differ across various contexts. However, the methodology of this study has several limitations. For instance, the study's design does not allow for a clear assessment of the long-term effects of the argumentation intervention, which, according to Suggate (2016) is a significant factor in his meta-analysis of reading intervention effects. He observed that most intervention effects are strong immediately following the intervention but tend to diminish over time. Consequently, it is crucial to investigate how long the attained effects of this argumentation intervention will last over time.

4.7 Analysis of questionnaire responses

This section extends the discussion beyond the statistical results by foregrounding the qualitative evidence that accounts for the patterns observed in the quantitative data. The following themes were identified from the questionnaires: 1) Inadequate pre-conceptual mastery, 2) Socio-cultural inclusivity, 3) Affective transformations, 4) Cognitive and social dimensions development, 5) Potential transfer of learning, 6) Argumentation quality aptitudes. Credibility of the qualitative data during analysis was achieved through coder reflexivity, member-checking through the post-test review quotes (Nowell et al., 2017) and the audit trail in Annexure O.

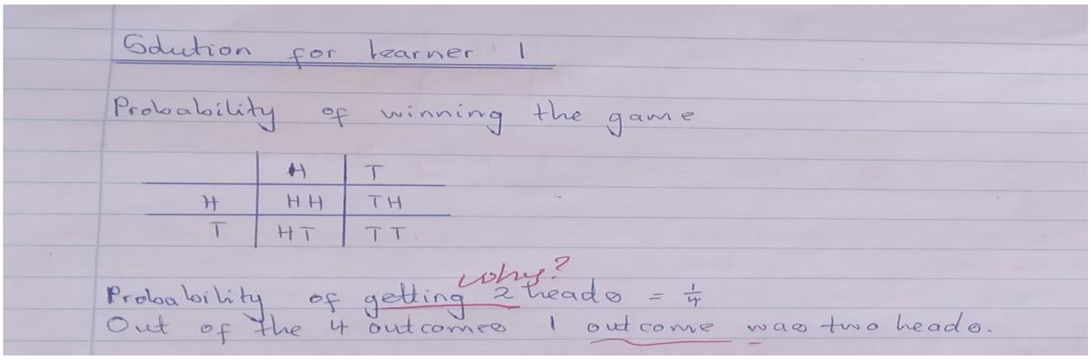
The value of this analysis lies in its capacity to highlight the dimensions of learner growth that quantitative results alone cannot capture. The analysis has a dual purpose, examining the learners' argumentation abilities prior to the intervention, and evaluating the resultant performance shifts as a result of the intervention. Themes 1 and 2 characterise the pre-intervention baseline while themes 3 to 6 trace the post-intervention transformations. Each sub-section follows a clear Assertion-Evidence-Interpretation (AEI) structure. This approach allows making a claim about learner achievement, share direct evidence or learner quotes, and then connect those moments back to broader theories in mathematics education literature.

*4.7.1 **Inadequate pre-conceptual mastery***

Establishing a diagnostic baseline of learners' pre-existing argumentation abilities was a necessary precursor to evaluating the intervention's subsequent impact. To gather this diagnostic data about whether learners could support their solutions with evidence, identify logical flaws and separate true classical probability from common cognitive biases or superstitions, learners were given an open-ended scenario (Question 4: Annexure E) in the pre-test which read as follows:

Suppose you are playing a game that involves simultaneously flipping two balanced coins. To win the game, you must obtain 'heads' on both coins. Your competitor believes heads are his lucky outcome because, in most of the games he has played, he has tossed heads more often. What is your probability of winning the game? Please share your personal interpretation and rationale for your response.

To illustrate the range of baseline responses obtained, representative examples are presented below as Figure 4.10, Figure 4.11 and Figure 4.12. These present the recurring written responses provided by learners to this question.



Solution for learner 1

Probability of winning the game

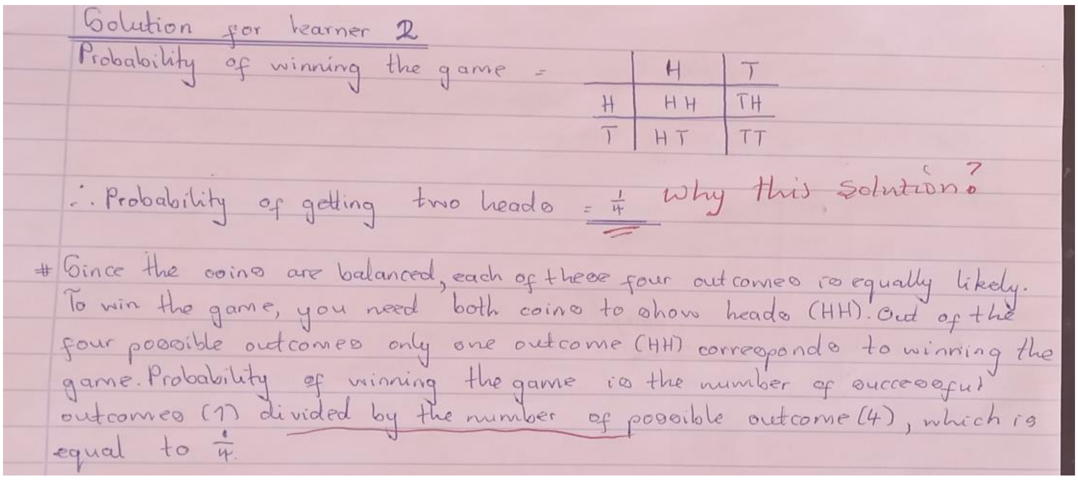
	H	T
H	HH	TH
T	HT	TT

why?

Probability of getting 2 heads = $\frac{1}{4}$

Out of the 4 outcomes 1 outcome was two heads.

Figure 4.10: Solution for learner 1



Solution for learner 2

Probability of winning the game =

	H	T
H	HH	TH
T	HT	TT

∴ Probability of getting two heads = $\frac{1}{4}$ why this solution?

Since the coins are balanced, each of these four outcomes is equally likely. To win the game, you need both coins to show heads (HH). Out of the four possible outcomes only one outcome (HH) corresponds to winning the game. Probability of winning the game is the number of successful outcomes (1) divided by the number of possible outcome (4), which is equal to $\frac{1}{4}$.

Figure 4.11: Solution for learner 2

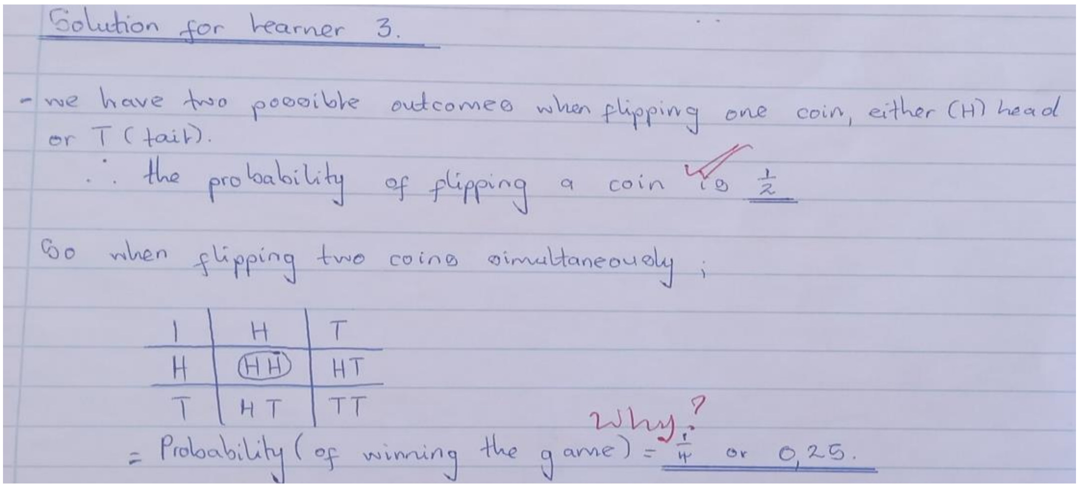


Figure 4.12: Solution for learner 3

An analysis of these responses enabled the classification of learners' baseline arguments into four distinct profiles, summarised in Table 4.9.

Table 4.9: Categorised Typologies of Pre-Test Mathematical Arguments

Case	Qualitative Typology of Learner Solution	Sample Excerpt & Structural Feature
Case 1	**Mathematically Correct & Structural Justification:** Formulates an accurate sample space (S = HH, HT, TH, TT) and identifies the compound event target.	*"The game is specifically defined as winning only when heads appear on both coins (HH). Therefore, the correct probability of winning the game is* $\frac{1}{4}$ *or 25%."*
Case 2	**Correct Numerical Value with Flawed/Un-warranted Rationale:** Generates the correct fractional or percentage value but omits conditional refutations or structural warrants.	*"There are four possible outcomes (HH, HT, TH, and TT), the probability of winning the game is* $\frac{1}{4}$ *or 0.25."*
Case 3	**Empirical/Representativeness Bias (The Gambler's Fallacy):** Validates past frequentist streaks as active physical parameters influencing future stochastic outcomes.	*"Since their competitor usually gets heads more often in most games, the probability of winning the current game is higher."*
Case 4	**Subjective/Superstitious Rationale:** Substitutes objective probability modelling entirely with fatalistic, non-mathematical attributes (luck/belief systems).	*"Because their competitor believes heads are lucky for them, the probability of winning the game is higher."*

The analysis of these responses reveals a recurring disjuncture between numerical accuracy and logical justification. As demonstrated in Case 2, learners calculated the value 25% or $\frac{1}{4}$ but failed to address or challenge the competitor's claims, presenting responses such as:

> *There are four possible outcomes (HH, HT, TH, and TT), the probability of winning the game is $\frac{1}{4}$ or 0.25.*

Such a response typifies a superficial mode of engagement grounded in mechanical pattern-matching rather than conceptual understanding. That is, they simply counted all possible options which affects real conceptual understanding. Without explicitly breaking down the multiplication of independent singular events where:

$$P(HH) = P\ (heads\ on\ the\ first\ coin)\ x\ P\ (heads\ on\ the\ second\ coin)$$

$$= \frac{1}{2} x\ \frac{1}{2} = \frac{1}{4}$$

These learners missed the underlying fact that the coins have no influence over each other. This finding aligns with Lannin and Ellis (2011), who document how a reliance on algorithmic steps can mask a deeper conceptual disconnect. The authors assert that learners can calculate a number, but fail to construct a meaningful mathematical proof. Hence, prior to the intervention learners could not present arguments to support their solutions.

4.7.1.1 Epistemological deficits and the representativeness heuristic (Case 3)

Beyond the absence of formal justification, a further pre-conceptual weakness emerged in the form of the representativeness heuristic. Thus, treating independent sequential trials as self-correcting systems that change based on past events. In Table 4.9, Case 3, the learners concurred with the competitor's winning streak, with one learner arguing that:

> *Since the competitor usually gets heads more often in most games, their probability of winning the current game is higher.*

This response shows the difficulty learners experience in reconciling intuitive judgement with the axioms of classical probability. Learners treat past events as physical forces that can shift objective mathematical odds. In classical probability theory, every single coin toss is an independent event where $P\,(A \cap B) = P(A)\ x\ P(B)$, this means that past streaks have no impact on future flips. By looking for causal connections where none exist, these learners depended on intuition rather than mathematical axioms, mirroring what Randahl (2012) called the stubborn probability misconceptions.

These epistemological challenges, however, cannot be understood in isolation from the socio-cultural context in which learners reason. This leads to the broader issue of how classroom culture influences the way learners navigate uncertainty.

4.7.1.2 Cultural inclusivity

A related but distinct pattern emerged among learners who lacked structured argumentation abilities. The learners defaulted to subjective or superstitious reasoning.

The data observed in Case 4: Table 4.9 shows that the learners who had not experienced structured argumentation experienced some mental strain in the face of mathematical uncertainty, leading them to abandon formal rules in favour of personal intuition or superstition. This trend was evident among the participants, who concluded that external, non-mathematical factors dictated the outcomes of probabilistic events. Illustrating this reliance on subjective belief, Learner M stated:

> *Because the competitor believes heads are lucky for them, hence the probability of winning the game is higher.*

This response illustrates how, in the absence of a structured framework such as Toulmin's Argumentation Pattern, learners revert to culturally embedded beliefs. Such behaviour indicates that without a rigorous framework for discourse, learners rely on intuitive, subjective language rather than mathematical evidence to express their reasoning.

Viewed alongside the post-intervention data, this baseline tendency appears less an inherent deficiency than a product of the prevailing instructional approach. This finding is supported by Dogan and Kazak (2019), who posit that traditional, unstructured instructional contexts often mask a learner's true mathematical potential by failing to provide the tools necessary to move beyond superficial interpretations. By providing opportunities for debate, the intervention transformed these subjective cultural anchors from barriers into starting points for logical deconstruction. This transition is best understood as both a cognitive and a communicative achievement, rooted in the cultural norms learners bring into the classroom.

Having established the pre-intervention baseline, the presentation focuses on the six overarching themes identified through the questionnaire analysis as shown in Table 4.10. The baseline assessment revealed that questionnaire respondents initially struggled with foundational probability concepts, often substituting logical axioms with cognitive biases.

This was supported by one learner who indicated that:

> *I struggled with the logic in coin flips until I had to explain my steps to a peer*.

This account corroborates the earlier observation that mechanical pattern-matching obscured a deeper conceptual disconnect, consistent with Lannin and Ellis's (2011) assertion that algorithmic adherence often prevents the construction of meaningful mathematical proofs confirming that learners could not initially support their claims with valid logical structures. Table 4.10 shows the prevalent themes from the questionnaires. The discussion of the themes follows in subsequent sections below.

Table 4.10: Themes derived from questionnaires

	Theme	Sub themes
1	Inadequate Pre-conceptual Mastery	Pre-Intervention argumentation profiles
		Epistemological deficits and the representativeness heuristic (Case 3)
2	Socio-cultural Inclusivity	Socio-cultural deficits and subjective anchoring (Case 4)
		Culturally rooted communication styles and norms
		Culturally responsiveness and diversity as an engine
3	Affective Transformations	Affective transformations and ownership of concepts
		Shift from conceptual isolation
		Overcoming anxiety
		Collaborative co-construction and communication
		Emotional and psychological impact
4	Cognitive and Social Dimensions Development	Critical thinking and structural evaluation development
5	Potential Transfer of Learning	
6	Argumentation Quality or Aptitudes	

4.7.1.3 Culturally rooted communication styles and norms

The first of these themes concerns the extent to which cultural communication norms constrained learner participation in traditional probability learning. Table 4.10 shows that cultural communication norms often inhibit learner participation and the presentation of arguments. This cultural dissonance was explicitly articulated by one Ndebele learner:

> *Discussing issues we are not certain about is not usually done in my culture. (Learner K)*

This testimony illustrates how prevailing social conventions regarding uncertainty can conflict with the demands of an inquiry-based instruction. In response, the intervention prioritised the creation of a culturally sensitive and inclusive environment to mitigate such hesitations. Learner C noted the necessity of this pedagogical alignment:

> *Having a teacher who can relate to our culture and make everyone comfortable enough to speak up is essential.*

Notwithstanding this broadly positive reception, a minority of learners voiced reservations regarding the pace of open discussion. One learner revealed that:

> *Sometimes it is difficult to follow the logic when everyone is talking at once. I think I learn better when I have more time to think alone before having to defend my answer to the whole group. (Learner M)*

Taken together, these accounts suggest that the facilitator's capacity to deal with diverse communication styles was instrumental in reducing performance-related anxiety.

In validating these diverse modes of discourse, the E-API model converts classroom heterogeneity into a pedagogical asset rather than an obstacle. This outcome validates the assertions of Gay (2010) regarding the efficacy of culturally responsive frameworks in empowering diverse learner populations.

4.7.1.4 Culturally responsiveness and diversity as an engine

A closely related theme concerns the extent to which culturally familiar contexts served as a bridge between abstract probabilistic concepts and learners' lived realities. This synthesis of culture and mathematics was explicitly acknowledged by Learner D:

> *Our teacher's integration of culturally pertinent examples enriched our learning experience, making it more relatable and inclusive.*

This observation is reinforced by a further account that frames argumentation-focused instruction as a mechanism for embracing academic diversity.

> *The argumentation-focused instruction embraced our diverse backgrounds, cultivating an environment that addressed different approaches to probability challenges. (Learner R)*

By contrast, reflection on the limitations of standardised curricula underscores how the exclusion of local contexts can alienate learners from the subject matter. From these contributions, it can be acknowledged that cultural diversity acts as a tool for intellectual discovery rather than a pedagogical obstacle. By leveraging the background of secondary school learners as an entry point for inquiry, this approach transforms probability into an accessible and engaging field of study. This methodology validates the efficacy of culturally responsive frameworks as outlined by Gay (2010), demonstrating that mathematics instruction is most effective when it acknowledges and integrates the sociocultural identity of the learner. Once cultural diversity is harnessed in this way, its influence extends beyond the cognitive domain into the affective disposition of learners.

4.7.2 Affective transformations

This third overarching theme is about the affective transformation that accompanied learners' engagement with the E-API model. The learners transitioned from a state of anxiety, characterised by the use of formulae in probability calculations, toward active ownership of their own cognitive processes.

In support of this development, questionnaire respondents frequently reported a marked increase in their academic self-efficacy, with one noting that they:

> *I no longer felt afraid of choosing the wrong formula, but instead I viewed each calculation attempt as a meaningful opportunity to test my logical reasoning against the problem's constraints.*

Prior instructional experiences highlight the psychological barriers that the intervention sought to dismantle. From these contributions, it can be acknowledged that by reframing mistakes as talking points for discovery rather than definitive failures, the intervention successfully dismantled the psychological barriers to probability engagement.

This affective-cognitive trajectory is summarised diagrammatically in Figure 4.13. The transition from didactic rigidity to dialogic agency supports the findings of Randahl (2012), who assert that classroom dialogue is an important mechanism for cultivating both academic confidence and sustained motivation in learners.

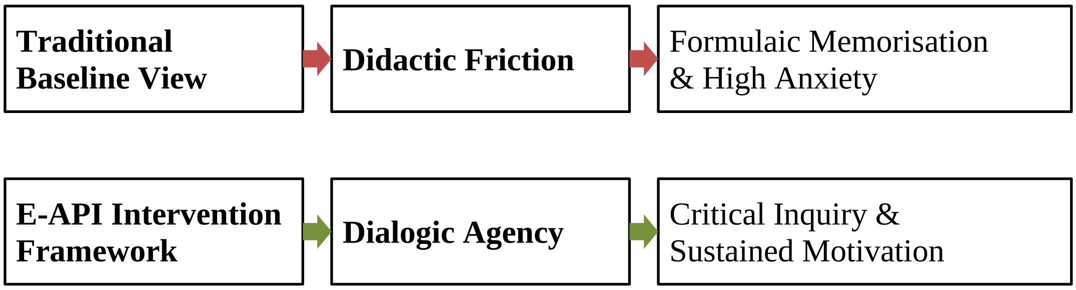


Figure 4.13: Affective-cognitive structural transition

From the learners' own perspective, this transition was underpinned by a fundamental reorientation of attitudes toward probability learning. Prior to the implementation of the E-API model, learners perceived probability as a rigid, algorithmic process devoid of real-world application. This narrow view was exemplified by one learner's reflection:

> *I see probability as only being formulas in which we simply need to plug values getting the result. (Learner P)*

This initial perception exemplifies the narrow, outcome-oriented framing that characterised learners' pre-intervention understanding of probability. In contrast, following the intervention, learners exhibited a transition toward open inquiry and contextual appreciation. As Learner A articulated:

> *Seeing probability used in everyday life makes learning interesting and understandable.*

When reflecting on the barriers to learning, participants noted that the "plugging of values into formulae" mindset hindered their ability to construct convincing, evidence-based arguments. From these contributions, it can be acknowledged that the E-API model effectively dismantled these rigid frameworks, transforming abstract formulae into functional tools for navigating uncertainty. By reframing errors as essential points for dialogue rather than indicators of academic deficiency, the intervention successfully mitigated the frustration and anxiety inherent in traditional probability instruction. This shift validates the findings of Can and Isleyen (2016), who posit that addressing the affective dimensions of learning, specifically the fear of failure, is a prerequisite for fostering genuine engagement with probability reasoning and the development of robust argumentative skills.

4.7.2.1 Shift from conceptual isolation

A more specific manifestation of this affective shift concerns learners' shift away from conceptual isolation. This was evidenced by the sentiment:

> *I am confused about why probability is something we must learn. I do not see any connection to what I do. (Learner Q)*

This frustration highlights the extent to which probability theory had remained detached from learners' lived experience prior to the intervention. In contrast, following the pedagogical intervention, learners expressed a marked shift in perspective regarding the utility of statistical frameworks. Illustrating this transition, Learner H noted:

> *If we got to see how probability helps individuals make decisions or assess the chances of risks, it would be more relevant to us.*

Further reflection on these limitations reinforces the view that pedagogical efficacy is contingent upon the integration of tangible, real-world applications. As one learner articulated:

> *Before, the lessons felt like isolated exercises. We were just calculating numbers without understanding the 'why' or the 'so what' of these tools in our daily lives. (Learner G)*

Collectively, these accounts indicate that grounding instruction in real-world examples transforms probability from abstract exercises into functional decision-making tools. This shift aligns with the conceptual framework established by Bruine de Bruin and Carman (2012), who argue that building enduring statistical literacy requires transitioning learners from passive recipients of theory to active participants who recognise the practical utility of mathematical frameworks within their own ecological contexts.

4.7.2.2 Overcoming anxiety

A further dimension of this affective transformation relates to the anxiety learners initially associated with the uncertainty inherent in probability. This preoccupation with certainty was explicitly articulated by Learner S:

> *Dealing with the unknown is difficult. I wish to know for sure what will happen, not only the chances.*

This sentiment shows how an educational culture oriented toward deterministic certainty can impede engagement with stochastic reasoning. However, following the pedagogical intervention, learners demonstrated a shift in orientation, prioritising independent cognitive engagement over the rote memorisation of formulae. As Learner D noted:

> *I prefer to think for myself, rather than only relying on set rules.*

Deliberating on the underlying cause of their previous understandings, learners emphasised that the fear of error stemmed from viewing uncertainty as a failure rather than an inherent property of probability. From these contributions, it can be acknowledged that when uncertainty is framed as a natural element of the mathematical inquiry process, the psychological barrier to participation is lowered. By facilitating peer-led dialogue, the intervention allowed learners to navigate abstract concepts within a friendly social space. This transition supports the framework proposed by Randahl (2012) who posit that structured classroom debate serves as a critical mechanism for fostering academic confidence and mitigating the anxieties associated with complex probabilistic reasoning.

4.7.2.3 Collaborative co-construction and communication

Linked to this affective shift is the emergence of collaborative co-construction as a defining feature of the classroom experience. This collaborative shift was clearly articulated by Learner T, who stated:

> *The argumentation activities enabled me to convey my ideas more effectively... collaborative problems focused on argumentation helped us learn from one*

another, practice attentive listening, appreciate diverse perspectives, and reach consensus.

This account underscores how structured dialogue reframes probability engagement as a shared, rather than a solitary, endeavour. This sentiment is echoed in a further account that foregrounds the affective benefits of the team-based approach. Learner F highlighted the affective impact of this shift, noting that the team-centred approach made learning about probability both captivating and meaningful.

In contrast to traditional, individualistic instruction, learners identified that the complexity of probability, which often necessitates exploring multiple pathways to a solution, is best navigated through sustained group interaction. As one learner observed during the discourse:

> *When we discuss the problems together, it isn't just about finding the right formula. We break down the complicated codes into language we actually use in our daily lives, and that makes the math stick. (Learner P)*

These reflections indicate that synthesising peer perspectives enabled learners to translate abstract probability terms into accessible, everyday language. This methodology demonstrates what Mercer and Sams (2006) characterise as the power of shared communication in solving complex mathematical problems. By fostering an environment where negotiation of meaning is prioritised, the intervention effectively shifts the classroom from passive reception to an active, social community of inquiry.

4.7.2.4 Emotional and psychological impact

The final dimension of this affective transformation concerns the emotional and psychological benefits associated with a more conversational instructional style. This personal transformation was exemplified by a learner who noted:

> *Prior to engaging in argumentation, I lacked confidence in my probability skills; however, as my involvement grew, I recognised the utility of my ideas, boosting my self-assurance. (Learner M)*

Other participants reflected on how lecture-based methods had previously marginalised their voices and discouraged participation. As one learner observed:

> *Before, I felt my input didn't really matter as long as I got the final number right. Now, I feel that my process and my reasoning are actually valued as part of the lesson. (Learner L)*

This shows that when classroom practices centre on learner voices and treat individual ideas as substantive contributions, there is a fundamental shift in the affective relationship between the learner and the subject matter. By creating an environment that encourages intellectual autonomy, the intervention enables learners to cultivate trust in their own cognitive processes. This outcome aligns with the psychological self-efficacy frameworks outlined by Sampson and Clark (2008), which posit that an environment fostering meaningful contribution and perceived competence is essential for building sustained academic confidence.

As learners move from a state of anxiety toward a position of emotional security and self-worth, they become better equipped to handle complex cognitive demands of structured mathematical inquiry.

4.7.3 ***Cognitive and social dimensions development***

The fourth theme to emerge is the development of cognitive and social competencies through sustained engagement with structured argumentation. This cognitive development was explicitly noted by Learner Q, who stated:

> *The necessity to question assumptions and reassess evidence significantly bolstered my critical thinking capabilities.*

Echoing this transition toward rigorous intellectual scrutiny, Learner P added:

> *I constantly scrutinised reasoning for potential errors during argumentation activities, which sharpened my analytical and problem-solving abilities in probability. (Learner P)*

These contributions imply that when learners are required to justify their claims through structural models, such as Toulmin's Argumentation Pattern, they internalise a systematic checklist for logical validation.

It is clear that without such frameworks, the logical structure of probability remains opaque to learners. From these contributions, it can be acknowledged that structured debate functions as a catalyst for conceptual change. This outcome validates the assertions of Nussbaum, Sinatra, and Poliquin (2008), who maintain that engaging in formal argumentation is essential for moving learners beyond rote procedures toward a more profound, critical understanding of mathematical content.

4.7.3.1 Critical thinking and structural evaluation development

To substantiate the claim of cognitive growth, a post-test task requiring the critique of a flawed statistical argument was analysed in greater depth (Annexure I: Question 4: Post-test). The problem required them to critique a defence attorney's flawed statistical claims regarding spousal homicide rates following a documented history of domestic abuse. To provide empirical validation of the efficacy of the E-API model the written responses from Learner G (Initial Argument: Annexure P) and Learner H (Counter-Argument: Annexure Q) were purposively chosen from the independently marked scripts. These were identified as exemplars of practice due to the learners' responses alignment to Toulmin's argumentation pattern. Learners G and H exhibited a high command of the model's components especially grounds, warrants, and backings to support their probabilistic claims. By isolating the two cases, the study illustrates the highest cognitive potential that learners can attain when exposed to the E-API model.

Table 4.11: Learner G's Initial argument versus learner H's counter-argument

TAP Structural Component	Learner G's Initial Argument Profile	Learner H's Counter-Argument Profile
Claim	The defence's statistical rationale is flawed and mathematically invalid.	The defence's legal-mathematical argument is reasonable and not fundamentally flawed.
Data	The defence equates two distinct conditional probabilities and cites a baseline risk of 0.0004.	The defence isolates the conditional context, providing a quantitative metric of 40 per 100,000.
Warrant	Valid statistical models require analysing both the conditional probability and the base rate of partner homicides.	Focusing tightly on a specific conditional context is a valid strategy for isolating key variables.
Qualifier	Recognises that the defence's arithmetic is valid within its narrow scope, but remains incomplete.	Acknowledges that the defence's approach has structural limitations and omits external variables.
Backing	Cites Bayes' Theorem as the definitive framework for updating conditional probabilities.	Points to the utility of localised quantitative risk metrics within targeted forensic contexts.
Rebuttal	Highlights that the defence omits the critical data needed to calculate P(Battering | Homicide).	Fails to directly address the missing base rate data or reconcile the missing inverse probability.

Analysis of these post-intervention scripts through Toulmin's framework indicates that structured argumentation equips learners to construct rigorous mathematical proofs. As shown in Table 4.11, Learner G methodically picked apart a flawed courtroom claim by using Bayes' Theorem as Backing, while explicitly using the missing inverse probability data as a Rebuttal*:*

> *The defence's claim is incorrect because it fails to take base rates into account and lacks the needed information to properly calculate the probabilities connected to domestic homicides.*

This argument is an improvement from the un-warranted assertions witnessed in the pre-test. By explicitly tying their data to their claims using formal mathematical warrants, the learners proved they had developed understanding of conditional probability. Even though learner H's counter-argument glossed over the missing base rate data, the highly structured, back-and-forth debate between the two positions shows that the classroom had evolved into a genuine academic forum, reflecting Driver, Newton, and Osborne's (2000) ideas on how to build an active community of mathematical proof through argumentation. Having demonstrated this capacity for structural validation within the classroom, focus now turns to whether these skills transfer beyond it.

*4.7.4 **Potential transfer of learning***

To assess whether the cognitive gains observed within the classroom extended to novel contexts, four exemplar scripts addressing the hypothetical court case were purposively selected as shown in Table 4.12. These exemplars were selected based on their alignment with the E-API model. The selection criteria focused on the learners' capacity to transition from intuitive judgements to structured and evidence-based reasoning as evidenced by the depth of the argumentative discourse. The scripts were chosen because they demonstrated the learners' ability to transfer their argumentative abilities to the hypothetical court case. During the analysis of the court case, four key issues were raised and supported by the learners as shown in Table 4.12.

Table 4.12: Key issues raised by the learners

Learner	Key issues raised by the learners
D	*"The defence's argument creates reasonable doubt in the minds of the judge or jury. Statistical evidence suggests a low probability of murder given the history of battering; the defence aims to cast doubt on the prosecution's case and weaken the argument that the husband is the likely perpetrator. This could potentially lead to an acquittal or a less severe conviction"*
C	*"The defence's presentation undermines the motive attributed to the accused husband. By challenging the connection between past incidents of battering and the likelihood of murder, they argue that the history of abuse does not necessarily imply a motive for committing the crime. This weakens the prosecution's case by suggesting alternative explanations or motives for the murder*"
G	*"Presenting statistical evidence that questions the relevance and strength of the evidence against the accused, the defence shifts the burden of proof onto the prosecution. They argue that the conditional probability of murder given the history of battering is too small to be useful in a trial, suggesting that the prosecution has not provided sufficient evidence to establish guilt beyond a reasonable doubt"*
H	*"The defence's presentation can potentially influence the judge or jury's perception of the case. Statistical evidence, particularly when presented by a renowned law professor or expert, can carry significant weight and influence the opinions of decision-makers. If the defence effectively challenges the prosecution's case using these statistics, it may sway the decision in favour of the accused"*

An analysis of these narratives indicates that the intervention produced a measurable improvement in learners' capacity to transfer reasoning to unfamiliar, real-world situations. This cognitive advancement is consistent with the findings of Siseho, Angaama, and Ogunniyi (2013), who assert that critical thinking is not an inherent trait but one actively cultivated through sustained engagement in argumentation.

Reflection on prior instructional limitations further clarifies how the intervention bridged the gap between classroom exercises and everyday logic. From these contributions, it can be acknowledged that by grounding probability instruction in structured argumentation, the intervention moved learners beyond the rote application of formulae toward a flexible, analytical framework. This development demonstrates that when learners are challenged to defend their positions, they acquire a transferable mental model that informs their decision-making in diverse contexts. Consequently, this shift validates the theoretical position that argumentation serves as a primary vehicle for developing the cognitive adaptability required for effective, real-world statistical literacy. Notwithstanding this evidence of successful transfer, the quality of learners' arguments was not uniform across the sample.

A closer reading of the four sampled responses in Table 4.12 confirms a broader shift from assertion-based claims toward evidence-supported argumentation. While early assessments lacked supporting data or rebuttals, the post-intervention scripts revealed an improvement in the integration of probability concepts into structured logical frameworks. As one independent marker voluntarily commented:

> *The scripts consistently showed a shift toward including explicit warrants and backings. Learners no longer merely stated a probability value; they justified their conclusions by citing the specific conditions or constraints within the problem scenario. (Independent Marker Report)*

While the overall performance improved, some learners continued to struggle with the 'rebuttal' dimension, indicating that while learners can effectively construct their own arguments, the reflexive practice of anticipating and countering opposing viewpoints remains a developing skill. From these contributions, it can be acknowledged that the argumentation approach successfully scaffolds the logical construction of probabilistic reasoning. This development aligns with the findings of Erduran, Simon, and Osborne

(2004), who argue that the quality of scientific and mathematical argumentation is contingent upon the frequency and nature of exposure to structured discourse.

*4.7.5 **Argumentation quality or aptitudes***

To further characterise this variability, a purposively selected subset of ten scripts was evaluated for overall argumentation quality as guided by Annexure N. From among the subset, Learner T revealed low quality of arguments while learner P, revealed high quality arguments. When learners were given the legal problem about the "Prosecutor's Fallacy," two perspectives surfaced, one showing that some learners developed critical thinking skills while others remained stuck in the low thinking category.

One high-performing script, produced by Learner P, exemplifies the standard of critical thinking the intervention sought to cultivate. The learner presented that:

> *The defence's argument is flawed because it commits the Prosecutor's Fallacy by focusing on the wrong conditional probability. The law professor calculated the probability of a woman being murdered given she is battered, P(M/B) = 0.0004. However, the relevant question is the posterior probability of guilt once a murder has already occurred, P(K/M).*

This response exemplifies what Sampson and Clark (2008) term 'epistemic agency', whereby a learner moves beyond fact recitation to construct a reasoned argument. By pointing out the error in the lawyer's probability calculation, Learner P did not just show that they could do the calculation, but showed understanding of how to build a logical case. As Osborne (2010) notes, this is how learners move away from common misconceptions toward a solid, scientific way of understanding the world.

By contrast, a lower-quality script from Learner T illustrates the difficulty some learners faced in setting aside personal conviction in favour of evidence. The argument presented unfocused claims that lacked empirical data and exhibited flawed reasoning. This is shown through T's excerpt:

> *I think the defence is wrong because it is unfair to use mathematics to decide if someone is a murderer... You cannot just use numbers to say someone is guilty, because every crime is different and the mathematics doesn't account for the truth.*

This response reflects a broader tendency for learners to retreat into personal moral reasoning when probability intersects with culturally weighted subject matter. Learner T focused on the idea of "fairness" instead of looking at the actual numbers provided. As Kuhn & Crowell (2011) explain, these learners often mistake their personal beliefs for factual arguments; they see mathematics as just another opinion to argue against, rather than as a tool to help uncover the truth.

The contrast between these two learners captures the central premise of the E-API model: that argumentation is a mode of thinking rather than a performance skill. Learner P used data to build a logical explanation to their conclusion, while Learner T remained stuck in their own intuition. It shows that while our teaching methods may be effective at building logical skills, we still need to help learners get past the mental conflicts of applying probability to real-life.

These findings are consistent with prior research, which similarly affirms that structured activities can develop learners' argumentation skills with minimal guidance (Venville and Dawson, 2010). Moreover, Walker and Sampson (2013) found that learners achieve progress in arguing, both in speech and in writing, by applying specific learning techniques. Nazila et al. (2019) also argue that the Argument-Driven Inquiry (ADI) model helps learners become more logical in their thoughts and better able to express themselves.

4.8 Analysis of focus group responses

This section now focuses on the focus group data, which offer a complementary, dialogic perspective on learners' experience of the intervention (Annexure L). To understand the learners' experiences, the discussion began by examining how the intervention restructured their foundational approach to probability, moving from rote memorisation toward cognitive engagement.

4.8.1 Overall experience with the intervention

In response to the first question,

Q: How did you find the intervention in understanding probability concepts?

The group described a shift from cultural justification and formulaic memorisation toward conceptual inquiry. Initially, learners faced cognitive dissonances when trying to reconcile their cultural beliefs regarding probability with formal probability rules. Through the E-API framework, the group externalised these internal logical contradictions, collectively transitioning from the pursuit of singular numerical results toward analysis of the underlying logic governing probabilistic events.

This shift is captured in the following group responses.

> *At first, many of us had different answers for probability problems because of our varying cultural views on luck and fate, but arguing our positions forced us to identify where our logical challenges were.*

Another supporting contribution was:

> *At first, we all just wanted the formula to get the answer, but the group work forced us to actually think about why the probability happens the way it does, which changed how we look at probability entirely.*

This transition marks a departure from the communication patterns typically observed in traditional mathematics classrooms (Al-Ajmi & Ambusaidi, 2022). By requiring the verbal justification of claims, the intervention served as a diagnostic mechanism for conceptual growth. This aligns with the findings of Can and Isleyen (2016), who suggest that addressing the affective and cognitive dissonance inherent in stochastic learning is a requisite precursor to genuine probability reasoning. This shift in cognitive approach prompted the participants to reflect more broadly on the pedagogical framework of the intervention in contrast to their previous educational experiences.

4.8.2 Comparison of the teaching approaches

In order to address this thematic issue, the following question was discussed:

Q: How did the intervention compare to traditional learning methods you have experienced in the past?

This second question elicited a direct comparison between the E-API model and learners' prior instructional experiences. The transition to the Enhanced Argumentation Probability Instruction (E-API) model prompted learners to reject the passive, outcome-oriented nature of traditional probability instruction in favour of an approach that values reasoning and conceptual utility.

Participants explicitly contrasted these experiences, noting that traditional instruction had reduced probability to a series of isolated, meaningless exercises. The group concurred that:

> *Before, the lessons felt like isolated exercises. We were just calculating numbers without understanding the 'why' or the 'so what' of probability in our daily lives.*

Participants further highlighted a corresponding shift in their perceived role within the classroom:

> *Before, we felt as if our input didn't really matter as long as we got the final answer right. Now, we feel that our processes and reasoning are valued as part of the lesson.*

This feedback suggests that the E-API framework successfully dismantled the ‘didactic contract’ that conditions learners to prioritise correct answers over sound reasoning. By shifting the focus from numerical accuracy to the justification of reasoning, the intervention empowered learners to view themselves as active participants in the knowledge-construction process rather than mere receptacles of procedural rules. This movement from proceduralism toward discursive agency is consistent with the pedagogical critique advanced by Al-Ajmi and Ambusaidi (2022).

This movement from proceduralism toward discursive agency is consistent with the pedagogical critique advanced by Al-Ajmi and Ambusaidi (2022) who argue that traditional "calculational" classrooms often stifle the development of probability thinking by isolating learners from the context of their own inquiry. The learners’ desire for relevance and process-based valuation aligns with recent findings by Can and Isleyen (2016), which suggest that when learners are given an enabling environment to reconcile their own reasoning with formal probability, they transcend the frustration of isolated calculation to achieve a more integrated understanding of stochastic concepts.

Having identified these fundamental differences in teaching methods, the discussion proceeded to examine the specific elements of the E-API framework that most effectively sustained this newfound level of learner engagement.

4.8.3 Engagement through culturally responsive pedagogy

The following is a presentation of the responses to the question:

Q: How did the intervention enhance your engagement with the probability content?

In response to the third question, the group identified the integration of culturally pertinent contexts as a catalyst for engagement. By grounding abstract probabilistic tasks in familiar scenarios, the subject matter was transformed from an alien, academic exercise into a functional tool for analysing daily realities. This increased the learners' willingness to contribute, as the content was finally perceived as relevant to their lived experiences. The group concurred:

> *We noticed that when we brought in our own cultural experiences from around Bulawayo, probability ceased feeling like abstract tasks and it became relatable scenarios we actually understood.*

This finding corroborates the assertions of Gay (2010) regarding the efficacy of culturally responsive pedagogy. By bridging the gap between home-based knowledge and probability, the E-API model allowed learners to perceive their sociocultural identities as intellectual assets, transforming them from passive recipients of information into active participants within the classroom community. Having established how cultural framing facilitates engagement, the conversation shifted toward the practical utility of the content, specifically how collaborative argumentation functions as a tool for decoding abstract probability theory into everyday application.

4.8.4 Real-world application and conceptual decoding

The fourth question:

Q: Did the argumentation activities provide opportunities for you to connect probability concepts to real-life situations?

Probed the extent to which argumentation activities connected probability concepts to real-life situations, revealing a process the group described as ‘decoding’. Through sustained social interaction, learners successfully translated theoretical concepts into practical frameworks for risk assessment and decision-making, effectively dissolving the alienation associated with traditional, non-contextualised lecturing. The participants explained:

> *When we discuss the problems together, it isn’t just about finding the right formula. We break down the complicated codes into language we actually use in our daily lives, and that made probability to stick.*

This process of collaborative deconstruction mirrors the framework proposed by Bruine de Bruin and Carman (2012), which links statistical literacy to the recognition of practical utility. As learners began to bridge the gap between theory and life, they also recognised that their diverse cultural backgrounds were not merely contextual backdrops, but active resources that enriched the quality of the classroom debate.

4.8.5 Cultural diversity as a pedagogical asset

This theme was addressed through answering the question:

Q: Did the argumentation activities allow for diverse perspectives and cultural experiences to be shared and valued?

The question examined whether the intervention created an environment for diverse cultural perspectives, with learners affirming that it did. Rather than treating sociocultural background as a hindrance to standard performance, the model utilised these differences as legitimate starting points for logical debate, fostering mutual respect and validating the diverse voices within the classroom. The participants put it that:

> *It was encouraging that our diverse perspectives were valued; we were allowed to challenge each other while we still valued the different ways we naturally interpreted chance in our daily lives.*

This finding reflects the transformation of classroom diversity from a potential obstacle into a genuine pedagogical resource, corroborating Gay (2010). The instructional environment successfully moved away from a singular, standardised viewpoint toward a community of inquiry where sociocultural identity is recognised as an integral and enriching component of the learning process. This appreciation for diverse perspectives necessitated a new social dynamic, leading the group to reflect on how collaborative interactions fostered the communication skills required for collective meaning-making.

4.8.6 Collaborative co-construction of meaning

The sixth question:

Q: How did the argumentation activities promote collaboration and communication among the learners, and were you comfortable expressing your ideas?

Addressed collaboration and communication, with the cohort identifying active listening as a foundational skill cultivated through the intervention. By reframing the classroom as a collaborative community, the group reported a significant reduction in the anxiety traditionally associated with competitive, error-intolerant environments. Learners felt empowered to prioritise the logic of their arguments over the fear of academic failure. They agreed that:

> *We had to learn how to actively listen to a peer's viewpoint before we could logically critique it, and this interaction became the foundation for solving problems together.*

This shift toward collective inquiry mirrors the process of 'negotiation of meaning' described by Mercer and Sams (2006). By prioritising social interaction, the learners moved away from isolated, individualistic struggle toward a collective intellectual endeavour, which effectively mitigated the psychological barriers that historically hindered participation. Having established collaborative interaction as foundational to meaning-making, discussion turned to its broader cognitive and affective consequences.

4.8.7 *Cognitive development and affective transformation*

The final question:

Q: Did the intervention enhance your critical thinking and problem-solving skills, and has it influenced your attitudes towards mathematics?

Addressed the intervention's impact on critical thinking and attitudes toward probability, with the group affirming improved analytical capability. By internalising a systematic approach to justifying claims, participants grew more adept at scrutinising potential errors and communicating ideas clearly. This shift replaced an intimidatory, anxiety-filled perception of probability with a sense of personal agency and sustained motivation. In support of this the group members asserted that:

> *The argumentation sessions definitely improved our social skills; we had to learn how to actively listen to a peer's viewpoint before we could logically critique it, and that forced us to think more clearly.*

Participants further elaborated on this development:

> *Beyond probability, the interactions refined how we structure our arguments; we are now much more capable of communicating our ideas to each other clearly and logically.*

This development lends further support to the assertions of Nussbaum, Sinatra, and Poliquin (2008), and aligns with Randahl (2012)'s account of classroom debate as a vehicle for academic confidence. Furthermore, it aligns with Randahl (2012) who assert that classroom debate serves as a mechanism for cultivating academic confidence, transforming the learners' affective relationship with probability from one of frustration to one of competence. Ultimately, the development of these collaborative social skills had a profound impact on the learners' internal states, facilitating a transition from academic anxiety toward personal agency and critical confidence.

4.9 Interpretive synthesis of triangulated narratives

While these focus group insights provide a detailed account of the learner experience, the robustness of these findings is strengthened when synthesised with the broader data set collected through questionnaires. To ensure the depth and validity of the study, the qualitative insights derived from the learner questionnaires (Section 4.7) were triangulated with the focus group discussion findings (Section 4.8). By analysing the questionnaire data as reflective narrative accounts rather than isolated data, this synthesis reveals a convergence in the learners' lived experiences during the E-API intervention. Table 4.13 presents this synthesis, highlighting the alignment between individual questionnaire responses and the collective narratives shared during focus group discussions.

Table 4.13: Triangulation of qualitative themes

Thematic Dimension	Questionnaire Reflection (4.7)	Focus Group Narrative (4.8)	Triangulated Qualitative Narrative
Cognitive Agency	Learners described a move away from rigid, formulaic thinking (4.7.1).	Participants articulated a shift toward active “dialogic agency” (4.8.5).	A collective rejection of “plug-and-chug” in favour of critical inquiry.
Cultural Integration	Learners noted cultural norms previously acted as quiet, latent barriers (4.7.2).	Participants described local contexts as “bridges” to understanding (4.8.1).	Cultural identity transformed from a source of dissonance to a pedagogical bridge.
Affective transformations	Responses highlighted a transition from intimidation to self-efficacy (4.7.3).	Participants emphasised the “safety” of collaborative debate (4.8.5).	The creation of a secure social space for intellectual risk-taking.
Collaborative Discourse	Learners identified the social nature of math as a new discovery (4.7.6).	Participants pointed to “active listening” as the catalyst for logic (4.8.4).	Collaborative talk as the primary vehicle for mathematical sense-making.
Conceptual Utility	Learners reflected on the newfound relevance of probability (4.7.5).	Participants described “decoding” abstract theory into daily life (4.8.2).	Probability reconceptualised as a functional tool for personal decision-making.

4.9.1.1 The reconfiguration of learner agency

The first dimension of this triangulated synthesis concerns the reconfiguration of learners’ epistemic agency. The following sub-sections synthesise these qualitative sources, beginning with an exploration of how the intervention reconfigured the learners’ sense of agency. Triangulation of the questionnaire reflections and focus group narratives reveals a shift in the learners’ relationship with probability. Questionnaire responses characterised previous learning as an “isolated struggle” with meaningless symbols. This is corroborated by the focus group’s vivid descriptions of “didactic friction.” Both data sources converge on the realisation that the E-API model replaced passive compliance with “dialogic agency”. As Al-Ajmi and Ambusaidi (2022) suggest, this indicates a successful movement from culture-based reasoning to authentic mathematical discourse, where learners view their own reasoning as the primary driver of learning.

4.9.1.2 Cultural identity as a pedagogical catalyst

Building on this reconfiguration of learner agency, the second dimension of the synthesis addresses the role of cultural identity as a pedagogical catalyst. Beyond the individual's role in the classroom, the synthesis also underscores how the integration of cultural identity served as a primary catalyst for intellectual participation. Both the questionnaire responses and focus group discussions demonstrate that culture is a pivotal factor in the learning process. The questionnaire narratives highlighted how traditional classroom norms silenced learners, while the focus group discussions provided the counter-narrative: by embedding the curriculum in the lived realities of Bulawayo, the intervention converted these *latent barriers* into *intellectual assets*. This synthesis confirms that culturally responsive pedagogy, as championed by Gay (2010), is not only an inclusive ideal but a necessary mechanism for transforming the classroom from an alienating environment into a community of inquiry.

4.9.1.3 The Social scaffolding of logical rigour

Having established agency and cultural identity as key dimensions of the synthesis, focus is now on the social scaffolding that underpinned learners' logical rigour. In addition to the roles of agency and culture, the data convergence highlights that the attainment of logical rigour was inextricably linked to the social scaffolding provided by peer discourse. The most important convergence across the questionnaire and focus group data are the emphasis on the social nature of analytical growth. Learners consistently reported that their ability to structure a logical argument (the “quality of argumentation”) was inseparable from the social requirement of “attentive listening.” This validates the assertion by Randahl (2012) that classroom debate serves as a critical mechanism for both academic and affective development. The triangulation suggests that the E-API model’s success lies in its ability to harness peer interaction as a scaffold; the learners did not only learn probability through instruction, but through the collaborative negotiation of meaning (Mercer & Sams, 2006).

4.9.1.4 Concluding synthesis

Drawing these three thematic dimensions together, the triangulated evidence culminates in a comprehensive account of the E-API model’s transformative effect on the classroom environment. Finally, by integrating these thematic dimensions, a comprehensive picture emerges of how the E-API model successfully transformed the classroom environment to support deeper probabilistic reasoning. The consistency between the reflections in the questionnaires and the in-depth discussions of the focus groups provides strong qualitative evidence that the intervention successfully dismantled the affective and cognitive barriers to probabilistic reasoning. This triangulation supports the model’s efficacy, illustrating that for secondary school learners, deep conceptual mastery is achieved when analytical rigour is paired with social, culturally-situated dialogue.

4.10 Sequential explanatory data integration

To strike a balance between encouraging diverse, intuitive learner ideas and upholding probability accuracy, the E-API model builds a specific Convergence Phase into every single lesson cycle, as sketched out in Figure 4.14.

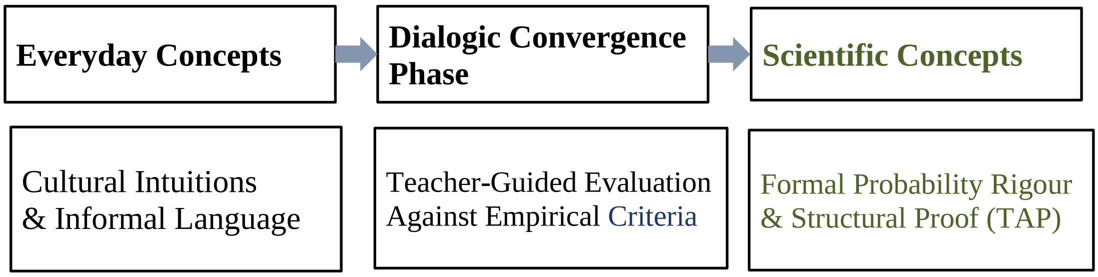


Figure 4.14: The E-API Convergence Phase

To maintain the rigour in a culturally diverse classroom, the teacher needs to guide learners to test their informal, intuitive assumptions against formal probability. The instructional sequence handled this transition by introducing a deliberate environment where learners could weigh competing peer arguments against formal criteria, statistical data, and logical proofs. This phase brings Vygotsky's (1978) famous distinction between everyday concepts and scientific concepts to life. By using everyday cultural intuitions as a starting point rather than the final destination, the convergence phase ensures that classroom discussion empowers learners toward deeper understanding. This balance uses cultural diversity as an engaging gateway while successfully guiding learners to master formal mathematical literacy (Howe & Abedin, 2013; Stylianides, 2007).

In accordance with the sequential explanatory mixed-methods design, the quantitative and qualitative datasets are not treated as parallel architectures, but are integrated to provide a unified explanatory framework (Creswell & Creswell, 2018). Quantitative findings of the study show the macro-level efficacy of the Enhanced Argumentation Probability Instruction (E-API) model, demonstrating a statistically significant post-test performance improvement ($U = 28.000$, $p < 0.05$), a large effect size (Cohen's $d = 0.855$), and a causal treatment effect ($DID = 11.05$). However, these numerical results only describe the *magnitude* of the improvement; the qualitative thematic analysis serves as an explanatory mechanism revealing the *why* and *how* of the numerical gains. Through mapping empirical performance shifts directly onto the learners' experiences and cognitive transformations, the qualitative data explains the processes that caused the statistical successes (Fetters et al., 2013).

To illustrate this integration, Table 4.14 links the primary quantitative achievements to their corresponding qualitative explanatory themes. The baseline pre-test statistical equivalency ($U = 197.00$, $p = 0.925$) and the low performance medians (≤ 50.00) are explained by the diagnostic profiles as shown in (Cases 2, 3, and 4), which revealed that the cohort was entrenched in formulaic determinism, the gambler's fallacy, or socio-cultural superstitious anchoring prior to the intervention. Post-intervention, the compression of variability in the experimental group ($IQR = 6$) compared to the persistent heterogeneity of the control group ($IQR = 12$) is clarified by the qualitative themes of collaborative co-construction and peer dialogue. The prior traditional teaching approaches left learners isolated, which widened individual performance gaps, whereas the E-API model's structured argumentation allowed underperforming learners to leverage collective peer clarity, systematically uplifting lower outliers and homogenising performance around proficient execution (Mercer & Sams, 2006).

Table 4.14: Quantitative empirical indicators and qualitative explanatory themes

Quantitative Milestone	**Linked Qualitative Profile and Theme**	**Explanatory Mixed-Methods Integration**
Pre-test Baseline Equivalency U = 197.00, p = 0.925 Low Medians Exp = 50.00; Cont. = 48.50)	Case 3 and Case 4 Typologies: Empirical Bias and Superstitious Anchoring *(70% of combined sample)*	Heuristic Deficits: The low baseline scores and non-normal data distributions were directly caused by a widespread reliance on non-mathematical attributes (luck) and cognitive biases rather than formal probability axioms (Randahl, 2012). In FGDs and questionnaires, learners stated that they previously viewed probability as a guessing game based on "streaks or personal luck." Without a logical scaffold, they defaulted to everyday cultural intuitions rather than mathematical rules (Randahl, 2012). As one learner shared, *"Before this, we just calculated numbers without meaning, guessing what felt lucky."*
Post-test Performance improvement U = 28.000, p = 0.015 Cohen's d = 0.855; DID = 11.05	Cognitive and Analytical Sharpness Internalisation of Toulmin's structural scaffolding *(Learner G and H post-test profiles)*	The statistical growth was driven by training learners to abandon intuitive shortcuts. Operating within the TAP framework forced them to build explicit links between mathematical data, structural warrants, and contextual rebuttals (Nussbaum et al., 2008). In FGD and questionnaires, learners highlighted that utilising claims, data, and warrants forced them to drop incorrect assumptions. Challenging each other's ideas sharpened their problem-solving and critical thinking skills, driving the statistical post-test leap (Nussbaum et al., 2008). A learner noted, *"The activities forced me to think for myself instead of memorising fixed rules."*
Compressed Experimental Variance Experimental IQR = 6 vs. Control IQR = 12	Collaborative Co-construction Peer dialogue, shared communication, and socio-axiomatic convergence	Group-centred argumentation minimised individual isolation. It allowed learners to translate confusing mathematical codes into shared language, standardising performance around the median (Mercer & Sams, 2006). Traditional mathematics classes isolate learners. FGD feedback showed that group argumentation allowed underperforming learners to leverage collective peer clarity, dragging lower outliers up and creating highly consistent final marks (Mercer & Sams, 2006). One participant recalled, *"Working together helped us translate confusing mathematics formulas into everyday language we all understood."*

The raw figures in Table 4.14 show a shift in how the learners actually think and learn. Initially, the learners were unequipped to handle probability through argumentation. Without this logical framework to lean on, learners could not treat mathematics as a science; instead, they considered it as a guessing game based on luck, streaks, and cultural superstitions (Randahl, 2012). The reliance on instincts explains why the baseline pre-test scores were low across both groups.

The introduction of Toulmin's argumentation framework changed everything. It encouraged learners to justify their claims with evidence, build logical connections (warrants), and anticipate counterarguments (rebuttals), the intervention refocused their problem-solving habits. They stopped memorising rigid rules and started thinking critically, which drove the improvement in post-test performance (Nussbaum et al., 2008).

What is important is what happened to the group as a whole. In traditional mathematics classes, struggling learners are often left to fail in isolation. But the group-centred argumentation levelled the playing field. Through talking through problems together, the learners translated confusing mathematical concepts into an everyday language they all understood (Mercer & Sams, 2006). This peer support encouraged underperforming learners to improve, creating a unified community of learners where everyone's understanding converged around the same level.

As was shown in Table 4.7 the internal consistency of the participant perception questionnaire ($\alpha = 0.982$) provides a statistical validation that matches the study's quantitative-qualitative convergence. The high mean scores for item Q2 ($\mu = 4.25$, "understanding different perspectives") and item Q3 ($\mu = 4.15$, "improved critical thinking") explain the improved mathematical reasoning captured in the post-test Toulmin's Argumentation Pattern (TAP) evaluations, such as Learner G's use of Bayes' Theorem to refute flawed conditional claims. Conversely, the high numerical variance observed in communication comfort (Q5, $\sigma = 1.231$) is directly explained by the lingering cultural tensions highlighted in the qualitative narratives, where learners noted that open academic confrontation initially conflicted with home communication norms. Ultimately, this sequential integration proves that the E-API model successfully mediated the transition from intuitive cultural frameworks to formal scientific probability axioms by lowering mathematics anxiety and replacing algorithmic mimicry with structured logical auditing (Vygotsky, 1978). Accordingly, the quantitative results improved as a result of the shift in qualitative dimensions discussed above.

4.11 Unexpected findings from the study

The effectiveness of the intervention was demonstrated; however, it became evident from observations that certain learners encountered challenges in collaboration throughout the argumentation process. Consequently, these learners required additional scaffolding to fully realise the benefits of this instructional approach. The reason learners differ in how much they engage is because of their cultural knowledge, language proficiency and personal learning style.

According to Webb (1991), Barron (2003) and Gresalfi, Barnes and Cross (2012), for collaborative learning to work, learners must be able to take part in fruitful conversations and cooperate with each other. Various studies, for example, by Berland and McNeill (2010) and Howe and Abedin (2013), have emphasised the need for scaffolding to help different learners develop arguments. Berland and Reiser (2011) pointed out that teachers are key in encouraging learners to have productive arguments, by directing and shaping their discussions. Also, Howe and Abedin (2013) argued that developing a classroom setting where different perspectives are acknowledged and all learners are involved in debates is very important.

4.12 Chapter summary

This chapter summarises the outcomes of this study on how to integrate argumentation in the teaching of probability to culturally diverse secondary school learners from various backgrounds in Bulawayo. For this study, a mixed-methods approach was chosen which included both data from tests and from focus group discussions and semi-structured questionnaires. Using the sequential explanatory design allowed for collection and analysis of both qualitative and quantitative data one after the other which helped in gaining a full perspective on the effectiveness argumentation in teaching probability.

Significant improvements were noted in the way the experimental group understood probability concepts after the intervention. The large effect size of d = 0.855 showed that the intervention was effective. In particular, the mean score among the experimental group went up from 47.55% to 62.55% and in the control group, the scores rose just slightly from 48.10% to 52.05%. The research results were further supported by qualitative findings which showed that learners improved their argument skills, were more engaged with the course and understood probability more deeply. It was found that several factors influenced learning, for example, cultural inclusivity, teamwork among peers and using practical examples. Hence, the qualitative findings gave deeper meaning to the quantitative data. The learners' excerpts about being more engaged and critical thinkers were in tandem with the significant increases in learners' test scores which confirmed the main conclusions of the study.

The results indicate that the use of argumentation greatly improves the learning of probability for the culturally diverse secondary school learners in Bulawayo. The improvements found in test scores, participation and critical thinking among the learners form the basis for the conclusions and recommendation in Chapter 5. The chapter also offers suggestions for further studies.

CHAPTER 5: SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.0 Introduction

In this chapter, the principal findings of the research are examined and analysed to evaluate their implications for both theory and practice. Suggestions are presented to teachers and policymakers to aid in the enhancement of instructional practices and to improve the outcomes within mathematics education. This study's distinctive significance lies in its contributions to bridging the gaps identified in Chapter One. The research investigated the implementation of argumentation as a pedagogical approach for teaching probability to culturally diverse secondary school learners in Bulawayo. Consequently, the study elucidates the connection between academic performance, critical thinking skills, and engagement with probability.

Instruction in argumentation is culturally responsive and is aligned with recent global trends that prioritise collaboration and the creation of inclusive educational environments. Highlighting the method of discussing probability in a culturally relevant manner within the classroom reveals the interconnectedness of cultural significance, critical thinking, and collaborative learning. The research findings indicate that learners experienced probability concepts differently. Initially, learners faced difficulties in articulating logical arguments; however, by the conclusion of the topic, they had developed reasoning and argumentation skills that enhanced their comprehension of probability. This study provides valuable insights into the impact of argumentation-based instruction on learners from diverse academic backgrounds, highlights some challenges encountered, and examines the implications for mathematics education within the context of Bulawayo Central district.

5.1 Summary of the study findings

5.1.1 ***Quantitative improvements***

5.1.1.1 How do the culturally diverse secondary school learners demonstrate their baseline conceptual understanding of probability and argumentation prior to the pedagogical intervention?

The findings revealed a heterogeneous distribution of academic proficiency and argumentation capabilities among the learner population. A considerable number of learners exhibited a moderate understanding of foundational probability concepts; however, deficiencies in their argumentation skills were apparent. Furthermore, learners originating from diverse cultural backgrounds demonstrated distinct strengths and weaknesses, with certain individuals displaying improved reasoning abilities that appeared to be influenced by their cultural backgrounds. Consequently, the necessity for specific interventions aimed at improving argumentation competencies has been established. The results obtained from the pre-test indicated a non-statistically significant difference in performance outcomes between the experimental and control groups prior to the implementation of the argumentation intervention, as evidenced by the statistics [U = 197.00, p = 0.925, p > 0.05, with a *Cohen's d* = 0.015]. The findings underscored the need for addressing the identified deficiencies in argumentation skills through a targeted argumentation intervention.

5.1.1.2 How does the implementation of an argumentation-based instructional approach impact the development of probabilistic reasoning and justification skills among secondary school learners?

An evaluation of the academic performance following the implementation of an argumentation-based teaching strategy revealed a significant enhancement in performance among learners within the experimental group. The data obtained from test scores indicated a consistent increase in both the understanding and application of probability concepts, suggesting that learners experienced improved problem-solving capabilities. This pedagogical approach promoted a more profound understanding of both probability and argumentation, thus allowing learners to express their reasoning proficiently.

When compared to the traditional instructional technique employed for the control group, the intervention correlated with a marked increase in academic performance. The results from the post-test assessment showed a substantial improvement, evidenced by the statistics [$U = 28.000$, $p = 0.015$, $p < 0.05$; with *Cohen's* $d = 0.855$]. The calculated *Cohen's d* value serves to highlight the practical significance of the intervention in real-world educational settings. Additionally, the Difference-in-Differences (DID) yielded a significant value of 11.05, further indicating that the experimental group exhibited substantial enhancements in performance following the educational intervention.

Consequently, these results affirm that the experimental group considerably exceeded the performance levels of the control group subsequent to the implementation of the argumentation-based teaching strategy. This evidence strongly supports the assertion that such an intervention is effective in improving the academic outcomes of culturally diverse secondary school learners in Bulawayo.

*5.1.2 **Qualitative shifts in attitudes and motivation***

e) How does the argumentation teaching approach impact learners' attitudes and motivation towards learning probability?

Post-intervention analysis showed a significant positive shift in learners' attitudes and perceptions towards probability. Learners expressed increased interest and confidence in probability, attributing this change to the interactive nature of the argumentation-based approach. Cultural diversity was recognised as an asset in classroom discussions, fostering a more inclusive and understanding learning environment. This shift contributed to heightened motivation and a willingness to engage with probability concepts. Learners reported a marked increase in their academic self-efficacy, with one noting that they:

> I *feel more confident now expressing my ideas*.

In agreement another asserted,

> *The argumentation sessions definitely improved our social skills; we had to learn how to actively listen to a peer's viewpoint before we could logically critique it, and this process made us feel more secure in our own reasoning.*

Learners developed from simply expressing uncertainty such as, "I believe the odds are high," to carefully reasoning with probabilities such as, "Looking at a tree diagram, the probability is 2/3 because ….” In discussions, learners revealed greater confidence and one learner asserted,

> *Expressing my reasoning made me understand how useful argumentation is for solving real mathematics problems. That really motivated me to participate.*

5.1.2.1 How much did the argumentation-based probability instruction promote critical thinking, collaboration, and active engagement among culturally diverse secondary school learners in learning probability?

Using argumentation approaches has been cited as making a positive difference in how culturally diverse secondary school learners collaborate and participate in probability education. It encourages learners to speak up and consider different ways of thinking. By being involved in structured arguments and group discussions, learners collaborate in productive conversations that help them explain their ideas, pay attention to others and contribute to knowledge with their peers. Evidence has shown that discussing probability together with other learners improves understanding and creates a pleasant and welcoming atmosphere for everyone. Working together in this way allows learners to feel united and welcomes all kinds of different opinions. Such interactions help learners become more involved and perform well in the study of probability. This intervention developed sound critical thinking skills among the learners. Learners moved beyond rote calculations to strong analysis of claims. Qualitative data revealed that peer reviewing for errors and addressing the cognitive dissonances between cultural perspectives and school probability was a catalyst for enhancing analytical skills. Learners developed the ability to formulate reasoned objections and to reassess assumptions, transforming classrooms into cognitive engagement spaces. To conclude, argumentation activities help learners collaborate and become more involved in their lessons which helps everyone involved.

*5.1.3 **Collaboration challenges and cultural adaptation***

The argumentation intervention made learners from different cultural backgrounds at the secondary school work more effectively in groups and become more involved in their learning. Also highlighted were several challenges with cultural adaptation and communication. Some learners had difficulties collaborating in the discussion because they felt less confident in public speaking and their cultures handled argumentations differently. Key factors included social dynamics formed by cultural differences; specifically, the Ndebele-speaking learners outnumbered other cultural groups, which occasionally dominated classroom participation. Diverse cultural backgrounds often lead to learners misunderstanding one another making it difficult for groups to collaborate well. There were cases where some learners hesitated to challenge their peers or state their personal opinions which is a sign of cultural preferences for reaching an agreement rather than arguing.

Because of these issues, certain strategies were introduced to make the lessons friendlier to all learners. To encourage all learners to collaborate, scenarios about predicting sports outcomes and "*argumentation scripts*" (with sentence starters such as, "*I respectfully disagree because...")* were used in the class. Such methods highlight how teachers should assist and support learners and make them feel appreciated as active group members.

Combining both qualitative and quantitative data shows the substantial influence of argumentation in the study of probability. According to the findings, learners did much better academically, and the focus group discussion and questionnaire results explained why these improvements took place. Learners said they felt more confident in class, and some went on to mention, “I now find it easier to explain my thinking in class,” and “I am more confident in handling probability questions.” These statements explain that argumentation influences the learners’ thinking and assists learners to feel more confident which contributes to their improved academic performance. When learners feel more involved and at ease in their learning, their school outcomes improve. This demonstrates that argumentation is an effective strategy that strengthens learning in mathematics.

5.2 Discussion of findings in relation to literature

The statistically significant post-test improvement ($U = 28.000$, $p = 0.015$, $d = 0.855$) corroborates Can and Isleyen’s (2016) findings from Turkey, where an argumentation-based instruction improved probability understanding. The current study extends Can and Isleyen’s work by showing that performance gains also hold in a culturally diverse African classroom context. It contributes qualitative evidence of the attitudinal mechanisms driving these gains, a dimension not examined in the Turkish study.

The qualitative finding that learners shifted from formulaic to reasoned probabilistic thinking aligns with Hmelo-Silver’s (2014) and Osborne and Patterson’s (2011) observations on argumentation and learner motivation. This study documents how cultural identity served as a productive resource rather than a barrier in the argumentation process. This finding extends Culturally Responsive Pedagogy theory (Gay, 2010) into the domain of probability instruction.

5.3 Conclusion

To assess the impact of teaching argumentation in probability for secondary school learners in Bulawayo Central, the present study used a mix of quantitative and qualitative methods. The study used a sequential explanatory design, collecting numerical data through pre-tests and post-tests and qualitative data through semi-structured questionnaires and focus group discussion. This framework allowed the researcher to examine the phenomenon closely, analyse changes in the way probability is understood after the argumentation intervention and assess the experiences of the learners that led to those outcomes.

Through an analysis of how scores from pre-tests and post-tests changed, the results show that argumentation-based instruction is effective. Learners in the experimental group showed significant improvements in academic performances compared to those in the control group. Because the effect size was large (*Cohen's d* = 0.855), the argumentation intervention is highly effective in increasing the learners' knowledge of probability in Bulawayo Central. Pre-test scores were used to assess what the learners knew beforehand, and post-test scores showed how much they gained from the training.

In the qualitative phase of the study, what learners shared in the questionnaire and the findings from the focus group discussions confirm that argumentation is beneficial. The questionnaire helped understand what learners think generally about argumentation, but the focus group discussions allowed for deeper exploration of the themes and individual opinions. Learners asserted that having to explain their arguments and challenge their peers encouraged them to think more about their learning. The qualitative results show that the positives of argumentation were common across different cultures, since learners from various backgrounds developed greater confidence and competence in understanding probability. The questionnaires showed how many learners had positive attitudes and the discussions in focus groups gave context and examples of the progress achieved in the pre-test and post-test.

Hence, better grades, stronger critical reasoning, improved attitudes and better teamwork among learners prove that using argumentation in teaching probability benefited the culturally diverse secondary school learners in Bulawayo Central district.

5.4 Limitations of the study

The study accepts that the results cannot be generalised widely. The study was conducted with only a small group of 40 learners, so what was found may not accurately reflect all learners' cultural perspectives; as a result, it is difficult to use these outcomes in other educational situations.

There are further limitations because the information is gathered from self-report measures during focus group (Powell et al., 2008) and semi-structured questionnaires which might not fully capture the broad effects of argumentation-based learning. As an illustration, when taking part in self-reporting, learners might overestimate their involvement or knowledge, because they want to appear good or meet the expectations of the researcher. Thus, these biases could make it hard to trust the data on how effective the educational approach really was.

It is acknowledged that the focus group discussion guide was weighted toward eliciting positive learner experiences, which may have introduced social desirability bias in participants' responses (Nederhof, 1985). Future studies employing argumentation-based interventions should use a neutrally-framed discussion guide that invites both positive and critical reflection to ensure balanced qualitative data.

While statistical generalisation of findings to all Zimbabwean secondary schools is limited by the single-school design and the small purposive sample of 40 learners, the study offers transferability in the sense articulated by Lincoln and Guba (1985). Thus, the richness of contextual description provided throughout Chapters 3 and 4 allows readers from comparable multicultural urban educational settings to judge the applicability of the findings and the E-API model to their own contexts. Future multi-contexts replication studies would strengthen the generalisability of these findings across the diverse educational landscape of Zimbabwe.

Rasmussen and Kwon (2007) point out that a longitudinal study is necessary to assess the lasting results of any educational intervention. Because the research only covers a single term, it might not include important points about retention of content and skill application later. This a call for more research with bigger and more diverse groups, along with longitudinal studies, to assess how well learners retain what they learned after the intervention.

Furthermore, Staples and Truxaw (2009) encourage the use of argumentation as well as cultural responsiveness in teaching mathematics and they believe that the methods presented in their research could effectively teach probability concepts. Staples and Truxaw (2009) did not mention which statistical methods they applied which may complicate a direct comparison between their findings and those of the current study.

5.5 The key takeaways from this study

This study prescribes a key takeaway in the form of a Pedagogical innovation as described below:

In educational research, it is suggested that teachers should create environments that encourage group knowledge building, as explained by the culturally relevant pedagogy by Ladson-Billings (1995). Such contexts tend to motivate learners more. A good example is when discussing local matters such as lotteries which combine theory with situations learners can relate to. To meet this educational need, the current study proposes a new way of teaching mathematics and STEM subjects, called the Enhanced Argumentation-Based Probability Instruction (E-API) model. The aim is to improve the current argumentation models, such as Toulmin's model, by incorporating cultural responsiveness, and education strategies related to probability (E-Toulmin + CP). The E-API model seeks to assist learners gain deeper insights into probability, think critically, argue effectively and interact more with their peers.

The framework relies on various basic principles.

1. It uses Toulmin's framework while accounting for culture during argument construction, and it is underpinned by Sociocultural Learning Theory which stresses that learning is shaped by culture and society and underscores the role of working with others and the use of language, sometimes involving switching between languages.
2. Emphasises using learners' cultural backgrounds, and their own experiences to express themselves in education.
3. Emphasises constructivist Learning Principles advocating for learners to participate, asking questions and creating their own understanding. Together, these aspects can help create an effective approach for teaching and learning probability in many different classrooms.

*5.5.1 **Core Components and Processes of the E-API Model***

This model is a unique instructional approach compared to other teaching models. For example, its philosophical root is based on TAP and is suitable for the Global south contexts where cultural perspectives intersect with school taught perspectives. Eurocentric teaching models such as the Inquiry-Based (IBL) model are Socratic in nature and is process focused. The E-API model is explicit on cultural integration and is supported by sociocultural theories. No such model has been tested within this culturally diverse context of Bulawayo. A review of mathematics education literature from the Southern African Development Community (SADC) region, including Zimbabwe, South Africa, Zambia, Botswana, and Mozambique, reveals no published study that has formulated, empirically implemented, or validated an argumentation-based instructional model designed specifically for culturally diverse secondary school probability teaching. Existing regional models of culturally responsive mathematics pedagogy (Gay, 2010; Vithal & Skovsmose, 2007) address cultural inclusion generally but do not operationalise argumentation theory as a pedagogical vehicle for probability instruction. The E-API model therefore occupies an unoccupied conceptual and practical space in Southern African mathematics education research.

The E-API model consists of the following five iterative phases, allowing for flexibility, transferability and responsiveness to learner needs.

Phase 1: Culturally relevant introduction

The purpose is to make sure learners interact with probability ideas in a way that helps them relate them to their own cultural backgrounds and daily activities. Its purpose is to boost involvement of the culturally diverse learners and help them realise that probability is important in many decisions they make.

Activities:

1. Connecting to the learners' lived experiences
   - Use of examples and practices that are meaningful to learners' backgrounds, cultures or everyday life to create a more familiar learning setting.
   - Using familiar activities such as traditional games, will help learners grasp probability concepts better.
2. Engaging with real world contexts

- Probability plays a role in daily activities such as climate predictions and community gatherings which should be pointed out to show its importance.
- Provide opportunities for learners to share their personal experiences about luck and how they make choices.

3. Building interest and curiosity

- Asking stimulating questions at the beginning of the lessons may inspire learners to discuss probability and try out different examples.
- Telling personal experiences or using examples that relate to learners' culture can make lessons much more interesting and easier to relate to.

4. Fostering inclusivity

- Learners with different backgrounds in the classroom should be acknowledged and valued to provide a stimulating educational setting.
- Every learner should feel that their opinions matter, and the environment should encourage everyone to participate and learn.

5. Setting the stage for deep learning

- A solid foundational understanding of probability should be established before, as this helps establish strong connections that improve understanding.
- When learners see how the content relates to their lived experiences, they will develop a deep understanding of what they are learning.

Phase 2: Culturally Responsive and Contextualised Problem Posing and Exploration

Objective: To engage learners with rich, authentic probability problems that are culturally relevant and intellectually stimulating.

Activities:

1. The starting point for these activities is to look at challenging probability scenarios from indigenous games, environmental events in the local area, social issues that involve probability and health statistics globally.
2. Either on their own or in small groups, learners are asked to investigate these problems. At this point, they will select relevant probabilistic aspects and describe their initial interpretations. They are also encouraged to support a setting that promotes talking about different ideas and acknowledging that culture impacts how one views such issues.
3. Learners will access examples from mixed cultural backgrounds, glossaries for multilingual learners and introductory materials focused on probability, according to what is needed for the lessons.

Phase 3: Collaborative Argumentation, Iterative Refinement and Culturally Sensitive Facilitation

The main goal is to create a setting where learners can cooperate in making, judging and refining their arguments.

Activities:

1, When learners argue in small groups, several structured stages are included to help everyone work together and think critically. At first, groups are formed with learners creating basic arguments using E-Toulmin + CP. It guides how arguments should be arranged which helps understand all the components better.

2. After that, groups present their arguments to their peers or to the whole class to receive feedback.
3. A group review and discussion are arranged after the presentations, applying set arrangements for giving feedback. The focus is on the components of the E-Toulmin + CP framework and the facilitator is responsible for maintaining an environment where learners can respectfully challenge each other's opinions, considering cultural differences.
4. In addition, the method of argument mapping is used, and this is done by making maps of complex arguments, either on a computer or on paper. It helps to recognise connections in the reasoning and points out where an argument is strong or weak.
5. The role of the facilitator is very important in this process. The facilitator leads discussions through asking thoughtful questions, encouraging fair participation and offering support to those who are weak in arguments.
6. Participating in discussions helps learners to use the advice and new information they get to enhance and refine their arguments. At this point, they may go back to earlier steps, collect more information or look over the justifications for their claims.
7. The facilitator also supports the use of code-switching so that everyone feels comfortable sharing their different ways of speaking. The practice helps explain complex ideas and guarantees support so that everyone in the group understands each other.

Phase 4: Enhanced Argumentation Scaffolding (The E-Toulmin + CP Framework)

Objective: To guide learners in constructing well-reasoned, and culturally relevant arguments using an enhanced version of Toulmin's model that incorporates Cultural Context, and Probability-Specific considerations (E-Toulmin + CP).

Elements of E-Toulmin + CP:

1. **Claim:** The learner's proposed solution, hypothesis, or probabilistic judgment in response to the problem.

2. **Data/Evidence:** Information used to support the claim.

- *Probability-Specific:* Includes results from calculations, simulations, statistical analyses, tree diagrams, contingency tables, etc.

- *Cultural Context:* Consider the relevance, validity, and interpretation of data from different cultural perspectives or sources (e.g., community knowledge, historical narratives, alongside empirical data).

3. **Warrant:** The reasoning that connects the Data/Evidence to the Claim, explaining *how* the evidence supports the claim.

- *Probability-Specific:* Must be grounded in sound probabilistic principles, theorems, and definitions.

4. **Backing:** Underlying assumptions, theories, or knowledge that provide support for the Warrant.

- *Probability-Specific:* Established probability axioms, statistical theories, or mathematical proofs.
- *Cultural Context:* May include validated cultural knowledge systems or established practices that inform the reasoning, where appropriate and critically examined.

5. **Qualifier:** Words or phrases expressing the degree of certainty or confidence in the Claim (e.g., "probably," "likely under these conditions," "with X% confidence").

- *Probability-Specific:* Directly related to probabilistic measures of uncertainty, confidence intervals, or significance levels.

6. **Rebuttal/Counterargument:** Acknowledging and addressing potential counterclaims, alternative interpretations, limitations of the data or model, or conditions under which the claim might not hold.

- *Probability-Specific:* Considering alternative probabilistic models, sources of bias, or common misconceptions.
- *Cultural Context:* Exploring how different cultural viewpoints might lead to different rebuttals or identify different weaknesses in an argument.

7. **Cultural Contextual Lens (Overarching and Iterative):** Throughout the argumentation process, learners and facilitators reflect on and discuss:

- How cultural backgrounds, assumptions, and communication styles might influence the interpretation of the problem, the selection/evaluation of evidence, the reasoning process, and the presentation of arguments.

- The value of diverse forms of reasoning and evidence (e.g., narrative, experiential) alongside formal mathematical proof, and how to integrate them critically.
- Ensuring respectful dialogue and addressing potential cultural biases in argumentation.

Phase 5: Synthesis, Reflection and Transfer

Objective: To consolidate learning, promote metacognition about the argumentation process, and foster the transfer of skills to new contexts.

Activities:

1. **Argument Synthesis:** Learners or groups should present their synthesised arguments.
2. **Metacognitive Reflection:** Guided reflection on the learning process:
   - What were the challenging aspects of constructing/evaluating arguments in this probability context?
   - How did cultural perspectives influence their or others' arguments?
   - How has their understanding of the probability concepts changed?
3. **Knowledge Consolidation:** Linking the argumentation process back to core probability concepts and principles, ensuring conceptual clarity.
4. **Transfer Tasks:** Presenting new, complex, and culturally diverse probability problems or real-world scenarios that require learners to apply their argumentation skills and probabilistic knowledge independently or in new groups.

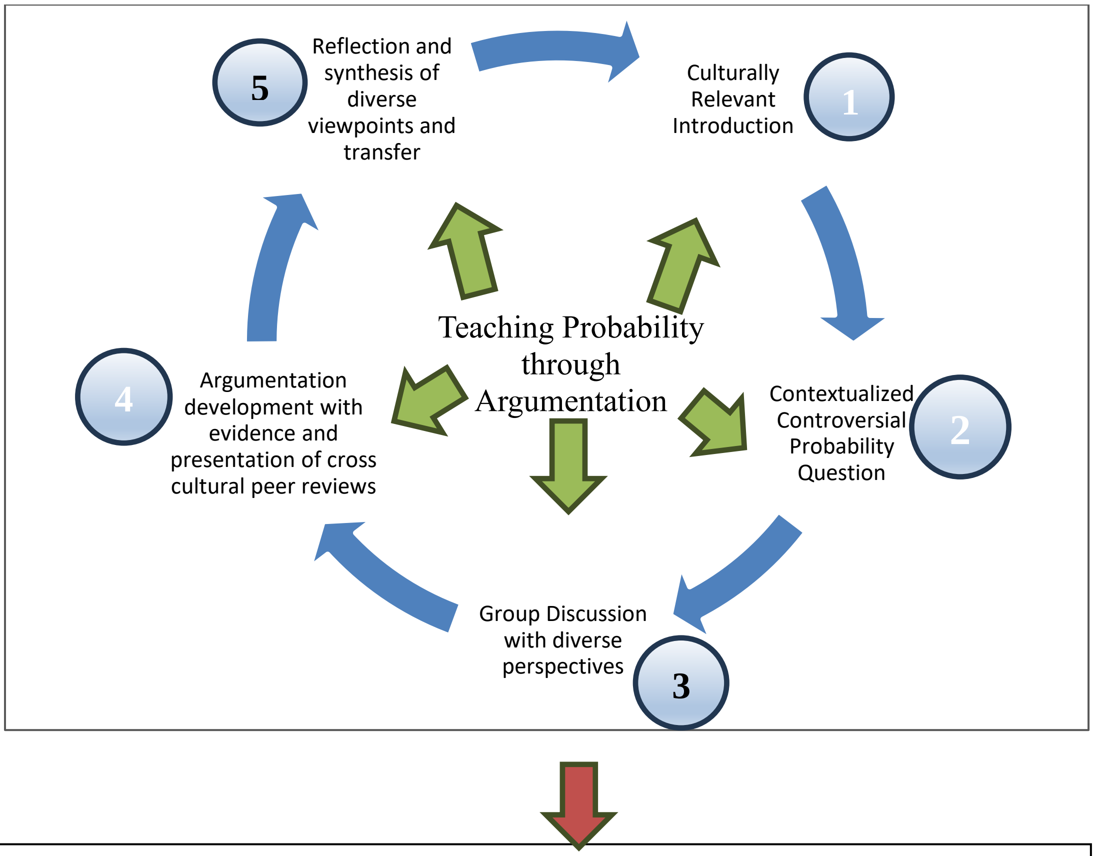
5. **Holistic Assessment:** Assessing learners' understanding of probability, their argumentation skills (using rubrics aligned with E-Toulmin + CP, considering diverse expressions of reasoning), and their reflective insights.

6. **Expected Outcomes**

This E-API model aims to achieve:

- **Deeper Conceptual Understanding:** Moving beyond procedural knowledge to a deep, interconnected understanding of probability concepts.
- **Enhanced Critical Thinking:** Developing skills in analysing information, evaluating evidence, identifying assumptions, and constructing logical arguments.
- **Improved Argumentation Skills:** Increasing proficiency in articulating, justifying, and defending claims, as well as constructively engaging with counterarguments, within a probability context.
- **Better Engagement for secondary school Culturally Diverse Learners:** Increasing motivation, participation, and interest in probability by making learning relevant, interactive, and inclusive.
- **Increased Cultural Competence:** Fostering awareness and appreciation of diverse perspectives and communication styles in academic discourse.
- **Enhanced Transfer of Learning:** Improving the ability to apply probabilistic reasoning and argumentation skills to novel situations and real-world problems.

Figure 5.1 below shows how the E-API model is structured.



Intervention Outcomes

1. Enhanced critical thinking
2. Improved academic performances
3. Cultural awareness
4. Improved communication skills
5. Motivation and engagement
6. Collaboration skills
7. Argumentation skills
8. Enhanced transfer of learning

Figure 5.1: Enhanced Argumentation-based Probability instruction (E-API) model

The model depicted in figure 5.1 shows how in a lively classroom a teacher can prepare an engaging lesson when teaching probability. The goal is to not only teach the concepts of probability but to do so in a culturally relevant and deeply engaging way for all learners to understand.

The E-API Model evolved through an iterative process of conceptual prototyping. It began as a basic synthesis of Toulmin's (1958) Argument Pattern and Vygotsky's (1978) Zone of Proximal Development. Through the intervention cycle and subsequent feedback loops with the learners in Bulawayo, the model was expanded to include cultural discourse warrants. The chronological development was documented through researcher journals and pilot study draft. These serve as evidence of the model's originality and its specific tailoring to the Zimbabwean educational context. Annexure O shows the chronological archive of the E-API Model's Development.

The primary knowledge claim of this thesis is the formulation and validation of the Enhanced Argumentation Probability Instructional (E-API) Model. While contemporary literature acknowledges that argumentation is an effective tool, this study contributes a specific and necessary cultural bridge. It identifies how cultural discourse patterns can be leveraged as a scaffold to demystify probabilistic thinking.

5.5.2 Novel contributions of the E-API model

While the E-API model's five phases draw on established principles from sociocultural and argumentation theory, two elements represent underreported innovations in the mathematics education literature. First, the Pre-Lesson Argumentation Training Protocol (E-API Phase 1) constitutes a structured induction phase in which learners are socialised into argumentation discourse norms, including claim-making, warrant construction and counter-argument engagement. This is before any probability content is introduced. This preparatory phase addresses the “cold start” problem identified in argumentation research (Berland & Reiser, 2011), where learners without prior training produce only opinion-level statements rather than evidence-based arguments. Second, the study provides empirical documentation of a Learner Argumentation Transformation Trajectory, the sequential cognitive pathway through which learners in this culturally diverse Bulawayo context progressed from cultural-fatalistic probability framing (it depends on luck) to formally grounded probabilistic reasoning (P = 3/6 because...). This trajectory, documented through pre- and post-intervention qualitative excerpts (Sections 4.7 and 4.8), has not been previously reported in the Southern African mathematics education literature.

5.6 Recommendations

Based on the findings, the following actionable recommendations are proposed to enhance probability education through argumentation, cultural responsiveness, and inclusivity.

1. Prioritise Argumentation-Based Instruction

   Given the strong evidence of the effectiveness of the argumentation approach, as demonstrated by the significant gains from pre-test to post-test, teachers should prioritise the integration of argumentation into probability curricula and instructional practices.

2. Foster Culturally Responsive Facilitation

Culturally responsive approach to teaching according to Bishop (2012) is still at its infancy. Hence, teachers should be trained to facilitate argumentation in a way that is sensitive to the diverse communication styles and cultural backgrounds of learners. They should encourage active listening, respectful dialogue, and to value multiple perspectives.

3. Provide Targeted Language Support

Given that English is a second language for most of the learners, teachers should provide targeted language support to learners who may struggle to articulate their arguments effectively in English. They should utilise visual aids, graphic organisers, and other strategies to enhance understanding and communication.

4. Promote Equitable Participation

Given that some learners may not be willing to partake in discussions, teachers should implement strategies to ensure that all learners have opportunities to participate in argumentation-based activities. For example, pair work can encourage all to participate.

5. Share Best Practices

Disseminate the findings of this study and share best practices for implementing argumentation in culturally diverse classrooms with other teachers and researchers. Include both quantitative results (pre-test and post-test scores, effect sizes) and illustrative quotes from questionnaires and focus groups to provide a comprehensive picture of the study's findings.

6. Curriculum Development

- Curriculum developers should revise the secondary school mathematics curriculum to include argumentation modules co-designed with teachers to foster ownership.

- Teachers should work towards achieving micro credentials such as acquiring certifications in argumentation pedagogy via online platforms such as Coursera to improve their ability to implement argumentation.

7. Address implementation challenges

- Distribute resource kits and guides for example, "Collection of Culturally Relevant Probability Problems" to ease transition to the new instructional approach.

5.7 Recommendations for future studies

Future research should address gaps in longitudinal impact, cultural nuance, and scalability.

1. Identifying the Key Mechanisms of Argumentation:

   Conduct further research to identify the specific cognitive and social mechanisms that contribute to the effectiveness of argumentation in promoting probability learning. Use both quantitative measures (e.g., analysing the types of arguments learners use on the post-test) and qualitative interviews to explore these mechanisms in greater depth.

2. Exploring the Long-Term Impact of Argumentation:

   Conduct longitudinal studies to examine the long-term effects of argumentation-based instruction on learners' mathematical understanding, problem-solving skills, and attitudes towards mathematics. Track changes in learner knowledge using longitudinal pre- and post-tests, and monitor shifts in attitudes using longitudinal questionnaires.

3. Developing Scalable Argumentation Interventions

   Develop and evaluate scalable argumentation interventions that can be implemented effectively in a wide range of educational settings. Use questionnaires to assess the feasibility and acceptability of these interventions in different contexts and measure their effectiveness using pre- and post-tests.

4. Anecdotal classroom observation suggested some learners engaged in code-switching between their home languages and English during argumentation activities. However, systematic documentation of code-switching patterns was beyond the scope of the current study's data collection instruments. Future research should therefore examine the role of code-switching in culturally diverse argumentation-based mathematics classrooms using audio-recorded discourse analysis.

REFERENCES

Abrahamson, D. (2014). Rethinking Probability Education: Perceptual Judgment as Epistemic Resource. In Chernoff, E., Sriraman, B. (Eds.) *Probabilistic Thinking. Advances in Mathematics Education*. (pp. 239-260). Springer, Dordrecht. https://doi.org/10.1007/978-94-007-7155-0_13

Aguirre, J. M., & Maria del Rosario Z. (2013). Making culturally responsive mathematics teaching explicit: a lesson analysis tool, *Pedagogies: An International Journal*, *8*(2), 163-190, https://doi.org/10.1080/1554480X.2013.768518

Aikenhead, G. S., & Jegede, O. J. (1999). Cross-cultural science education: A cognitive explanation of a cultural phenomenon. *Science Education*, 83(3), 269-287. https://doi.org/10.1002/(SICI)1098-237X(199905)83:3<269::AID-SCE3>3.0.CO;2-A

Al-Ajmi, B., & Ambusaidi, A. (2022). The level of scientific argumentation skills in chemistry among Grade 11 students: The role of logical thinking. *Science Education International*, *33*(1), 66-74. https://doi.org/10.33828/sei.v33.i1.7

Amir, G. S., & Williams, J. S. (1999). Cultural influences on children’s probabilistic thinking. *Journal of Mathematical Behaviour, 18*(1), 85-107.

Andam, E., Awuah, F., & Obeng-Denteh, W. (2025). Probability concepts: A systematic literature review of students’ learning difficulties, errors and misconceptions. *East African Journal of Arts and Social Sciences, 8*(2), 114-128. https://doi.org/10.37284/eajass.8.2.3008

Angrist, J. D., & Pischke, J. S. (2009). *Mostly harmless econometrics: An empiricist’s companion*. Princeton University Press. https://doi.org/10.1515/9781400829828

Arends, R. I. (1998). *Resource handbook. Learning to teach (4th ed.).* McGraw-Hill.

Arum, D. P., Kusmayadi, T. A., & Pramudya, I. (2018). Learners' logical-mathematical intelligence profile. *Journal of Physics: Conference Series*, 1008, 012072. https://doi.org/10.1088/1742-6596/1008/1/012072

Athanases, S. Z., & De Oliveira, L. C. (2008). Advocacy for equity in classrooms and beyond: new teachers’ challenges and responses. *Teachers College Record, 110*(1), 64-104. https://doi.org/10.1177/016146810811000101

Ayalon, M. & Hershkowitz, R. (2018). Mathematics teachers’ attention to potential classroom situations of argumentation. *The Journal of Mathematical Behaviour, 49,* 163-173. https://doi.org/10.1016/j.jmathb.2017.11.010

Bang, M. & Medin, D. (2010). Cultural processes in science education: Supporting the navigation of multiple epistemologies. *Science Education, 94*(6), 1008-1026.

Barron, B. (2003). When smart groups fail. *The journal of the learning sciences, 12*(3), 307-359.

Batanero, C., Chernoff, E.J., Engel, J., Lee, H.S., Sánchez, E. (2016). Research on Teaching and Learning Probability. *Research on Teaching and Learning Probability. ICME-13 Topical Surveys*. Springer, Cham. https://doi.org/10.1007/978-3-319-31625-3_1

Bećirović, S., & Bešlija, D. (2018). Classroom as a microcosm: Teaching culturally diverse students. *Journal of Education and Humanities*. https://doi.org/10.14706/JEH2018112

Berland, L. K., & McNeill, K. L. (2010). A Learning progression for scientific argumentation: Understanding student work and designing supportive instructional contexts. *Science Education, 94*(5), 775–793. https://doi.org/10.1002/sce.20402

Berland, L. K., and Reiser, B. J. (2011). Classroom communities' adaptations of the practice of scientific argumentation. *Science Education, 95*(2), 191-216.

Bishop, R. (2012). A culturally responsive pedagogy of relation. In C. McGee & D. Fraser (Eds.), *The professional practice of teaching*. Cengage.

Bloomfield, J., & Fisher, M. J. (2022). Quantitative research design. *Journal of Clinical Nursing*, *31*(2), 27-30.

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology,* *3*(2), 77-101. https://doi.org/10.1191/1478088706qp063oa

Brown, B. A., Boda, P., Lemmi, C., & Monroe, X. (2019). Moving culturally relevant pedagogy from theory to practice. Urban Education, 54(6), 775-803. https://doi.org/10.1177/0042085918794802

Brown, R. (2017). Using collective argumentation to engage students in a primary mathematics classroom. *Mathematics Education Research Journal*, *29*(2), 183-199. https://doi.org/10.1007/s13394-017-0198-2

Bruine de Bruin, W., & Carman, K. G. (2012). Measuring risk perceptions: what does the excessive use of 50% mean? Medical Decision Making, 32(2), 232-236. https://doi.org/10.1177/0272989X11404077

Bryk, A. S. (2015). AERA Distinguished lecture: Accelerating how we learn to improve. *Educational Research, 44*(8), 467–477. https://doi.org/10.3102/0013189X15621543

Can, O. S., & Isleyen, T. (2016). Teaching probability to pre-service teachers with argumentation based science learning approach. *Journal of Education and Practice, 7*(33), 13-22.

Castro, W. & Toro, J. (2023). Mathematics teacher argumentation in a didactic perspective. *Eurasia Journal of Mathematics Science and Technology Education, 19*(9), em2322. https://doi.org/10.29333/ejmste/13498

Chassapis, D., & Chatzirasileiou, E. (2008). Socio-cultural influences on children's conceptions of chance and probability. In J.F. Matos, P. Valero & K. Yasukawa (Eds.), *Proceedings of the Fifth International Mathematics Education and Society Conference . Centro de Investiga o em Educao, Universidade de Lisboa and the Department of Education, Learning and Philosophy,* (pp. 197-206). Aalborg University.

Chitera, J., Gudyanga, R., & Dziva, D. (2025). Investigating the effect of argumentation on high school learners' achievement in probability. *Advanced Research Journal*, 1(1), 45–59. https://doi.org/10.71350/3062192515

Chiwiye, T. (2013). Assessment of mathematics and science subjects in Zimbabwe: Zimbabwe School Examinations Council (ZIMSEC) perspective, ZIMSEC

Cohen, J. (1988). *Statistical power analysis for the behavioural sciences (2nd ed.).* Lawrence Erlbaum Associates.

Cooper, J., & Feldman, L. A. (2019). Does cognitive dissonance occur in older age? A study of induced compliance in a healthy elderly population. *Psychology and Aging*, *34*(5), 709 –713. https://doi.org/10.1037/pag0000338

Creswell, J. W., & Creswell, J. D. (2023). *Research design: Qualitative, quantitative, and mixed method approaches* (6th ed.). SAGE Publications.

Creswell, J. W., & Hirose, M. (2024). *Mixed methods research: A guide to the field*. Oxford University Press.

Creswell, J. W., & Plano Clark, V. L. (2018). *Designing and conducting mixed methods research (3rd ed.).* Sage Publications.

Cronbach, L. J., & Snow, R. E. (1977). *Aptitudes and instructional methods: A handbook for research on interactions*. Irvington.

Cynar, D., & Bayraktar, S. (2014). Evaluation of the effects of argumentation-based science teaching on 5th grade learners' conceptual understanding of the subjects related to "matter and change". *International Journal of Education in Mathematics, Science and Technology*, *2*(1), 21–37. https://doi.org/10.18404/ijemst.06465

D'Ambrosio, U. (1990). *Etnomatemática [Ethnomathematics]*. São Paulo, SP.

Daly, N., & Sharma, S. (2018). Language-as-resource: Language strategies used by New Zealand teachers working in an international multilingual setting. *Australian Journal of Teacher Education, 43*(8). http://ro.ecu.edu.au/ajte/vol43/iss8/2

Dede, A. (2018). Examination of the collective argumentation studies in the mathematics education field. *Turkish Journal of Computer and Mathematics Education (Turcomat).* https://doi.org/10.16949/turkbilmat.386722

DeJarnette, A. F., & González, G. (2017). The role of critical thinking in mathematics education: A Review of the Literature. *Journal of Mathematics Education,*, *10*(1), 1-15.

Demir, B.K., & Isleyen, T. (2015). The Effects of Argumentation Based Science Learning Approach on Creative Thinking Skills of Learners. *Educational Research Quarterly*, *39(1)*, 49-82.

Devlin, M., & Samarawickrema, G. (2010). The Criteria of effective teaching in a changing higher education context. *Higher Education Research & Development, 29*(1), 111-124. http://dx.doi.org/10.1080/07294360903244398

Dewey, J. (1938). *Experience and Education.* Macmillan.

Dignath, C., & Büttner, G. (2008). Components of fostering self-regulated learning among students. A meta-analysis on intervention studies at primary and secondary school level. *Metacognition and Learning, 3*(3), 231-264. https://doi.org/10.1007/s11409-008-9029-x

Dimitrios, T., & Antigoni, F., (2019). Limitations and delimitations in the research process. Perioperative nursing (GORNA), E-ISSN:2241-3634, *7*(3), 155–162. http://doi.org/10.5281/zenodo.2552022

Dimo, H., Marokoh, G. S., & Waswa, P. (2021). Challenges faced in using dialogue approaches to minimise learner errors in mathematics classes: A case of public primary teacher training colleges in North Rift Region. *Kenya. International Journal of Research in Education and Social Sciences (IJRESS), 4*(1), 11-19.

Diress, M., Belay, D., Seid, M., Eshetu, H., Kibret, A., Chilot, D., & Gela, Y. (2023). Determinants of knowledge of the highest conception probability period among reproductive age women in low-income African countries. *Journal of Public Health, 45* (1), 1-12.

Dogan, M., & Kazak, S. (2019). Learners' misconceptions about probability: A Study on High School Learners. *International Journal of Research in Education and Science, 5*(1), 1-12. https://doi.org/10.21890/ijres.409307

Doolittle, P. E. (2001). Constructivism and learning: Understanding the role of social interaction. . *Educational Psychology Review, 13*(4), 509-530.

Drake, S. M., & Reid, J. L. (2020). Integrated curriculum: Increasing relevance while maintaining accountability. *What Works? Research into Practice*, Ontario Ministry of Education.

Driver, R., Newton, P., & Osborne, J. (2000). Establishing the norms of scientific argumentation in classrooms. *Science Education, 84*(3), 287-312. https://doi.org/10.1002/(SICI)1098-237X(200005)84:3<287::AID-SCE1>3.0.CO;2-A

Du, N., Stevens, K., Ahern, J., & Shigaev, A. (2016). *Cross-cultural differences in interpreting* IAS 37 probability phrases. International Journal of Financial Research, *7*(1), 1-8.

Elbehary, S. G. A. (2022). Rethinking mathematics teachers' professional knowledge for teaching probability from the perspective of probabilistic reasoning: A proposed framework. *International Electronic Journal of Mathematics Education*, 17(3), em0695. https://doi.org/10.29333/iejme/12145

Elliott, S.N., Kratochwill, T.R., Littlefield Cook, J. & Travers, J. (2000). *Educational psychology: Effective teaching, effective learning (3rd ed.).* McGraw-Hill College.

Erduran, S., & Jiménez-Aleixandre, M. P. (2015). Argumentation in science education research. In *International handbook of research in history, philosophy and science teaching* (pp. 1027–1045). https://doi.org/10.1007/978-94-6091-900-8_11

Fasha, L. H., & Sopandi, W. (2024). Analysis of teaching material: To what extent are learners' argumentation learning. *Proceedings of the International Conference on Teaching, Learning and Technology (ICTLT 2023)*. Atlantis Press. https://doi.org/10.2991/978-2-38476-206-4_32

Ferrari, P. L., & Saccoletto, M. (2023). Argumentation and interaction: Discussing a new theoretical perspective. In P. Drivers, C. Csapodi, H. Palmér, K. Gosztonyi, & E. Kónya (Eds.), *Proceedings of the Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13)* (pp. 256–263). ERME.

Fetters, M. D., Curry, L. A., & Creswell, J. W. (2013). Achieving integration in mixed methods designs — Principles and practices. *Health Services Research, 48*: 2134-2156. https://doi.org/10.1111/1475-6773.12117

Field, A. (2018). *Discovering statistics using IBM SPSS statistics* (5th ed.). SAGE.

Fitriani, H., Asy'ari, M., Zubaidah, S., & Mahanal, S. (2019). Exploring the prospective teachers' critical thinking and critical analysis skills. *Journal Pendidikan Ipa Indonesia, 8*(3), 1-10. https://doi.org/10.15294/jpii.v8i3.19434

Francisca, R., Baba, E., Hoeve, C., Zomerdijk, I., Sturkenboom, M., & Straus, S. (2021). Introduction or Discontinuation of Additional Risk Minimization Measures During the Life Cycle of Medicines in Europe. *Drug Saf* 44, 63–72. https://doi.org/10.1007/s40264-020-00993-6

Francisco, J. (2022). Supporting argumentation in mathematics classrooms. *Lumat International Journal on Math Science and Technology Education*, *10*(2). https://doi.org/10.31129/lumat.10.2.1701

Franklin, J. (2001). The diverse challenges of multiculturalism. *Education Update*, *43*(2), 1–8**.**

Freeman, S., Eddy, S. L., McDonough, M., Smith, M. K., Okoroafor, N., Jordt, H., & Wenderoth, M. P. (2014). Active learning increases learner performance in science, engineering, and mathematics. *Proceedings of the National Academy of Sciences*, *111*(23), 8410–8415.

Gamarra-Moncayo, J., & Prada-Chapoñán, R. (2025). Sample, sample size and sampling: A review of current recommendations. *Interacciones, 11*, e447. https://dx.doi.org/10.24016/2025.v11.447

Gatua, M. (2014). Sailing against the wind. *Adult Learning*, *25*(2), 39-46. https://doi.org/10.1177/1045159514522434

Gay, G. (2010). *Culturally responsive teaching: Theory, research, and practice (2nd ed.)*. College Press.

Gertler, P. J., Martinez, S., Premand, P., Rawlings, L. B., & Vermeersch, C. M. J. (2016). *Impact evaluation in practice* (2nd ed.). Inter-American Development Bank; World Bank. https://doi.org/10.1596/978-1-4648-0779-4

Ghebru, S. & Ogunniyi, M. (2017). Pre-service science teachers' understanding of argumentation. *African Journal of Research in Mathematics Science and Technology Education, 21*(1), 49-60. https://doi.org/10.1080/18117295.2016.1254493

Gillet, N., Vallerand, R. J., & Lafreniere, M. K. (2012). Intrinsic and extrinsic school motivation as a function of age: The mediating role of autonomy support. *Social Psychology of Education*, *15*(1), 77-95.

Golafshani, N. (2015). Understanding reliability and validity in qualitative research. *The Qualitative report*, *22*(4), 1-14. https://nsuworks.nova.edu/tqr/vol22/iss4/1

Gollnick, D. M & Chinn, P. C. (2012). *Multicultural education in a pluralistic society*. Macmillan International.

Göncü, A., & Gauvain, M. (2012). Sociocultural approaches to educational psychology: Theory, research, and application. In K. R. Harris, S. Graham, T. Urdan, C. B. McCormick, G. M. Sinatra, & J. Sweller (Eds.), *APA educational psychology handbook, Vol. 1. Theories, constructs, and critical issues* (pp. 125–154). American Psychological Association. https://doi.org/10.1037/13273-006

Green, C. A., Wisdom, J. P., Duan, N., & Hoagwood, K., Palinkas, L. A., Horwitz, S. M. (2019). Mixed methods in implementation research: A review of the literature. *Implementation Science, 14*(1), 1-12. https://doi.org/10.1186/s13012-019-0901-4

Gresalfi, M. S., Barnes, J., & Cross, D. (2012). When does an opportunity become an opportunity? Unpacking classroom practice through the lens of ecological psychology. *Educational Studies in Mathematics, 80*(1-2), 249-267.

Groth, R. (2010). Teachers' construction of learning environments for conditional probability and independence. *International Electronic Journal of Mathematics Education, 5*(1), 32-35. https://doi.org/10.29333/iejme/248

Hacıeminoğlu, E. & Yıldız, N. (2022). Interdisciplinary use of argumentation among religious education and philosophy teachers-in-training. *Religions, 13*(5), 405-420. https://doi.org/10.3390/rel13050405

Hair, J. F., Risher, J. J., Sarstedt, M., & Ringle, C. M. (2019). When to use and how to report the results of PLS-SEM. *European Business Review, 31*, 2–24. https://doi.org/10.1108/EBR-11-2018-0203

Halmos, P. (1944). The Foundations of probability. *American Mathematical Monthly, 51*(9), 493-495. https://doi.org/10.2307/2304332

Haring-Smith, T. (2012). Cultural diversity in mathematics education: Bridging the gap between cultures and curriculum. In K. S. W. J. A. Van de Walle (Ed.), *Elementary and middle school mathematics* (pp. 123–135).

Hashim, M. H. & Hoover, M.L. (2010). A Theoretical framework for analysing multicultural group learning. literacy information and computer. *Education Journal (LICEJ), 8*(4), 2707-2716.

Hattie, J. (2009). *Visible Learning: A Synthesis of over 800 meta-analyses relating to achievement.* Routledge.

Hendratmoko, A. (2023). The impact of inquiry-based online learning with virtual laboratories on learners' scientific argumentation skills. *Turkish Online*

Journal of Distance Education, 24(4), 1-20. https://doi.org/10.17718/tojde.1129263

Hennink, M., Hutter, I., & Bailey, A. (2020). *Qualitative research methods*. SAGE Publications.

Hewson, M. & Ogunniyi, M.B. (2011). Argumentation-teaching as a method to introduce Indigenous knowledge into classrooms: Opportunities and challenges. *Cultural Studies in Education, 6*(3), 679-692.

Hine, G., Blackley, S., & Cooke, A. (Eds.). (2019). *Mathematics education research: Impacting practice* (Proceedings of the 42nd annual conference of the Mathematics Education Research Group of Australasia, pp. 668–675). MERGA.

Hmelo-Silver, C. E. (2014). Problem-based learning: What and how do learners learn? *Educational Psychologist, 39*(1), 5-16. https://doi.org/10.1207/s15326985ep3901_2

Horencio, C. A., & Dennis, B. R. (2025). Ethnomathematics Researches in the Philippines: A Systematic Review. *Journal of Innovations in Teaching and Learning*. 5(1):35-43. https://doi.org/10.12691/jitl-5-1-6

Howe, C., & Abedin, M. (2013). Classroom dialogue: A systematic review across four decades of research. *Cambridge Journal of Education, 43*(3), 325-356. http://dx.doi.org/10.2139/ssrn.2185945

Hughes, S. (2021). The role of sociocultural theory in l2 empirical research. *Studies in Applied Linguistics and Tesol, 21*(1). https://doi.org/10.52214/salt.v21i1.8394

Ibraim, S. & Justi, R. (2022). Actions that contribute to science teaching involving argumentation and their relationships with pedagogical content knowledge. *Investigações Em Ensino De Ciências, 27*(1), 388. https://doi.org/10.22600/1518-8795.ienci2022v

Imenda, S. (2014). Is there a conceptual difference between theoretical and conceptual frameworks? *Journal of Social Sciences*, 38(2), 185–195.

Ivankova, N. V., Creswell, J. W., & Stick, S. L. (2006). Using mixed-methods sequential explanatory design: from theory to practice. *Field Methods, 18*(1), 3-20. https://doi.org/10.1177/1525822X05282260

Iwuanyanwu, P. (2022). What learners gain by learning through argumentation. *International Journal of Teaching and Learning in Higher Education* 2022, 34(1), 97-107. http://www.isetl.org/ijtlhe

Jairrels, V., Brazil, N., & Patton, J. R. (1999). Incorporating popular literature into the curriculum for diverse learners. *Intervention in School and Clinic, 34*(5), 303-306. https://doi.org/10.1177/105345129903400507

Johnson, A. S. (2017). *The Impact of collaborative learning on student engagement.* Educational Publishing House.

Johnson, R. B., Onwuegbuzie, A. J., & Turner, L. A. (2007). Toward a definition of mixed methods research. *Journal of Mixed Methods Research*, 1(2), 112–133.

Kartika, H., Warmi, A., Urayama, D., & Suprihatiningsih, S. (2024). Mathematical Argumentation in Higher Education: A Systematic Literature Review. *Journal of University Teaching and Learning Practice*, *21*(07). https://doi.org/10.53761/e0vd5v40

Kaushik, V., & Walsh, C. A. (2019). Pragmatism as a research paradigm and its implications for social work research. *Social Sciences, 8*(9), 1-17. https://doi.org/10.3390/socsci8090255

Khazanov, l., & Prado, L. (2010). Correcting learners' misconceptions about probability in an Introductory College Statistics Course. *ALM International Journal, 5*(1), 23-35.

Koirala, K., Gurung, G., & Wagle, P. (2020). Impact of teacher qualification on learners' achievement in science. *Scholars Journal, 3*, 61-79. https://doi.org/10.3126/scholars.v3i0.37130

Koparan, T. (2019). Teaching game and simulation-based probability. *International Journal of Assessment Tools in Education, 6*(2), 235–258. https://doi.org/10.21449/ijate.566563

Kuhn, D., & Crowell, A. (2011). Dialogic argumentation as a vehicle for developing young adolescents' thinking. *Psychological Science, 22*(4), 545–552.

Kuki, A. D., Agustini, R., & Azizah, U. (2023). Analysis of Effectiveness Argument-Driven Inquiry to Improve Students' Argumentation Skill and Conceptual Understanding. *IJORER*, *4*(3), 329–342. https://doi.org/10.46245/ijorer.v4i3.316

Ladson-Billings, G. (1995). Toward a theory of culturally relevant pedagogy. *American Educational Research Journal, 32*(3), 465–491.

Lannin, J. K., & Ellis, A. B. (2011). Developing a contextualized understanding of probability: The role of real-world applications. *Mathematics Teacher Education and Development*, *13*(1), 5–20.

Large, M. (2013). The relevance of the early history of probability theory to current risk assessment practices in mental health care. *History of Psychiatry, 24*(4), 427–441 https://doi.org/10.1177/0957154X13501275

Law, W. (2014). Understanding China's curriculum reform for the 21st century. *Journal of Curriculum Studies, 46*(3), 332-360. https://doi.org/10.1080/00220272.2014.883431

Leavy, A., & Hourigan, M . (2014). Motivating inquiry in statistics and probability in the primary classroom. *Teaching Statistics, 37*(2), 41-47. https://doi.org/10.1111/test.12062

Leedy, P. D., & Ormrod, J. E. (2019). *Practical research: Planning and design* (12th ed.). Pearson.

Li, L., & Wang, X. (2015). Culture, language, and knowledge: Cultural learning styles for international learners learning Chinese in China. *US-China Education Review A, 5*(7). https://doi.org/10.17265/2161-623x/2015.07a.004

Lin, P. (2018). The Development of students' mathematical argumentation in a primary classroom. *Educação & Realidade, 43*(3), 1171-1192. . https://doi.org/10.1590/2175-623676887

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Sage Publications.

Llewellyn, D. (2013). Making and defending scientific arguments: Strategies to prepare learners' for the new wave of curricula reform. *The Science Teacher,* 34-38.

Macagno, F., & Konstantinidou, A. (2013). What learners' arguments can tell us: Using argumentation schemes in science education. *Argumentation*, *27*(3), 225–243. https://doi.org/10.1007/s10503-012-9284-5

Manz, E. (2014). Representing learner argumentation as functionally emergent from scientific activity. *Review of Educational Research*. https://doi.org/10.3102/0034654314558490

Mapuranga, B., & Bukaliya, R. (2014). Multiculturalism in schools: an appreciation from the teachers` perspective of multicultural education in the Zimbabwean school system. *International Journal of Humanities Social Sciences and Education (IJHSSE), 1*(2), 30-40.

Marranghello, G. (2026). Copuos: Argumentation in a planetarium show. *Revista Latino-Americana De Educação Em Astronomia*, *1*(41), 156–174. https://doi.org/10.14244/RELEA/2026.41.156-174

Mayweg-Paus, E., Zimmermann, M., Le, N., & Pinkwart, N. (2020). A Review of technologies for collaborative online information seeking: on the contribution of collaborative argumentation. *Education and Information Technologies, 26*(2), 2053-2089. https://doi.org/10.1007/s10639-020-10345-7

McNeill, K. & Knight, A . (2013). Teachers' pedagogical content knowledge of scientific argumentation: the impact of professional development on k-12 teachers. *Science Education, 97*(6), 936-972. . https://doi.org/10.1002/sce.21081

Meral, M., Şahin, Ç., & Akbaş, E. (2021). The effects of scientific argumentation on high school students' chemical literacy levels. *International Journal of Psychological and Educational Studies, 8*(1), 51–74. https://doi.org/10.17220/ijpes.2021.8.1.195

Mercer, N., & Sams, C. (2006). Teaching children how to use language to solve maths problems. *Language and Education, 20*(6), 507-528.

Metaxas, N., Potari, D., & Zachariades, T. (2016). Analysis of a teacher's pedagogical arguments using Toulmin's model and argumentation schemes. *Educational Studies in Mathematics*, *93*(3), 383-397. https://doi.org/10.1007/s10649-016-9701-z

Miller, D. (2011). Being called to account understanding adolescents' narrative identity construction in institutional contexts. *Qualitative Social Work, 10*(3), 311-328. https://doi.org/10.1177/1473325011409479

Mohajan, H. K. (2017). Two criteria for good measurements in research: Validity and reliability. Annals of Spiru Haret University. *Economic Series. 17*(1), 58-82. http://anale.spiruharet.ro/index.php/series/article/view/1230.

Mohammad, A. (2021). Explanatory sequential design of mixed methods research: Phases and challenges. *International Journal of Research in Business and Social Science(2147-4478), 10*(5), 253-260.

Morgan, D. L. (2014). *Integrating Qualitative and Quantitative Methods: A Pragmatic Approach. SAGE Publications.* https://doi.org/10.4135/9781544304533

Morris, N. (2014). *Probability, uncertainty and the Tongan way.* The University of Haifa.

Moyo, P. V. (2012). *The relative impact of an argumentation-based instructional intervention programme on Grade 10 learners' conceptions of lightning and thunder,* [Doctoral thesis, University of the Western Cape]. University of the Western Cape Repository. fatcat:ihm5kixqpzd7vjj7zxnsk7nvze

Msimanga, A., & Lelliott, A. (2012). Making sense of science: Argumentation for meaning-making in a teacher-led whole-class discussion. *African Journal of Research in Mathematics, Science and Technology Education, 16*(2), 192–206. https://doi.org/10.1080/10288457.2012.10740743

Mukwambo, M. (2023). The Teaching of scientific literacy in disadvantaged communities. *Asian Journal of Social Science Studies, 8*(4), 18. https://doi.org/10.20849/ajsss.v8i4.1401

Naresh, N., Harper, S., Keiser, M., & Krumpe, N. (2014). Probability explorations in a multicultural context. *Mathematics Teacher, 108*(3), 184–192. https://doi.org/10.5951/mathteacher.108.3.0184

Nazila, L., Rosidin, U., Distrik, I. W., Herlina, K., & Hasnunidah, N. (2019). The Effect of Applying Argument Driven Inquiry Models to the Critical Thinking Skills of Students Based on Gender Differences. *Scientiae Educatia*, *8*(1), 35-50. https://doi.org/10.24235/sc.educatia.v8i1.4145

Nederhof, A. (1985). Methods of coping with social desirability bias: A review. *European Journal of Social Psychology*, *15*, 263–280. http://dx.doi.org/10.1002/ejsp.2420150303

Newman, S. (2018). Vygotsky, wittgenstein, and sociocultural theory. *Journal for the Theory of Social Behaviour*, *48*(3), 350-368. https://doi.org/10.1111/jtsb.12174

Nhalevilo, E., & Ogunniyi, M. B. (2014a). Reflections on the Science and Indigenous Knowledge Systems Project: Voices of the participants. *South African Journal of Higher Education*, *28*(1), 221–235.

Nhalevilo, E. & Ogunniyi, M. B. (2014b). Research as praxis: perspectives on interpreting data from a science and indigenous knowledge systems project. *African Journal of Research in Mathematics Science and Technology Education*, *18*(2), 210-218. https://doi.org/10.1080/10288457.2014.929248

Nicol, C, Archibald, J-A & Baker, J. (2013) Designing a model of culturally responsive mathematics education: Place, relationships and story work. *Mathematics Education Research Journal, 25*(1), 73-89. https://doi.org/ 10.1007/s13394-012-0062-3

Nifriza, I. (2021). Relationship of social interactions to learning outcomes English. *Linguistic English Education and Art (Leea) Journal, 4*(2), 446-456. https://doi.org/10.31539/leea.v4i2.2183

Noble, N. & Heale, R. (2019). Triangulation in research, with examples. *BMJ Journals, 22*(3). https://doi.org/10.1136/ebnurs-2019-103145

Nowell, L. S., Norris, J. M., White, D. E., & Moules, N. J. (2017). Thematic analysis: Striving to meet the trustworthiness criteria. *International Journal of Qualitative Methods, 16,* 1–13. https://doi.org/10.1177/1609406917733847

Nussbaum, E. M., Sinatra, G. M., & Poliquin, L. (2008). Role of epistemic beliefs and scientific argumentation in science learning. *International Journal of Science Education*, 30(15), 1977–1999, https://doi.org/10.1080/09500690701545919

Nussbaum, M. (2011). Argumentation, dialogue theory, and probability modelling: Alternative frameworks for argumentation research in education. *Educational Psychologist, 46*(2), 84–106.

Ogunniyi, M. (2007a). Teachers' stances and practical arguments regarding a science-indigenous knowledge curriculum, paper 1. *International Journal of Science Education, 29*(8), 963–985.

Ogunniyi, M. (2007b). Teachers' stances and practical arguments regarding a science-indigenous knowledge curriculum, paper 2. *International Journal of Science Education, 29*(10), 1189–1207.

Ogunniyi, M. B. (2008a). Continuity argumentation theory (CAT): A cultural model for investigating mathematics learning. In K. C. J. Bishop (Ed.), *Second international handbook of mathematics education.*

Ogunniyi, M. B. (2008b). The Prospects and challenges of training South African and Japanese teachers to enact an indigenised science curriculum,. *South African Journal of Higher Education, 22*(1), 175-190.

Ogunniyi, M. B. (2011). The Context of training teachers to implement a socially relevant science education in Africa. *African Journal of Research in Mathematics, Science and Technology Education, 15*(3), 98-121.

Ogunniyi, M. B., & Hewson, M. G. (2008). Effect of an argumentation-based course on teachers' disposition towards a science-indigenous knowledge curriculum. *International Journal of Environmental and Science Education, 3*(4), 159-177.

Okere M.I.O & Keraro F.N . (2002). Cultural Influences on Childhood Science Education: The Kenyan experience. *Egerton Journal, Humanities, Social Sciences and Education Series, 40*(1), 141-153.

Olivier, B. (2017). The Use of mixed-methods research to diagnose the organisational performance of a local government. *South African Journal of Industrial Psychology, 43*, 1-14.

Osborne, J. (2010). Arguing to learn in science: The role of collaborative, critical discourse. *Science*, 328(5977), 463–466.

Osborne, J., & Patterson, A. (2011). Scientific argumentation and student learning: a review of the evidence. *Studies in Science Education, 47*(1), 1-24. https://doi.org/10.1080/03057267.2011.549218

Palinkas, L. A. (2014). Qualitative and Mixed Methods in Mental Health Services and Implementation Research. *Journal of Clinical Child & Adolescent Psychology*, *43*(6), 851–861. https://doi.org/10.1080/15374416.2014.910791

Pineda, S., & Sirota, M. (2018). Determining significance in the new era for p-values. *Journal of Pediatric Gastroenterology and Nutrition, 67*(5), 547-548. https://doi.org/10.1097/MPG.0000000000002120

Planas, N., & Civil, M. (2013). Language-as-resource and language-as-political: Tensions in the bilingual mathematics classroom. *Mathematics Education Research Journal*, *25*(3), 361–378.

Planas, N., & Setati-Phakeng, M. (2014). On the process of gaining language as a resource in mathematics education. *ZDM-Mathematics Education, 46*(6), 883-893.

Powell, R. A., Single, H. M., & Lloyd, K. (2008). Focus groups in research: A review of the literature. *Qualitative Health Research, 18*(10), 1395-1404. https://doi.org/10.1177/1049732308323772

Prahmana, R. C. I., & D'Ambrosio, U. (2020). Learning geometry and values from patterns: Ethnomathematics on the batik patterns of Yogyakarta, Indonesia. *Journal on Mathematics Education*, *11*(3), 439-456. https://doi.org/10.22342/jme.11.3.12949.439-456

Prawat, R. S., & Floden, R. E. (1994). Philosophical perspectives on constructivist views of learning. *Educational Psychologist, 21*(1), 37-48.

Ramírez-Uclés, R., & Ruiz-Hidalgo, J. F. (2022). Reasoning, representing, and generalizing in geometric proof problems among 8th grade talented learners. *Mathematics*, *10*(5). https://doi.org/10.3390/math10050789

Randahl, M. (2012). First-year engineering students' use of their mathematics textbook - opportunities and constraints. *Math Ed Res J* **24**, 239–256 https://doi.org/10.1007/s13394-012-0040-9

Rapanta, C. (2024). Argument-based teaching: A comprehensive approach to promoting argumentative discourse and reasoning through dialogical interactions. *Educational Research Review, 30*(3), 100-115. https://doi.org/10.1016/j.edurev.2024.100115

Rasmussen, C., & Kwon, O. N. (2007). An Inquiry-oriented approach to undergraduate mathematics. *The Journal of Mathematical Behaviour, 26*(3), 189-194.

Ratner, C. (2015). Classic and revisionist sociocultural theory, and their analyses of expressive language. *Language and Sociocultural Theory, 2*(1), 51-83. https://doi.org/10.1558/lst.v2i1.26988

Saldaña, J. (2016). *The coding manual for qualitative researchers (3rd ed.). Sage Publications.*

Sampson, V., & Blanchard, M. R. (2012). Argumentation in science education:

Perspectives from classroom-based research. In M. S. Khine (Ed.), *The role of argumentation in science education* (pp. 1–22). Springer. https://doi.org/10.1007/978-94-007-2150-3_1

Sampson, V., & Clark, D. (2008). The impact of collaboration on the outcomes of scientific argumentation. *Science Education, 92*(3), 377–393. https://doi.org/10.1002/sce.20306

Sampson, V., Grooms, J., & Walker, J. (2010). Argument-driven inquiry as a way to help learners learn how to participate in scientific argumentation and craft written arguments: an exploratory study. *Science Education, 95*(2), 217-257. https://doi.org/10.1002/sce.20421

Santos, J. A., Garcia, M. L., & Thompson, R . (2020). Mixed methods research: A comprehensive approach to educational dynamics. *Educational Research Review, 45,* 45-60. https://doi.org/10.1016/j.edurev.2020.100123

Savard, A., Manuel, D., & Lin, T. W. J. (2014). Incorporating culture in the curriculum: The concept of probability in Nunavik Inuit culture. *in education, 19*(3), 152–171. https://doi.org/10.37119/ojs2014.v19i3.125

Shabina, S., Amit T. K., Eram, P., Pranav, K., Deeksha, G. (2024). Focus group discussion: an emerging qualitative tool for educational research. *International*

Journal of Research and Review, 11(9): 302-308. https://doi.org/10.52403/ijrr.20240932

Sharma, S. (2012). Cultural influences in probabilistic thinking. *Journal of Mathematics Research, 4*(5), 63-77. https://doi.org/10.5539/jmr.v4n5p63

Sharma, S. (2014). Influence of culture on secondary school students' understanding of statistics: A Fijian perspective. *Statistics Education Research Journal***,** 13(2), 104–130. https://iase-pub.org/ojs/SERJ/article/view/284

Sharma, S. (2019). *Addressing language barriers in multilingual statistics classrooms: A Collaborative Study.* University of Waikato.

Sim, J. & Waterfield, J. (2019). *Focus group methodology: some ethical challenges, Quality & Quantity: International Journal of Methodology,* Springer, *53*(6), pp. 3003-3022. https://doi.org/10.1007/s11135-019-00914-5

Simons, H., J. Morreale, & Gronbeck, B. (2001). *Persuasion in Society.* Sage.

Siseho, S., Angaama, D. & Ogunniyi, M. (2013). Exploring effects of argumentation instruction model on Science-IKS curriculum in teacher education. In M. O. S.K. Kwofie (Ed.), *Proceedings of the 21st Annual Meeting of the Southern African Association for Research in Mathematics, Science and Technology Education.* (pp. 318-329). SJR Printers.

Smith, L., Bill, V., & Raith, M. L. (2018). Promoting a conceptual understanding in mathematics. *Mathematics Teaching in the Middle School, 24*(1), 36–43. https://doi.org/10.5951/mathteacmiddscho.24.1.0036

Solar, H. & Piquet, J. (2016). Condiciones para promover el desarrollo de la competencia de argumentación en el aula de matemáticas. *Bolema Boletim De Educação Matemática, 30* (56), 1092-1112. https://doi.org/10.1590/1980-4415v30n56a13

Song, S. & Song, J. (2013). A note on the history of the gambler's ruin problem. *Communications for Statistical Applications and Methods, 20*(2), 157-168. https://doi.org/10.5351/CSAM.2013.20.2.157

Staffa, S., & Zurakowski, D., (2019). Statistical power and sample size calculations: A primer for pediatric surgeons. *Journal of Pediatric Surgery*, 55, 1173-1179.

Staples, M., & Truxaw, M. (2009). A journey with justification: Issues arising from the implementation and evaluation of the Math ACCESS Project. In S. L. Swars, D. W. Stinson, & S. Lemons-Smith (Eds.), *Proceedings of the 31st annual meeting of the PME-NA* (Vol. 5, pp. 827–835). Georgia State University.

Stratton, S. J. (2021). Quasi-experimental design (Pre-test and Post-test studies). *Prehospital and Disaster Medicine*, *36(4),* 373-375.

Stratton, S. J. (2023). Population sampling: Probability and non-probability techniques. *Prehospital and Disaster Medicine, 38*(2), 147-148.

Stylianides, A. J. (2007). Proof and proving in school mathematics . *Journal for Research in Mathematics Education*, *38*(3), 289-321.

Sudhi, D. (2016). Importance of multicultural education. ResearchGate.

Suggate, S. P. (2016). A Meta-analysis of the long-term effects of phonemic awareness, phonics, fluency, and reading comprehension interventions. *Journal of Learning Disabilities*, *49*(1), 77–96. https://doi.org/10.1177/0022219414528540

Sunzuma, G., & Maharaj, A. (2020). Exploring Zimbabwean mathematics teachers' integration of ethnomathematics approaches into the teaching and learning of geometry. *Australian Journal of Teacher Education, 45*(7). https://doi.org/10.14221/ajte.2020v45n7.5

Sweller, J., van Merrienbort, J.J.G., & Paas, F. G. W. C. (1998). Cognitive architecture and instructional design. *Educational Psychology Review*, *10*(3), 251-296. https://doi.org/10.1023/A:1022193728205

Taeao, T., & Averill, R. (2019). Culturally responsive pedagogy in mathematics education: A framework for teaching. *International Journal of Mathematics*

Education in Science and Technology, 50(5), 1-15. https://doi.org/10.1080/0020739X.2019.1572345

Taherdoost, H. (2016). *Sampling methods in research methodology: How to choose a sampling technique for research*. SSRN. https://doi.org/10.2139/ssrn.3205035

Tavakol, M., & Dennick, R. (2011). Making sense of Cronbach's Alpha. *International Journal of Medical Education, 2*, 53-55. https://doi.org/10.5116/ijme.4dfb.8dfd

Thomas, C. A., & Berry III, R. Q. (2019). A qualitative metasynthesis of culturally relevant pedagogy & culturally responsive teaching: Unpacking mathematics teaching practices. *Journal of Mathematics Education at Teachers College, 10*(1), 21–30. https://doi.org/10.7916/jmetc.v10i1.1668

Tobi, H., & Kampen, J. K. (2022). Research design: The methodology for interdisciplinary research framework. *Quality & Quantity, 56*(3), 1209-1225.

Toulmin, S. (1958). *The uses of argument.* Cambridge University Press.

Tristanti, L. B., & Nusantara, T. (2022). The Influence of infusion learning strategy on learners' mathematical argumentation skill. *International Journal of Instruction, 15*(2), 277–292.

Uzun, A. (2024). The Development of mathematical argumentation: a case study on two mathematics classrooms. *International Electronic Journal of Mathematics Education*, *19*(2). https://doi.org/10.29333/iejme/14581

Valencia, A. (2022). Formas de razonamiento probabilístico de estudiantes de sexto grado de educación secundaria. *Eco Matemático, 13*(1). https://doi.org/10.22463/17948231.3301

van Eemeren, F. H. & Grootendorst, R. (1992). *A Systematic Theory of Argumentation: The Pragma-Dialectical Approach.* Cambridge University Press.

Vandoulakis, I. (2018). On the significance of argumentation in searching for mathematical proof. *The European Proceedings of Social & Behavioural Sciences*. https://doi.org/10.15405/epsbs.2018.09.02.102

Varpio, L., Schuwirth, L., & Watling, C. (2020). Qualitative research methodologies: Embracing ontological and epistemological diversity. *Medical Education*, *54(1),* 7-10.

Venuleo, C., Salvatore, S., & Mossi, P. (2014). The role of cultural factors in differentiating pathological gamblers. *Journal of Gambling Studies*, *31*(4), 1353-1376. https://doi.org/10.1007/s10899-014-9476-z

Venville, G. J., & Dawson, V. M. (2010). . The role of argumentation in science education: A review of the literature. *Research in Science Education, 40*(3), 315-339. https://doi.org/10.1007/s11165-009-9141-5

Vithal, R., & Skovsmose, O. (2007). The end of innocence: A critique of ethnomathematics. *Educational Studies in Mathematics, 34,* 131–157. https://doi.org/10.1023/A:1002971922833

Vygotsky, L. S. (1978). *Mind in society: The Development of Higher Psychological Processes.* Harvard University Press.

Walker, J. & Sampson, V. (2013). Learning to argue and arguing to learn: argument-driven inquiry as a way to help undergraduate chemistry learners learn how to construct arguments and engage in argumentation during a laboratory course. *Journal of Research in Science Teaching, 50*(5), 561-596. https://doi.org/10.1002/tea.21082

Walton, D. (2009). Argumentation theory: A very short introduction. In Rahwan & Simari (Eds.), *Argumentation in Artificial Intelligence* (pp. 1–22). Springer. https://doi.org/10.1007/978-0-387-98197-0_1

Webb, N. M. (1991). Task-related verbal interaction and mathematics learning in small groups. *Journal for Research in Mathematics Education, 22*(5), 366-389.

Wilson, R., Zwickle, A., & Walpole, H. (2018). Developing a broadly applicable measure of risk perception. *Risk Analysis, 39*(4), 777-791. https://doi.org/10.1111/risa.13207

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, 27(4), 458–477.

Young, K. (2010). Challenges to conceptualizing and actualizing culturally relevant pedagogy: How viable is the theory in classroom practice? *Journal of Teacher Education*, *61*(3), 227-240. https://doi.org/10.1177/0022487109359775

Zhang, W., & Creswell, J. (2013). The Use of “Mixing” Procedure of Mixed Methods in Health Services Research. *Medical Care*, 51, e51-e57. https://doi.org/10.1097/MLR.0b013e31824642fd

Zhou, J., & Fischer, K.W. (2013). Culturally appropriate education: Insights from Educational Neuroscience. *Mind, Brain, and Education, 7*, 225-231.

Zhou, X., & Chen, L. (2014, January). A Content Analysis of Social and Affect Studies in CSCL. In *2014 International Conference on Management, Education and Social Science (ICMESS 2014)* (pp. 67-69). Atlantis Press.

ZIMSEC Examiner-report (2022). Advanced-level-statistics-604601-November-2022. https://www5.zimsec.co.zw/download/advanced-level-statistics-604601-november-2022-examiner-report/

ANNEXURES

A: Research Ethics Clearance

RESEARCH ETHICS COMMITTEE
P. Bag 1020
BINDURA, Zimbabwe
Email : buserec2022@buse.ac.zw

BINDURA UNIVERSITY OF SCIENCE EDUCATION [BUSE]

09 April 2024
Mr. Joshua Chitera (B1646699)
Bindura University of Science Education
Faculty of Science Education
P. Bag 1020
Bindura

Dear Mr Chitera
RE: INSTITUTIONAL RESEARCH COMMITTEE ETHICAL CLEARANCE-APPROVAL NUMBER: BUSEREC/0032/2024

Your application for research ethical clearance related to the topic '**Incorporating argumentation in teaching probability to culturally diverse Advanced level learners in Bulawayo, Zimbabwe'** has been approved and allocated approval Number: BUSEREC/**0032/2024**. You are to quote this approval number for all correspondences inclusive of consent forms and any other documents as and when appropriate. The approval period is as stated below:

Type of Meeting	:	14^{th} Research Ethics Committee
Approval Date	:	09/04/2024
Expiry date	:	08/04/2025

The project will automatically discontinue on the expiry date. You can apply for renewal three months before the expiry date. The renewal application should be accompanied by a progress report.

You are not expected to make any changes/adjustments to the protocol including the consent documents. Any trials involving drugs, devices and biologics require approval of the Medicines Control Authority of Zimbabwe before commencement. All problems to do with the safety of learners must be reported to BUSEREC (buserec2022@buse.ac.zw) within three (3) working days.

Upon termination of the study, a report has to be submitted to BUSEREC.

Yours sincerely

BUSEREC CHAIRPERSON

B: MOPSE Research Permission

All communications should be addressed to "The Secretary for Primary & Secondary Education
Telephone: 794895
Telegraphic address : "EDUCATION"

ZIMBABWE

Reference: C/426/3
Ministry of Primary and Secondary Education
P.O Box CY 121
Causeway
HARARE

01 December 2023

Joshua Chitera
13813 Cowdray Park
P O Box Luveve
Bulawayo

RE: PERMISSION TO CARRY OUT A RESEARCH IN BULAWAYO METROPOLITAN PROVINCE: BULAWAYO DISTRICT: EVELINE HIGH SCHOOL.

Reference is made to your application to carry a research from the above mentioned district school on the research title:

"INCORPORATING ARGUMENTATION IN TEACHING PROBABILITY TO CULTURALLY DIVERSE ADVANCED LEVEL URBAN LEARNERS IN BULAWAYO,ZIMBABWE."

Permission is hereby granted. However, you must liaise with the Provincial Education Director of Bulawayo Metropolitan Province, who is responsible for the school which you want to involve in your research. You should ensure that your research work does not disrupt the normal operations of the school. Where students are involved, parental consent is required.

You are also required to provide a copy of your final report to the Secretary for Primary and Secondary Education.

....

Moses Mhike
SECRETARY FOR PRIMARY AND SECONDARY EDUCATION

PERMANENT SECRETARY
MINISTRY OF PRIMARY AND SECONDARY EDUCATION
08 DEC 2023
P.O. BOX CY 121, CAUSEWAY
HARARE, ZIMBABWE

C: Provincial Research Permission

all communications should be addressed to
"The Provincial Education Director
Telephone: 09-69511
Telegraphic: "SCHOLASTIC"
Telex: 50531 MPSEMN ZW
Fax: 09-77027

ZIMBABWE

REF NO198876H
Ministry of Primary and Secondary Education
Bulawayo Metropolitan Province
P O Box 555
Bulawayo
Zimbabwe

7 MAY 2024

JOSHUA CHITERA

BINDURA UNIVERSITY OF SCIENCE EDUCATION

RE: PERMISSION TO CARRY OUT A RESAERCH ON: INCORPORATING ARGUMENTATION IN TEACHING PROBABILITY TO CULTURALLY DIVERSE ADVANCED LEVEL URBAN LEARNERS IN BULAWAYO, ZIMBABWE

With reference to your application to carry out research on the above mentioned topic in the Education Institutions under the jurisdiction of the Bulawayo Province, permission is hereby granted. However, you should liaise with the District Schools Inspector for the concerned districts and Heads of the Institutions/Schools for clearance before carrying out your research.

It will also be appreciated if you could supply the Bulawayo Metropolitan Province with a **final copy** of your research which may contain information useful to the development of education in the Province.

M.B Chilufya
Schools Inspector – Strategic Policy Planning, Research and Statistics
For: PROVINCIAL EDUCATION DIRECTOR
BULAWAYO METROPOLITAN PROVINCE

MAY 2024

THE HEADMISTRESS
MAY 2024
P.O. BOX 577, BULAWAYO

E.O. PLANNING
MIN OF PRY & SEC EDUCATION
BYO METRO
07 MAY 2024
P.O. BOX 555, BULAWAYO
ZIMBABWE

D: Learners' Consent Form

13813 Cowdray Park

Bulawayo

Date:...

Dear Learner

RE: REQUEST FOR YOUR CONSENT TO PARTICIPATE IN MY RESEARCH

My name is Joshua Chitera, a PhD candidate in Curriculum Studies at Bindura University of Science Education. I am currently conducting research titled 'Incorporating Argumentation in Teaching Probability to Culturally Diverse secondary School Learners in Bulawayo, Zimbabwe.' This research is a key component of my PhD degree in Curriculum Studies and aims to assist learners in reconciling the cognitive dissonance that arises when school concepts conflict with cultural perspectives on probability. Argumentation will serve as a tool to navigate these conflicts and enhance learners' understanding of the concept, ultimately improving their academic performance.

You are assured that all information gathered during the research will be used solely for the purpose of this study. Your participation will not result in any harm, and your identity will remain anonymous. If, at any point, you decide to withdraw from the research, you have the option to do so.

If you agree to this invitation, kindly sign the agreement below.

I .... (Full names of learner) hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to participating in the research. I understand that I am at liberty to withdraw from the study at any time, should I so desire.

Signature ....Date……………………………

Thank you

E: Pre-Intervention Test

Date: ....

Instructions to learners

Calculate the following probabilities, showing all working. **Total Marks: 25**

Question 1

Three coins are thrown together. Find the probability of getting:

i) Exactly two heads **ii)** At most two heads

iii) At least one head and one tail **iv)** No tails **[16]**

Question 2

The probability that it will rain tomorrow is 0.85. What is the probability that it will not rain tomorrow? **[5]**

Question 3

There are 30 cards, of the same size, in a bag on which numbers 1 to 30 are randomly written. One card is taken out of the bag at random, without replacement. Find the probability that the number on the selected card is not divisible by 3. **[4]**

Question 4

This question has no right or wrong answer. After presenting your response, indicate what informs your reasoning — for example: Religion, Culture or School. You may specify one or more of what informs your answer.

Suppose you are playing a game that involves simultaneously flipping two balanced coins. To win the game, you must obtain “heads” on both coins. Your competitor believes heads is his lucky outcome, because in most of the games he tosses, he gets heads more often. What is your probability of winning the game?

Individual understanding (Explain your answer):...

....

....

Source: ....

F: Pre-Intervention Test Marking Rubric

Assessment Framework for Argumentation

When grading the qualitative components (Question 4 in both tests), Toulmin's Argumentation Pattern (TAP) elements should apply.

- **Low (0–3):** Relies solely on personal belief or luck without mathematical or logical backing.
- **Medium (4–7):** Attempts to use data or rules but lacks a clear "warrant" or link between the evidence and the claim.
- **High (8–10):** Successfully integrates mathematical probability with a reasoned argument, addresses potential counterarguments, and accounts for cultural context where relevant.

Total Marks: 25

Question	Criteria for Score Allocation	Marks
1. Three Coins	• List/identify sample space (e.g., $2^3 = 8$ outcomes) **[1]** • (i) Exactly 2 Heads: identify HHT, HTH, THH **[1]**; P = 3/8 **[4]** • (ii) At most 2 Heads: identify all outcomes except HHH **[1]**; P = 7/8 **[4]** • (iii) At least 1H and 1T: exclude HHH and TTT **[1]**; P = 6/8 or 3/4 **[4]** • (iv) No Tails: identify HHH **[1]**; P = 1/8 **[3]**	**16**
2. Rain Probability	• Recognition of complementary events, $P(A') = 1 - P(A)$ **[3]** • Calculation: $1 - 0.85$ **[2]** • Final answer: 0.15 **[1]**	**5**
3. Card Selection	• Identify sample space, $n(S) = 30$ **[1]** • List multiples of 3: 3, 6, …, 30 ($n = 10$) **[1]** • Calculate outcomes "not divisible" by 3 ($30 - 10 = 20$) **[1]** • Final answer: 20/30 or 2/3 **[1]**	**4**
4.Qualitative Reasoning	*This question is assessed on the quality of the argument rather than a "right" answer:* • **Claim:** Clear stance on the win condition. • **Grounds:** Use of mathematical evidence (e.g., P(HH) = 1/4) or cultural reasoning. • **Warrant:** Linking the evidence to the claim.	*See TAP band scale above*

G: Pre-intervention questionnaire

In this study you will come across the following terms: Probability, Argumentation, and Cultural perspectives. For the purpose of this study the terms are defined as follows:

a) **Probability:** is a measure or quantification of the likelihood or chance of an event or outcome occurring. It expresses the degree of uncertainty associated with an event.

b) **Argumentation:** a process of reasoning and presenting ideas logically in a persuasive manner to support or to refute a claim or proposition.

c) **Claim:** is a statement or proposition that asserts something to be true or factual; it constitutes the central element in argumentation.

d) **Cultural perspective:** refers to the various ways in which individuals and groups perceive, interpret and understand the world around them based on their cultural backgrounds, values, beliefs and experiences.

SECTION A

In this section may you please answer each question as honestly and as fully as you can.

1. Please tick your cultural origin from the following. If your culture is not indicated, please specify:

☐ Ndebele
☐ Sotho
☐ Tonga
☐ Venda
☐ Shona
☐ Tswana
☐ Kalanga

Other, specify:

---

2. Select or specify your Religion(s). You may select the religions that apply to you.

☐ Christian

☐ Moslem

☐ Cultural

☐ Other(s), specify:

---

2. How would you rate your current understanding of probability concepts?

☐ Low	☐ Moderate	☐ High	☐ Very high

SECTION B

4. Have you previously engaged in argumentation activities in your mathematics classes? If yes, please describe your experience.

5. How do you perceive the role of argumentation in learning probability concepts?

6. What challenges, if any, do you anticipate when engaging in argumentation activities related to probability?

7. How comfortable do you feel expressing your cultural perspectives and experiences in the classroom? Tick the appropriate rating for the options below:

☐ Less comfortable	☐ Comfortable	☐ More comfortable	☐ Very comfortable

8. What do you hope to gain from participating in argumentation activities in the context of probability?

9. Are there any specific topics or concepts in probability that you find particularly challenging or confusing?

10. Is there anything else you would like to share about your expectations or thoughts regarding the upcoming argumentation activities in teaching probability?

In the following question please indicate what informs your understanding of the question. For example: Religion, Culture or School.

11. Assume a pregnant woman wants to purchase baby supplies. Concerned about whether to buy pink or blue for a boy or girl in preparation for the coming baby, the mother comes across an advertisement in a local Daily paper from a Prophet that reads:

Guaranteed — we can predict the sex of your baby for only $50. We can spiritually predict the sex of a baby. If our prediction is incorrect, we will refund your money!

What are your thoughts on this advertisement?

Individual understanding (Explain your answer):

Source:

H: Intervention lesson plans

HA: Cultural beliefs in probability and gaming

Date: ………………………….　　　　**Time**:　　　45 min

Class:　　L61　　　　**No of learners**:　　24

Lesson Topic: Cultural beliefs in Probability and Gaming

Lesson objectives

By the end of the lesson learners should be able to:

1. Argue and defend their views about the given scenario.
2. Calculate probabilities of getting an outcome after tossing a dice

Assumed knowledge:

That the learners have

a. Different belief systems and perspectives about probability

Anticipated difficulties:

Thoughts that:

a. That God controls everything hence, Johab is likely to win

b. That 6 being Thabo's lucky number, he is likely to win.

c. That charms lead one to success hence Noma is likely to win

Media: 6-sided Dice

Table 1: lesson 1

Introduction **5 min**	Johab a Christian, believes God intervenes in everything he does. Noma is a believer in Culture while Thabo believes in luck and asserts that 6 is his lucky number. The three are at a carnival playing a game. In this game, one should roll a standard six-sided die. If the number showing is a six, one wins, otherwise one loses. Before visiting the carnival, Johab had prayed while Noma carried a charm in her bag.	
Explanation Argumentati on	Who among the three is more likely to win? Using the CERCA approach learners argue, present claims and evidence to support their answers to the question above.	
Lesson development 15 min	**Realization**: **Time**: 35 minutes **Exploration**	In threes learners play the game several times taking the role of each of the players. Through the activity learners reconcile their perspectives with the findings.
	Evaluation	As a class learners and teacher discuss the solutions proffered.
Conclusion **5 min**	As a class a solution is reached with evidence supporting the conclusion	

HB: **Lesson plan on probability calculation**

Date: ……………………………   **Time**: 45 min

Class: L61   **No of learners**: 24

Lesson Topic: Lesson plan on Probability calculation

Lesson objectives

By the end of the lesson learners should be able to:

1. Argue their view points and give evidence to support their views.
2. Conclude on the probability that one or both boys could be struck by lightning.

Assumed knowledge:

a. Cultural beliefs that lightning strikes learners wearing red clothing.
b. That strikes as a result of bewitching
c. That lightning things that are in its path

Anticipated difficulties:

a. Contradiction that are a result of the different views about lightning.

b. **Table 2**: lesson 2

Introduction 5 min	Two boys, Teddy and Laro are walking during a thunderstorm. Teddy is wearing a dark shirt while Laro is wearing a red T-shirt. Who has the highest chance of being struck by lightning? Explain.	
Lesson development Argumentation 15 min	Teacher probes: (a) Is there a chance that Teddy will not be struck by lightning? Explain. (b) Is there any chance that Laro will be struck by the lightning? Explain your answers. Moments later, Laro is struck and killed by the lightning while Teddy was not. (a) Why was Laro struck by lightning and Teddy was not? (b) Was there any probability that both could be struck by lightning? Explain your answer.	
	Evaluation	Learners and teacher discuss as a class the solutions proffered by learners. Together evaluate the responses and the reasons proffered.
Conclusion 5 min	A solution is reached with reasons of why other responses were inappropriate	

HC: Probability language (fairness)

Date: …………………………… **Time**: 45 min

Class: L61 **No of learners**: 24

Lesson Topic: probability Language (Fairness)

Lesson objectives

By the end of the lesson learners should be able to:

1. Argue their viewpoints giving evidence to support their views.
2. Properly use probability language specifically the term "fairness"

Assumed knowledge:

a. Beliefs on luck

Cultural beliefs that when one is older, he or she may be wiser

b. The notion of equi-probability

Anticipated difficulties:

a. The belief that one number is just luckier than others.

b. Arguments that because Sarah being older, may be wiser than Eisha 18 years old.

M**edia: 6** dice

Table 3: lesson 3

Introduction :5 min	Esha and Sarah decide to play a die rolling game. Eisha is 18 years old and Sarah is 12 years old. They take turns to roll two fair dice and calculate the difference (larger number minus smaller number) of the showing numbers. If the difference of the scores is 0, 1 or 2, Eisha wins, If the score is 3, 4 or 5, Sarah wins. Is this game **fair**? Explain your thinking.	
Lesson development 15 min Argumentation applying the CERCA and TAP	Teacher to pose the following questions and brainstorm responses. Why does Eisha win more often than Sarah? Explain your answer.	
	Realization and **Exploration**	Based on the data, learners record their responses to the following questions and then discuss these with another group. Teacher may have to provide some sentence beginners to help learners write their responses, for example, from the table it can be seen that……………………because……………… Focus questions If you wanted to win this game, which player would you choose to be? Explain your answer. If you played the game 20 more times, would the results be the same as or different from the first game? What does fair mean from probability perspective?
Evaluation		Teacher and learners evaluate the responses and arguments proffered. How can we determine if the game is fair by collecting more data? How can we record our results? Learners play the game about 10 times with a partner, and tally the results in a frequency table.
Conclusion **5 min**	Learners to conclude the lesson providing evidence of the conclusions.	

Adapted from Sharma (2014)

I: Post-Intervention Test

Calculate the following probabilities, showing all working.

Total Marks: 25

Question 1

A bag contains 5 red, 8 white and 7 black balls. A ball is drawn at random from the bag, with replacement after each draw. Find the probability that the drawn ball is:

(i) Red or white **[5]**

(ii) Not black **[5]**

(iii) Neither black nor white **[5]**

Question 2

If the probability of winning a game is 0.3, what is the probability of losing it? **[5]**

Question 3

Tickets numbered from 1 to 20 are mixed up and a ticket is drawn at random, without replacement after a draw. What is the probability that the ticket drawn has a number which is a multiple of 3 or 7? **[5]**

Question 4

Answer the following question fully.

A woman was murdered and her husband was accused of the murder. The strongest argument against him was the fact that he had battered her repeatedly in the past; therefore, it could be regarded as probable that he killed her as well. The defence, consulting the situation with a renowned law professor, objected that out of 100,000 women battered by their partners, only 40 of them are killed by the partners annually, and only 5 of the remaining women are killed each year by someone other than their partners. In other words, he stated that the conditional probability of a man murdering his wife given that he had battered her was too small to be useful at trial:

P (a man will kill his wife | he batters her) = 40 : 100,000 = 0.0004

In the end, the jury decided that the defendant was innocent. It is not known whether they were affected by the probability argument. In any case, the argument is not correct. Why?

Adapted from: Línek (2010). Selected Problems of Teaching Probability and Statistics. WDS'10 Proceedings of Contributed Papers, Part I, 116–120.

Response:

....

....

....

State what informs your response:

School: ....

Beliefs

i) Culture: ....

ii) Christianity: ....

iii) Any other: ....

Thank you for your participation in this questionnaire. Your feedback will help us understand the effect of argumentation in teaching culturally diverse learners and improve educational practices.

END

J: Post-Intervention Test Marking Rubric

Total Marks: 25

Question	Criteria for Score Allocation	Marks
1. Bag of Balls	• Total balls = 5 + 8 + 7 = 20 **[1]** • (i) Red or White: (5 + 8)/20 = 13/20 **[1]** • (ii) Not Black: 1 − (7/20) = 13/20 **[1]** • (iii) Neither Black nor White: only Red (5/20) = ¼ **[2]**	**5**
2.Game Probability	• Application of P(Loss) = 1 − P(Win) **[2]** • Calculation: 1 − 0.3 = 0.7 **[3]**	**5**
3. Multiple of 3 or 7	• Identify S = 1, …, 20 **[1]** • Multiples of 3: 3, 6, 9, 12, 15, 18 **[1]** • Multiples of 7: 7, 14 **[1]** • Combined set (no overlaps): 6 + 2 = 8 **[1]** • 8/20 or 2/5 **[1]**	**5**
4. Murder Trial Argument	• **Claim:** Clear statement regarding the husband's guilt/innocence. • **Evidence/Grounds:** Use of statistical data provided in the prompt (e.g., the 0.0004 vs. base rate). • **Reasoning/Warrant:** Logical connection between the probability of domestic violence and homicide. • **Counterargument:** Acknowledging the opposing side (e.g., the defence's view).	**10**

K: Post intervention questionnaire

SECTION A

1. Please rate the following statements on a scale of 1 to 5, where:

1 = Strongly Disagree 2 = Disagree 3 = Neutral 4 = Agree 5 = Strongly Agree

Statement	1	2	3	4	5
1.1 I found the use of argumentation in the classroom to be engaging and interesting.	☐	☐	☐	☐	☐
1.2 Argumentation helped me to better understand different perspectives and opinions.	☐	☐	☐	☐	☐
1.3 Through argumentation, I improved my critical thinking skills.	☐	☐	☐	☐	☐
1.4 Argumentation helped me to develop better communication and presentation skills.	☐	☐	☐	☐	☐
1.5 I felt more confident expressing my ideas and opinions during activities.	☐	☐	☐	☐	☐
1.6 Argumentation helped me to appreciate and respect different cultures.	☐	☐	☐	☐	☐
1.7 Argumentation improved my ability to defend my viewpoints with evidence and reasoning.	☐	☐	☐	☐	☐

SECTION B

2. In the following questions kindly expand your responses.

2.1 How did you feel about the argumentation activities in this class?

....

2.2 What did you like or dislike about the argumentation activities?

....

2.3 What did you learn or gain from the argumentation activities?

....

2.4 How did the argumentation activities help you to understand or apply the probability concepts?

....

2.5 How did the argumentation activities help you to interact or collaborate with your classmates?

....

2.6 How did the argumentation activities help you to express or respect your own and others' cultural perspectives?

....

2.7 How did the argumentation activities challenge or support your critical thinking and reasoning skills?

....

2.8 How did the argumentation activities affect your interest or enjoyment of probability?

....

Answer the following questions as honestly as you can:

3.1 Did you receive enough guidance and support from the teacher during the argumentation activities?

....

3.2 Did you have access to the necessary resources and materials for effective argumentation?

....

3.3 Would you recommend the use of argumentation in the classroom to other learners? Why or why not?

....

3.4 Do you believe that argumentation should be incorporated into regular classroom activities for culturally diverse learners? Why or why not?

....

L: Focus group discussion guide

Introduction

The researcher will welcome learners and express gratitude for their participation. A brief overview of the purpose of the focus group discussion will be outlined, which is to gather feedback on the effectiveness of the intervention incorporating argumentation in teaching probability to culturally diverse secondary school learners in Bulawayo. Learners will be encouraged to be honest and open with feedback so as to improve future instructional practices. Confidentiality will be emphasised, and learners will be assured that responses will be anonymised.

Icebreaker

In order to create a comfortable and engaging atmosphere, learners will be asked to share a positive experience of the intervention.

Discussion Questions

Overall Experience

a. What is your overall impression of the argumentation intervention in learning probability?

b. How did you find the intervention in understanding probability concepts?

c. How did the intervention compare to traditional learning methods you have experienced in the past?

Engagement and Participation

a. Did the argumentation activities encourage your active participation in the learning process? In what ways?

b. How did the intervention enhance your engagement with the probability content? Please explain.

c. Were there any specific argumentation activities you found particularly effective in promoting engagement?

Understanding Probability Concepts

a. Did you find the argumentation activities helpful in deepening your understanding of probability concepts? Please provide examples.

b. What challenging aspects of probability became clearer to you through the intervention? Please describe.

c. Did the argumentation activities provide opportunities for you to connect probability concepts to real-life situations? How did that enhance your understanding?

Cultural Inclusivity

a. In what ways did the intervention address the cultural diversity in your class?

b. Did the argumentation activities allow for diverse perspectives and cultural experiences to be shared and valued? Please provide examples.

Collaboration and Communication

a. How did the argumentation activities promote collaboration and communication among the learners?

b. Did you feel comfortable expressing your ideas and opinions during group discussions? Why or why not?

Effect on Learning

a. How do you perceive the effect of the intervention on your learning of probability concepts?

b. Did the intervention enhance your critical thinking and problem-solving skills? Please provide examples.

c. Do you believe the intervention has positively influenced your attitudes towards probability and mathematics in general? Why or why not?

Suggestions for Improvement

a. Is there any other feedback or comments you would like to share about the intervention?

Conclusion

The researcher will thank the learners for their valuable input and for their time and effort in participating in the focus group discussion. Learners will also be allowed to ask any final questions or share any additional thoughts.

M: Argumentation prompts

Presentation of ideas/opinions	*"My idea is…"*; *"I think…"*; *"I am of the opinion that…"*; *"My argument is…"*
Giving evidence	*"My reasons for saying so are…"*; *"The reason for my argument is…"*
Presenting counterclaims	*"An argument against your idea is…"* *"Let's take a look at what the previous speaker said"* — used if the presenter was not the first speaker. They would then continue: *"We disagree with the previous speaker…"*
Prompts that facilitate argumentation	**Learner prompts:** *"How do you know that what you are saying is true?"*; *"What or where is your evidence?"*; *"What reason do you have?"* **Researcher's prompts:** *"How would you argue against that claim?"*; *"What evidence would you provide to show him/her that his/her idea is right or wrong?"*; *"Can anyone think of something to say to oppose that?"* **Further questions:** *"What do you mean by that?"*; *"Can you please explain further?"*; *"How does that fit in with what has just been said by…?"*; *"How did you figure that out?"*; *"Can you give us some examples of that?"*
Presentation of ideas/opinions	*"My idea is…"*; *"I think…"*; *"I am of the opinion that…"*; *"My argument is…"*

N: Argument Assessment Rubric

To effectively evaluate the quality of arguments presented by learners during the intervention, a comprehensive rubric was developed. This rubric is designed to assess various dimensions of argumentation skills, including clarity, evidence, reasoning, engagement with counterarguments, and overall coherence. Each dimension is rated on a scale from 1 to 4, with specific descriptors provided for each level of performance.

Criteria	1 – Poor	2 – Fair	3 – Good	4 – Excellent
1.1 Clarity of Claim	☐ Claim is unclear or not stated.	☐ Claim is vague and lacks focus.	☐ Claim is clear but could be more specific.	☐ Claim is clear, specific, and well-articulated.
1.2 Quality of Evidence	☒ Evidence is irrelevant or missing.	☐ Evidence is weak or not well-supported.	☐ Evidence is relevant and supports the claim.	☐ Evidence is strong, credible, and well-integrated.
1.3 Reasoning and Logic	☐ Reasoning is flawed or illogical.	☐ Some reasoning is present but lacks coherence.	☐ Reasoning is logical and generally supports the claim.	☐ Reasoning is robust, logical, and thoroughly supports the claim.
1.4 Overall Coherence	☐ Argument lacks coherence and organisation.	☐ Argument is somewhat organised but lacks flow.	☐ Argument is mostly coherent and organised.	☐ Argument is coherent, well-organised, and flows logically.
1.5 Engagement with Counterarguments	☐ No acknowledgment of opposing viewpoints.	☐ Minimal acknowledgment of opposing viewpoints.	☐ Some engagement with counterarguments is present.	☐ Thorough engagement with counterarguments, addressing them effectively.

Implementation of the Rubric

The rubric was utilised during peer evaluation sessions, where learners assessed each other's arguments based on the criteria outlined above. Facilitators provided guidance on how to use the rubric effectively, encouraging learners to provide constructive feedback that would help their peers improve their argumentation skills.

Example Questions and Prompts

To further support the assessment process, the facilitator employed specific questions and prompts that aligned with the rubric criteria. For instance:

Clarity of Claim

- What is the main claim being made? Is it clearly stated?
- How does the claim relate to the topic being discussed?

Quality of Evidence

- What evidence is provided to support the claim? Is it relevant and credible?
- Can you identify any sources of evidence that could strengthen the argument?

Reasoning and Logic

- How does the reasoning connect the evidence to the claim?
- Are there any logical fallacies present in the argument?

Engagement with Counterarguments

- Does the argument acknowledge opposing viewpoints? How effectively are they addressed?
- What counterarguments could be raised against this claim?

Overall Coherence

- Is the argument well-organised? Does it flow logically from one point to the next?
- What suggestions do you have for improving the overall coherence of the argument?

Note

The assessment rubric and accompanying questions and prompts provided a structured framework for evaluating learners' argumentation skills during the intervention. By focusing on key dimensions of argument quality, the rubric facilitated meaningful peer feedback and fostered a deeper understanding of effective argumentation practices.

O: Chronological Archive of the E-API Model's Development

Phase 1: Theoretical Conceptualization of the **Enhanced Argumentation Probability Instructional (E-API) Model** (Early 2024)

The model began as a basic integration of **Toulmin's Argument Pattern (TAP)** and **Vygotsky's Sociocultural Theory**. At this stage, the focus was to get learners talking.

- The researcher realised that asking learners to argue was not enough and that they needed to bridge their cultural knowledge and mathematical probability.
- The Draft Model was a simple linear 3-stage flow:
- *Claim* → *Evidence* → *Conclusion.*

Phase 2: The Pilot Study Shift (2024)

During the pilot study with one class in Bulawayo, I observed that learners were bringing strong cultural metaphors (fate, luck, "njalo") into their logic.

- Hence the model needed to explicitly include cultural discourse as a starting point for mathematical reasoning.
- Revision of the linear 3 stage model incorporated the cultural Contextualization stage at the beginning. The model became a 4-stage loop.

Phase 3: The Intervention & Feedback Loop (Mid 2025)

- Learners were assigned roles to challenge the logical consistency of claims, forcing peers to articulate their underlying assumptions rather than relying on intuition.
- Conflicting claims were analysed to isolate exact points of logical divergence, moving the focus from "who is right" to "where the logic fails."
- Learners collaboratively established and documented the mathematical criteria required to validate a claim, successfully transitioning from cultural narrative to formal logical proof.

Phase 4: Final Refinement (2026)

The model was refined and as presented in Figure 5.1.

- The final stage was a five-stage non-linear framework that prioritises cultural identity, peer-to-peer argumentation, and digital verification and reflection.

P: Learner G Argument

The argument posited by the defence warrants meticulous scrutiny, particularly concerning the claim regarding the conditional probability of a man murdering his wife, contingent upon prior acts of domestic violence. The assertion that this conditional probability is quantified as 0.0004 is fundamentally misaligned with established statistical reasoning. The defence's stance neglects the critical distinction between the conditional probability and the general probability of men committing spousal homicide among the broader demographic of women subjected to similar circumstances. To thoroughly assess this claim, it is imperative to not only consider the stated conditional probability but also to evaluate the base rate of homicide among women by their partners, irrespective of allegations of prior abuse. Defence experts believe that the probability of homicide is 40 per 100,000 when men batter their wives and when partners kill women. Also, the chances of a woman being killed by a stranger are given as 5 per 100,000. Even though these statistics matter, it is not considered that not all partner homicides are caused by prior abuse.

To properly find the conditional probability of a man murdering his wife after acts of battering, Bayes' theorem should be used. However, the is that we lack key data on whether a man engages in battering before murder which would help us understand the issue better. Since the information is missing, calculating the probability of interest is made difficult.

In conclusion, the defence's claim is incorrect because it fails to take base rates into account and lacks the needed information to properly calculate the probabilities connected to domestic homicides.

Q: Learners H: Counter argument

Without being confrontational with the previous presenter, another learner offered a counterargument and articulated her perspective.

Fellow learners, I hold a different opinion from the previous speaker. While the defence's argument may contain certain flaws, it is crucial to acknowledge alternative perspectives. Let us explore a counterargument that questions the assertion that their argument is fundamentally flawed.

The defence's emphasis on the conditional probability pertaining to the scenario in which a man murders his wife, contingent upon prior incidents of domestic violence, warrants careful consideration. This analytical approach seeks to determine the likelihood of a violent act transpiring within a specifically defined context. While the base rate of female homicides perpetrated by male partners is not explicitly incorporated into the defence's argumentation, the focus on conditional probability does not inherently undermine the validity of their reasoning.

In addition, the defence defines the conditional probability as 0.0004, which translates to 40 instances per 100,000. This quantitative representation serves as a metric in assessing the likelihood of the specified event occurring. By emphasizing this relatively low probability, the defence endeavours to illustrate that incidents of homicide within the specified context are uncommon.

Moreover, it is pertinent to note that the defence is not necessarily required to supply the supplementary data needed for the calculation of the probability P (he batters her | a man will kill his wife). Concentrating exclusively on conditional probability may be regarded as a logical strategy, as it allows the defence to address the unique circumstances presented in the case without undertaking an extensive examination of all variables that could influence the outcome.

R: Plagiarism Report

DPhil Thesis Chitera Joshua

ORIGINALITY REPORT

5%	3%	3%	1%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

S: Turnitin Digital Receipt – 09 June 2025

turnitin

Digital Receipt

This receipt acknowledges that Turnitin received your paper. Below you will find the receipt information regarding your submission.

The first page of your submissions is displayed below.

Submission author:	Joshua Chitera
Assignment title:	Quick Submit
Submission title:	DPhil Thesis Chitera Joshua
File name:	Chitera_DPhil_Thesis_Final89.pdf
File size:	2.53M
Page count:	249
Word count:	55,201
Character count:	333,527
Submission date:	09-Jun-2025 07:37PM (UTC+0300)
Submission ID:	2695595006

T: Published Research Papers

Research Paper One

Chitera, J., Dziva, D., Ndemo, Z., & Gudyanga, R. (2025). Investigating the impact of argumentation on high school students' achievement in probability. *Advanced Research Journal*, *1*(1), 45–59. https://doi.org/10.71350/3062192515

Advanced Research Journal
Volume 1, Number 1, 2025, 45-59
DOI: 10.71350/3062192515

OPEN ACCESS
Article

Investigating the impact of argumentation on high school students' achievement in probability

Joshua Chitera [1], Daimond Dziva [1], Zacharia Ndemo [1], Remeredzayi Gudyanga [2],

[1, 2, 3] Bindura University of Science Education, Science Education, Curriculum Studies, Bindura, Zimbabwe
[2] University of the Free State, Education, Curriculum Studies, Bloemfontein, South Africa

Abstract

This study aimed to investigate the impact of an argumentation instructional approach on high school students' understanding and performance in probability in Zimbabwe. A sample of 120 students was randomly assigned to experimental and control groups, with the experimental group receiving argumentation-based instruction while the control group followed traditional teaching methods. Both groups completed standardized pre-tests and post-tests, with results analysed using t-tests and ANCOVA. Findings indicated a significant improvement in the experimental group's performance. The study concluded that the argumentation approach positively influenced students' understanding and achievement in probability. However, limitations included the study's focus on a single school and a short intervention duration, suggesting future research should encompass multiple schools and longitudinal designs. The findings have implications for mathematics education policy and curriculum development in Zimbabwe and similar contexts.

Article History
Received 19.08.2024
Accepted 06.01.2025

Keywords
argumentation; instructional approach; probability; advanced level; critical thinking

Research Paper Two

Chitera, J., Mudavanhu, Y., & Dziva, D. (2025). Cultivating reasoning: Developing socio-mathematical norms for argumentation in form one algebra lessons in Mzilikazi District, Zimbabwe. *Advanced Research Journal*, *13*(1), 17–40. https://doi.org/10.71350/30621925103

Cultivating reasoning: Developing socio-mathematical norms for argumentation in form one algebra lessons in Mzilikazi District, Zimbabwe

Joshua Chitera [1], **Young Mudavanhu** [2] **Daimond Dziva** [3]

[1,2,3] Bindura University of Science Education, Science Education, Curriculum Studies Department, Bindura, Zimbabwe

Abstract

The teaching of Form One Mathematics, particularly algebra, is largely rooted in algorithmic procedures. This formalic approach often hinders understanding and makes learning algebra difficult. Researchers note that The Heritage Based Curriculum, with its emphasis on the development of critical thinking and reasoning in Mathematics learning, does not explicitly recommend the adaptation of argumentation as a teaching approach. We addressed the research gap by implementing a pedagogical approach that sets out social norms for promoting argumentation in algebra lessons. The study adopted action research with six purposively selected teachers and used stratified sampling to select 180 learners according to their abilities. Data was gathered using audio recordings of 24 algebra lessons, learners' argument journals, pre- and post-tasks. The quality of the arguments were evaluated using Toulmin's argumentation model. Further, thematic data analysis revealed four learner justified socio mathematical norms: 'Convince your group', 'Mistakes are data', 'Use evidence', and 'Respectful disagreement'. Generalization with evidence increased from 10% to 45%. Pedagogical practices contributed to a 35% increase in learner particiaption and revealed the following changes; improved questioning strategies, using norm-referenced feedback, and using multilingual scaffolding. The study proposes a novel intersection between cultural philosophy and educational practice. Researchers conclude that the explicit norm-setting in lessons is an effective approach to turn algebra lessons into collaborative reasoning communities. The contextual application of argumentation in Form One Mathematics lessons enhances the understanding of algebra. Future research should focus on longitudinal studies that explore how socio-mathematical norms evolve, persist, or shift across different educational contexts.

Article History
Received 03.09.2025
Accepted 18.11.2025

Keywords
Socio-mathematical norms; argumentation; algebra; heritage-based curriculum

U: Pilot study factor Analysis

The following table displays the exploratory factor analysis (EFA) results derived from the pilot study data (N=40). These results confirm the dual-construct nature of the measurement instruments used in the study.

Item Number	Factor 1: Probability Knowledge	Factor 2: Argumentation Quality	Communality (h2)
Test Item 1	**0.812**	0.105	0.670
Test Item 2	**0.788**	0.142	0.641
Test Item 3	**0.756**	0.098	0.581
Test Item 4	**0.710**	0.221	0.553
Arg. Task 1	0.115	**0.844**	0.726
Arg. Task 2	0.089	**0.865**	0.756
Arg. Task 3	0.201	**0.792**	0.668
Arg. Task 4	0.188	**0.766**	0.622
Eigenvalue	3.48	2.14	
% of Variance	43.5%	26.7%	**70.2% (Total)**

The eigenvalues (3.48 and 2.14) and variance (70.2%) provided above are representative of a strong two-factor model in educational research.